On the Emergence of Quantum Chromodynamics as the Theory of Strong Interactions

Jean-Philippe Lansberg
IPN Orsay, CNRS/IN2P3, Paris-Sud U., France

ABSTRACT

This lecture note is a historical account on how Quantum Chromodynamics (QCD) was progressively accepted as the theory of strong interactions, governing how hadrons appear, scatter and decay. We first analyse a number of experimental data and theoretical ideas presented in the sixties and early seventies which QCD stemmed from. In particular, we analyse how the idea that hadrons are made of quarks, namely spin-1/2 fermions carrying colour and interacting via the exchange of coloured spin-1 gluons, emerged. For instance, we discuss in detail how the Bjorken scaling in Deep Inelastic electron-proton Scattering (DIS) can be accounted for by the existence of point like particles in the hadrons.

We then move on to the experimental data and theoretical ideas which confirmed QCD as the theory of strong interactions. We discuss the running of the strong coupling constant and the concepts of asymptotic freedom and confinement, necessary for the scaling to hold in an interacting theory such as QCD. We also report on the J/ψ discovery in November 1974 – also known as the November revolution – which provided a further evidence of QCD with coloured quarks and gluons being the correct theory of strong interactions. Indeed, it constituted the proof of the existence of heavy quarks, whose bound-state spectroscopy could be explained by a coulombic potential compatible with the asymptotic freedom plus a linear contribution accounting for the confinement; the property of QCD which prevents quarks and gluons to be free at large distances. Finally, we review some important topics related to the so-called R-value and the angular distribution of two-, three- and four-jet events in electron-positron annihilation. Indeed, these happened to be ultimate proofs that quarks and gluons can be "seen" as particles, with respectively a spin 1/2 and 1, interacting through a non-Abelian theory, that is QCD.

CONTENTS

I. THE THESIS: QUANTUM CHROMODYNAMICS

According to chapter 9 of the Particle Data Group [1],

> *Quantum Chromodynamics (QCD), [is] the gauge field theory that describes the strong interactions of colored quarks and gluons, [it is] the SU(3) component of the SU(3) x SU(2) x U(1) Standard Model of Particle Physics.*

In pedestrian language, this means that

- Strong interactions are mediated by 8 massless spin-1 bosons: the gluons;
- The (spin 1/2) quarks are point like and can exist in 3 colours;
- The gluons self-interact, as opposed to photons, but QCD is otherwise rather similar to QED;
- The quarks are the only other particles to interact with the gluons; the leptons and the electroweak bosons do not…

We will try to explain how it became clear that such a picture was the most accurate way to describe the strong interactions binding the nucleons, and in particular the protons, in the nuclei. Note that all the particles believed to be sensitive to strong interactions are named hadrons, from the Greek "$\alpha\delta\rho o\varsigma$", "strong".

This also means that the protons, the neutrons and other pions are not elementary particles as it has been long believed in the first half of the twentieth century. These are actually made of quarks and gluons. For instance, the proton is minimally made of 2 quarks "up" of charge +2/3 associated with a quark "down" of charge -1/3. Each quark has a baryonic number 1/3. Hence, one naturally gets a baryonic number and a charge +1 for the proton. Further static properties of the hadrons, such as the magnetic moment and the charge radius should be completely understood within QCD as well as all the aspects of their scattering with the leptons or between each others.

II. HADRONS ARE NOT ELEMENTARY PARTICLES

Nearly each new experimental discovery accumulated in the first half of the last century was in fact a further hint that hadrons could not eventually be elementary particles. Retrospectively, the situation is actually drastically different from that of leptons, for which no real anomaly was found.

As expected from spin-1/2 charged particles, the nucleons should exhibit a magnetic moment $\mu_N = \frac{e_N \hbar}{2m_N}$. Since 1933 [2] and 1940 [3], we know that this is not so. One indeed has $\mu_p = 2.79\mu_N$ for the proton and $\mu_n = -1.9\mu_N$ for the neutron. This was the first indication that neither the proton nor the neutron could be structureless/elementary particles.

Another indication that hadrons are not elementary particles was also the growing size of the zoo of particles belonging to the hadron family. The situation was actually strongly reminiscent of the set of atoms before their classification by Mendeleev using the periodic table. In the early fifties, 20 hadrons were already discovered; including the proton and neutron, 3 pions, spin-3/2 baryons, as well as "strange" particles.

Quoting G. Zweig in his "Memories of Murray and the Quark Model" [4],

> *On April 15 [1963], Physical Review Letters published a paper titled "Existence and Properties of the ϕ Meson". [...] By then over 25 "credible" meson resonances had been reported.*

A first success among the attempts to classify them was the so-called Gell-Mann – Okubo mass formulae [5, 6], fixing the sum of the masses of either baryons or mesons within a given multiplet[1]. Their properties followed the so-called eightfold way imagined by Gell-Mann in a paper which was never published [7]. The same symmetry was also independently identified by Ne'eman [8]. This also worked for some excited states [9].

The baryon decuplet formula by Gell-Mann, for spin-3/2 baryons, allowed him to successfully predict the mass of the yet undiscovered Ω^-. This prediction earned him a Nobel prize in 1969.

This idea of an $SU(3)$ symmetry of hadrons was pushed further by Gell-Mann and Zweig who introduced the concept of fractionally charged "quarks", as Gell-Mann called them[2]. The existing hadrons were then either baryons made of 3 quarks or mesons made of one quark and one anti-quark. This worked for 3 –the 3 of $SU(3)$– quarks of different flavour [10, 11].

However, in Gell-Mann's mind, it was not at all obvious that quarks were particles. He considered them mostly as mathematical entities entering his classification in terms of mass and spin. A picture similar to that of atoms made of a nucleus and electrons was not what Gell-Mann expected:

> *Such particles [quarks] presumably are not real but we may use them in our field theory anyway. [12]*

Even further, the last paragraph of his paper introducing the quarks reads [10]:

> *It is fun to speculate about the way quarks would behave if they were physical particles of finite mass [...] Ordinary matter near the earth's surface would be contaminated by stable quarks as a result of high energy cosmic ray events*

---

[1] A multiplet is a set of hadrons with the same spin. For the baryons, one has an octet for spin 1/2 and decuplet for spin 3/2. For the mesons, one has two nonets, one for spin-0 (pseudoscalar) and one for spin-1 (vector) mesons.

[2] Zweig christened them instead "aces". For non-scientific reasons, most probably Gell-Mann's notoriety, the former name was retained, despite the fact that Zweig's viewpoint on the quarks, being most probably real particles, was indeed the most correct one.

throughout the earth's history, but the contamination is estimated to be so small that it would never have been detected. A search for stable quarks of charge $-1/3$ or $+2/3$ and/or stable di-quarks of charge $-2/3$ or $+1/3$ or $+4/3$ at the highest energy accelerators would help to reassure us of the non-existence of real quarks.

Nonetheless, he does not completely rule out their possible existence, which should however be tangible. Here is what he says in 1967:

Now what is going on? What are these quarks? It is possible that real quarks exist, but if so they have a high threshold for copious production, many GeV [13].

This was indeed the situation in 1967: quarks had never been produced in isolation; if there existed a production threshold for associated production (quark + anti-quark), it had not been identified in any experimental data and should be above a couple of GeV.

One of the possible solutions could be that they are indeed very massive, with the mass of a single quark in the GeV range, above the nucleon masses, despite the fact that they are made of 3 of them. However, this means that they should be strongly bound to explain the very low mass of the pion, about 140 MeV. Such a strong binding would then conflict with the existing hadron-hadron scattering.

III. HADRONS ARE MADE OF QUARKS, BUT WHAT ARE THESE QUARKS ?

Despite the elegance of the "quark" model of Gell-Mann and Zweig based on the symmetry group $SU(3)$, further issues blurred the picture.

The first is the existence of the Δ^{++} resonance. It is the lowest-lying spin-3/2 doubly charged baryon. We expect it to be a ground state, hence with a symmetric radial wave function for its constituents. Since only 3 u quarks can form a spin-3/2 doubly charged baryon in the Gell-Mann–Zweig quark model, we know its flavour wave function. We can as well deduce the spin of the quarks, all aligned to form a spin-3/2 particle. The issue is that both the spatial, the spin and the flavour wave functions are symmetric, in conflict with the Pauli exclusion principle applicable to the spin-1/2 quarks. Clearly, something went wrong there.

Another issue was the non-observation of fractionally charged objects. By virtue of the electric-charge conservation, the lightest quark(s) should be stable since it (they) cannot decay into the lighter leptons which all have integer charges. We should also expect bound states involving fractionally charged quarks. Searches for quarks in atomic physics were indeed carried out. The way it was done could appear as a bit odd nowadays, but it was taken quite seriously at that time, as seriously as searches in cosmic rays. Indeed, Gell-Mann wrote about a friend of his who was doing atomic spectroscopy:

And since most things with curious chemical in the ocean eventually are eaten by oysters, he is grinding up oysters and looking for quarks in them. He has not found any, nor any have been found at very high energies in cosmic rays. So we must face the likelihood that quarks are not real. [14]

Not only do we find it surprising nowadays that researchers could be looking for quarks in oysters in the late sixties, we consider it astounding that somebody like Gell-Mann could put results for searches for quarks in oysters and in cosmic rays on the same footing. This quote is most probably archetypal of the situation in particle physics in those years.

IV. ARGUMENTS FOR COLOUR AND ITS INTRODUCTION

Soon after the introduction of the quarks and their flavour, another quantum number appeared in the literature, namely the colour. In 1964-1965, Nambu and Han [16] introduced a new quantum number, with Greenberg independently doing likewise [15], albeit with different motivations. Nambu's (and Han's) motivation was to explain why not all particle allowed by $SU(3)$ symmetry are observed, in particular the absence of the coloured ones which are heavier since unbound and to allow for integer-valued charged quarks. Greenberg's motivation was to explain the strange statistics of non-relativistic quark models (this motivation was in fact shared by Nambu). While Greenberg's point was well taken, the dynamical interpretation of its approach was not clear [17].

The Δ^{++} resonance was indeed puzzling with its completely symmetric wave function. It required the introduction of a new degree of freedom. Allowing the quark to be in any state among 3 of this newly introduced colour was solving the issue. It was realised later on that at least two factors "3" were clearly missing in other predictions, namely in the decay rate of $\pi^0 \to 2\gamma$ derived from current algebra [18] as well as in the ratio $R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$ discussed later.

However, it took some more years before the dynamical role of this new quantum number became comprehensible. This came with the advent of Quantum Chromodynamics. It is not completely clear to whom one should attribute the paternity of "colour" as we know it to be today. We would not be too mistaken by saying that the situation started to clarify with a paper by H. Fritzsch, M. Gell-Mann and H. Leutwyler [19] in 1973 following the proceedings by H. Fritzsch, M. Gell-Mann in 1972 [20].

These proceedings were, for instance, cited in an interesting paragraph in the paper of Gross and Wilczek [21] written in 1973 – for which they would receive the Nobel Prize for the discovery of asymptotic freedom of non-Abelian theories; among them QCD :

> *One particularly appealing model is based on three triplets [20] of fermions, with Gell-Mann's SU(3) ⊗ SU(3) as a global symmetry and an SU(3) "color" gauge group to provide the strong interactions. That is, the generators of the strong interaction gauge group commute with ordinary SU(3) ⊗ SU(3) currents and mix quarks with the same isospin and hypercharge but different "color."*

V. BJORKEN SCALING: POINT LIKE AND QUASI NON-INTERACTING CONSTITUENTS

We are still in the sixties. Quarks have been introduced, be them particles or mathematical objects; a new quantum number, colour, seems to be needed. But, so far there is no clear sign of the existence of sub-particle inside the hadron. We have a zoo of hadrons, but no single observational evidence of quarks as separate particles.

As we will see now, the situation will start to clarify thanks to the experiment of Deep Inelastic Scattering (DIS) using electron beams at SLAC. In reading the introduction of Friedman's Nobel lecture [22] we have the feeling that this fundamental discovery, the scaling, was not completely expected:

> *In the latter half of 1967 a group of physicists from the Stanford Linear Accelerator Center (SLAC) and the Massachusetts Institute of Technology (MIT) embarked on a program of inelastic electron proton scattering [..] This work was done on the newly completed 20 GeV Stanford linear accelerator. The main purpose of the inelastic program was to study the electro-production of resonances as a function of momentum transfer. It was thought that higher mass resonances might become more prominent when excited with virtual photons, and it was*

our intent to search for these at the very highest masses that could be reached. For completeness we also wanted to look at the inelastic continuum since this was a new energy region which had not been previously explored. The proton resonances that we were able to measure showed no unexpected kinematic behaviour. Their transition form factors fell about as rapidly as the elastic proton form factor with increasing values of the four momentum transfer, q. However, we found two surprising features when we investigated the continuum region (now commonly called the deep inelastic region).

We will now explain what these "two surprising features" were.

A. Point like objects in the proton: *ep* scattering at high energy deviates from Rutherford scattering

1. *Reminder on elastic scattering*

Let us first start with a bit of kinematics to describe elastic scattering. We start with an electron with a momentum $k = (E, \vec{k})$ which scatters on a proton at rest $(M, \vec{0})$. The scattered electron is then detected with a momentum $k' = (E', \vec{k}')$. The momentum transfer carried by the virtual photon is then obviously $k - k' = q$ and we define $Q^2 = -(k - k')^2$ and $\nu = E - E'$. These are depicted on Fig. 1 as well as the scattering angle θ.

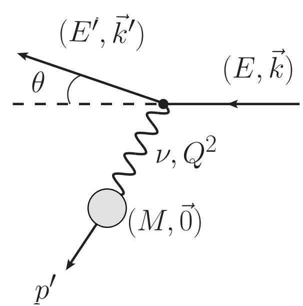


FIG. 1. Kinematics of elastic electron-proton scattering in the proton rest frame (*i.e.* the laboratory frame in the case of a fixed-target experiment).

Let us first remind the Mott cross section for the scattering of a point like charge where we neglect the target recoil for the moment. It is an useful reference to quantify the departure of a reaction from such a point like scattering. It reads:

$$\frac{d\sigma^{\text{point}}}{d\Omega} = \frac{4\alpha^2}{E^2} \frac{\cos^2 \frac{\theta}{2}}{\sin^4 \frac{\theta}{2}}. \tag{1}$$

For an elastic scattering on a finite size particle, one expect a fall-off in Q^2. Introducing the form factor $F(\vec{q})$ just as the Fourier transform in $\vec{q}$ of the spatial charge distribution in the target, one indeed gets

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma^{\text{point}}}{d\Omega} |F(Q^2)|^2. \tag{2}$$

To be more accurate, one should as well consider the magnetic moment of the target and its recoil explicitly in the Mott cross section. For the proton, we introduce two form factors,

$G_E(Q^2)$ and $G_M(Q^2)$, related to the proton charge and the magnetic moment distribution. One has the following differential cross section:

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\alpha^2 E'^2}{q^4}\{\dots\} \text{ with } \{\dots\}_{ep\to ep} = \Big(\frac{G_E^2 - \frac{q^2}{4M^2}G_M^2}{1-\frac{q^2}{4M^2}}\cos^2\frac{\theta}{2} - \frac{q^2}{2M^2}G_M^2\sin^2\frac{\theta}{2}\Big)\delta(\nu + \frac{q^2}{2m}) \tag{3}$$

which can be usefully compared to that in QED for elastic $e\mu \to e\mu$:

$$\{\dots\}_{e\mu\to e\mu} = (\cos^2\frac{\theta}{2} - \frac{q^2}{2m^2}\sin^2\frac{\theta}{2})\delta(\nu + \frac{q^2}{2m}). \tag{4}$$

These coincide for $G_E^2 = G_M^2 = 1$.

2. Generic cross section for inelastic scattering

If one assumes that DIS is mediated by a single off-shell photon, one can easily derive a generic expression of the cross section. Let us first factorise it into two independent pieces, one accounting for the photon emission from the electron, $L^e_{\mu\nu}$ and the other accounting for the interaction between the photon and the hadron target $W^{\mu\nu}$. Overall, we have $d\sigma^{DIS} \sim L^e_{\mu\nu}W^{\mu\nu}$. Gauge invariance and symmetries only allow one to write $W^{\mu\nu}$ as function of two independent Lorentz structures multiplied by 2 functions of the Lorentz invariant quantities Q^2 and ν:

$$W_{\mu\nu}(Q^2,\nu) = (-g_{\mu\nu} - \frac{q_\mu q_\nu}{Q^2})W_1(Q^2,\nu) + \frac{\mathcal{P}_\mu\mathcal{P}_\nu}{P.q}\frac{W_2(Q^2,\nu)}{M^2} \text{ with } \mathcal{P} = P_\mu + \frac{P.q}{Q^2}q_\mu. \tag{5}$$

From $L^e_{\mu\nu}$, one then obtains for the inelastic scattering $ep \to eX$:

$$\frac{d\sigma}{dE'd\Omega} = \frac{4\alpha^2 E'^2}{q^4}\{\dots\} \text{ with } \{\dots\}_{ep\to eX} = W_2(Q^2,\nu)\cos^2\frac{\theta}{2} + 2W_1(Q^2,\nu)\sin^2\frac{\theta}{2} \tag{6}$$

It is instructive to compare Eqs. 3 and 6. If the inelastic scattering at a W^2 tuned to the production threshold of a baryon resonance was like an elastic scattering, one would expect the angular coefficients of the inelastic cross section to behave similarly to a (transition) form factor with a corresponding strong Q^2 fall-off.

The first results for the Q^2-dependence of the cross section for inelastic electron-proton scattering at a fixed W^2 are shown on Fig. 2 and do not show the steep Q^2 fall-off expected from an elastic scattering on a extended object. As Kendall emphasised in his Nobel lecture [24]:

> *Results from the inelastic studies arrived swiftly: the momentum transfer dependence of the deep inelastic cross sections was found to be weak [...]*

B. Scattering on point like partons

At this moment, J.D. Bjorken pushed forward the great idea that the soft Q^2 fall-off could be attributed to a scattering on point like particles –partons– constituting the protons. Let us analyse the expected behaviour of the inelastic cross section in such a case.

From the cross section for $e\mu \to e\mu$, we can extract the contributions W_1 and W_2 of an elastic scattering on a point like particle in the proton by setting $\{\dots\}_{e\mu\to e\mu} = \{\dots\}_{ep\to eX}$

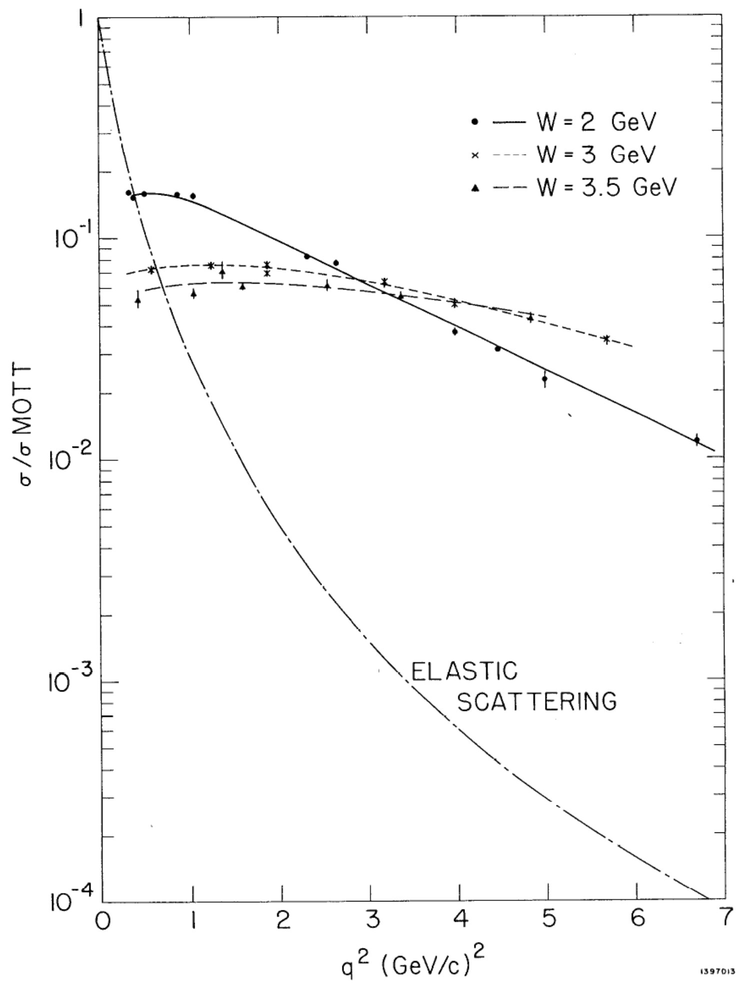


FIG. 2. Q^2-dependence of the cross section for inelastic electron-proton scattering for three values of the electron-beam energy normalised to the Mott cross section for point like elastic scattering. It is clear that the experimental data do not show the steep Q^2 fall-off expected (dashed line) from an elastic scattering on a extended object, except maybe for $W = 2$ GeV. From [23].

(with proper mass replacements and for partons of charge 1) and by identifying the angular coefficients. From the coefficient of $\sin^2\frac{\theta}{2}$ (resp. $\cos^2\frac{\theta}{2}$), one gets

$$2mW_1^{point}(\nu, Q^2) = \frac{Q^2}{2m\nu}\delta\left(1 - \frac{Q^2}{2m\nu}\right) \quad \left(\text{resp. } \nu W_2^{point}(\nu, Q^2) = \delta\left(1 - \frac{Q^2}{2m\nu}\right)\right). \tag{7}$$

In such a case, W_1 and νW_2 are now only functions of $\frac{Q^2}{2m\nu} \equiv \omega$. Irrespective of the experimental value of Q^2, the values of $W_{1,2}$ would remain constant as long as ω does not change; they scale in ω. This was Bjorken's finding. Kendall indeed wrote [24]:

> *During the analysis of the inelastic data, J. D. Bjorken suggested a study to determine if νW_2, was a function of ω alone. [Figure 3] shows $F_2 = \nu W_2$, for 10 values of Q^2, plotted against ω. Because R was at that time unknown, F_2 was shown for the limiting assumptions, $R = 0$ and $R = \infty$. It was immediately clear that the Bjorken scaling hypothesis was, to a good approximation, correct.*

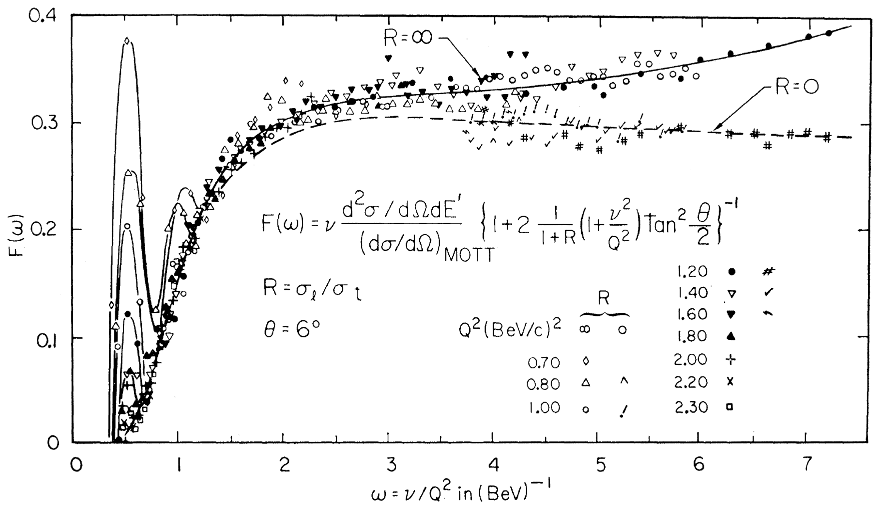


FIG. 3. Experimental values of the so-called structure function $F_1(\omega)$ for two otherwise extreme assumption on R, $R = 0$ and $R = \infty$. For $\omega \geq 3$, $F_1(\omega)$ is rather independent of ω. From [25].

More data were needed to extract $R \equiv \sigma_l/\sigma_t$ and thus $F_1 = \nu W_2$ without anymore ambiguity. The "Bjorken scaling" was then found, see Fig. 4. J.I. Friedman, H.W. Kendall and R.E. Taylor were awarded the Nobel prize in 1990 for this discovery. In fact, the Q^2 independence seemed so clear that it discouraged the introduction of QCD, for which the scaling was not expected – until the discovery of the asymptotic freedom which we discuss later.

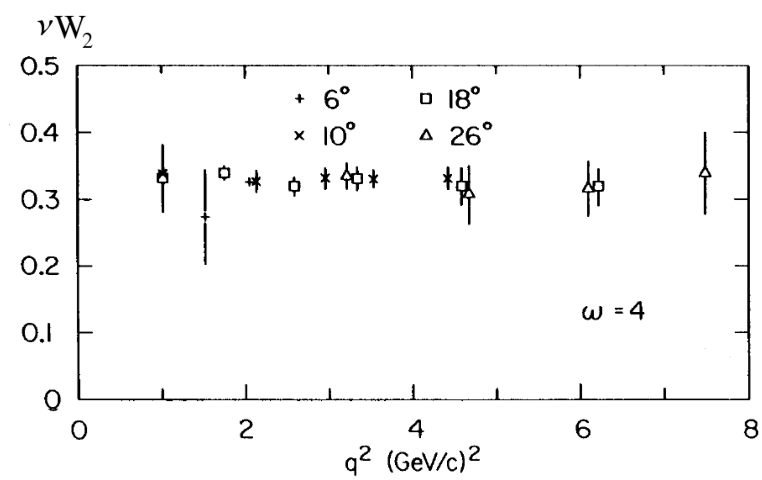


FIG. 4. Early data showing the independence of νW_2 w.r.t Q^2 for fixed ω. From [26].

About the understanding of scaling in 1970, Gross wrote [17]:

> *[...] once one introduced interactions into the theory, scaling, as well as my beloved sum rules, went down the tube. Yet the experiments indicated that scaling was in fine shape. One could hardly turn off the interactions between the quarks, or make them very weak, since then one would expect hadrons to break up easily into their quark constituents, and no one ever observed free quarks.*

C. Putting the partons together in the proton

So far, we have seen how a scattering on a point like particle exhibits the scaling in ω. A natural question which would then arise is "What if this particle is in a proton ?", like the partons. Is the scaling kept intact if the scattered particle is moving ?

The answer is yes if

- the scattered particle is moving independently of the other particles in the proton. This means that one scattering involves only one single parton;
- beforehand, the scattered particle moves along the proton direction with a momentum fraction x, without transverse momentum and with a negligible mass;
- the structure of the proton is fixed once we know the probability to find a parton with a momentum fraction x. This means that it does not depend on the absolute momentum of the partons (only on its fraction), neither[3] on the resolution –Q^2– with which we probe the parton;

This is Feynman's parton model [27, 28] !

Let us now explain this in detail and show how the momentum fraction x identifies to the (inverse of the) scaling variable ω ($\omega \equiv \frac{Q^2}{2M\nu}$). Let us proceed with some definitions; First we define the limit of $MW_1(\nu, Q^2)$ and $\nu W_2(\nu, Q^2)$ for large Q^2 :

$$MW_1(\nu, Q^2) \overset{large\ Q^2}{\longrightarrow} F_1(\omega) \quad \& \quad \nu W_2(\nu, Q^2) \overset{large\ Q^2}{\longrightarrow} F_2(\omega), \tag{8}$$

then the kinematics of the proton and the partons:

	Proton	Parton
Energy	E	xE
Momenta	P_L	xP_L
	$P_T = 0$	$P_T = 0$
Mass	M	$m = \sqrt{x^2E^2 - x^2P_L^2} = xM$

Let us now work out the structure functions for the scattering of one photon with one parton with a momentum xP (see Fig. 5). We simply use Eq. 7 (keeping m as the mass of the struck particle), the definitions of ω and $F_{1,2}(\omega)$ for a proton and $m = xM$. This gives:

$$\begin{aligned} F_1(\omega) &= MW_1^{point}(\nu, Q^2) = \frac{m}{x}\frac{Q^2}{4m^2}\delta(1 - \frac{Q^2}{2m\nu}) = \frac{1}{2x^2\omega}\delta(1 - \frac{1}{\omega x}) \\ F_2(\omega) &= \nu W_2^{point}(\nu, Q^2) = \delta(1 - \frac{Q^2}{2m\nu}) = \delta(1 - \frac{1}{\omega x}) \end{aligned} \tag{9}$$

The scaling is indeed preserved: as long as ω is fixed, F_1 and F_2 do not vary. In addition, momentum conservation tells us that ω – a quantity only involving "external" quantities, Q^2, ν and M – is equal to the inverse of the momentum fraction of the struck partons – the point like particle which eventually interacts with the photon. This means that a scan in ω would allow one to probe the partons with different momentum fraction x !

[3] . . . in a first approximation. The scaling was subsequently found to be logarithmically violated in Q^2 due to gluon radiations. Instead of being an issue, this became a success of QCD when quantitative predictions of this violation were found to agree with experiments.

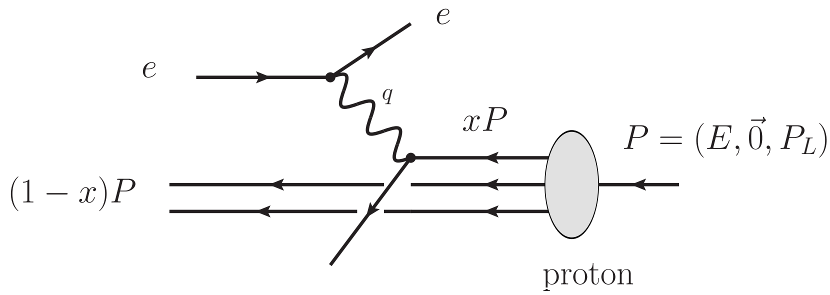


FIG. 5. Illustration of deep-inelastic electron-proton scattering in the parton model.

D. Parton Distribution and Callan-Gross relations

For the scattering on one parton of proton-momentum fraction, x, we have obtained a first evaluation of the structure functions $F_1(\omega) = \frac{1}{2}\delta(x - \frac{1}{\omega})$ & $F_2(\omega) = x\delta(x - \frac{1}{\omega})$. Let us now consider the proton as a collection of independently moving partons and define the probability to select a parton of type i with momentum xP, referred to as $f_i(x)$. Naturally, we have a normalisation constraint: $\sum_i \int dx\, f_i(x) = 1$.

$f_i(x)$ are called Parton Distribution Functions (PDF), defined for all quark flavour and also gluons. They give the probability to have a parton (quark or gluon) with a momentum fraction x in the proton.

F_1 and F_2 are then obtained by folding this probability with the values for a point like scattering summed over the available types i with their respective charge squared. We have:

$$\begin{aligned} F_1(x) &= \sum_i e_i^2 \int dx\, f_i(x) \frac{1}{2} \delta\left(x - \frac{1}{\omega}\right), \\ F_2(x) &= \sum_i e_i^2 \int dx\, f_i(x)\, x\delta\left(x - \frac{1}{\omega}\right). \end{aligned} \tag{10}$$

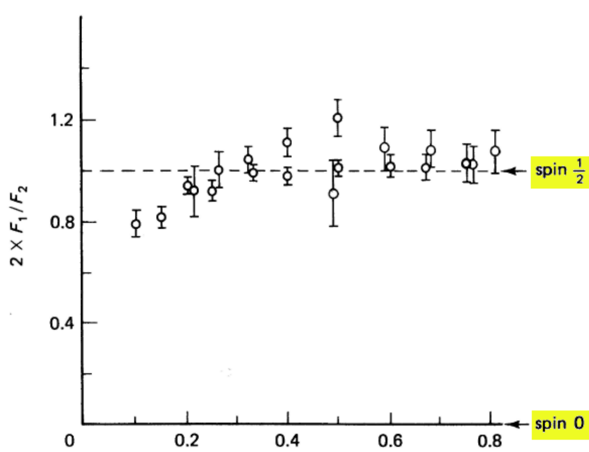


FIG. 6. Experimental verification of the Callan-Gross relation for spin-1/2 constituents in the proton. The ratio $2xF_1/F_2$ is compatible with unity. The idea that DIS occurs by scattering on spin-less constituent is completely ruled out. From [30] with data from [32].

We thus have the relation

$$F_1(x) = \frac{1}{2x} F_2(x), \tag{11}$$

known as the Callan-Gross relation derived in 1968. Such a relation only holds for spin 1/2 partons [29]. Its experimental confirmation in the seventies (see Fig. 6) was a further indication that proton are mostly made of spin 1/2 charged quarks, and not only of spin 0 and 1 particles – at least as regards particles interacting with the electromagnetic current. The Callan-Gross relation was also confirmed by early results on neutrino (deep) inelastic scattering at CERN by the Gargamelle detector [31]. They also provided evidences that quarks were fractionally charged and that they were only carrying about half of the proton momentum – something else was to be in the proton. Nowadays we know that the rest of the momentum is actually carried by the gluons.

VI. BJORKEN SCALING AND NON-INTERACTING CONSTITUENTS

What we have obtained so far is very appealing and sounds like a confirmation that protons are compound objects made of point like partons whose dynamics can be studied by deep-inelastic scattering. It is however important to recall the limit of applicability of this parton model proposed by Feynman as well as the important assumptions made in deriving the expressions for the structure functions.

First, we need to emphasise that the kinematics used here (esp. $m = xM$) only makes sense in the infinite momentum frame. As it should be for any real particle, the parton mass is fixed. One may argue on pertinent values of the (rest) quark mass ($m^2 = p^2$) entering the kinematics, be it the current mass ($\sim 5 - 10$ MeV for light quarks) or the constituent mass (~ 300 MeV for light quarks), but it is clear that it is fixed and independent of its momentum and thus of x. The relation $m = xM$ is virtually admissible if one can neglect the masses of all initial particles, that is in the high energy limit, at large ν .

Second, time is supposed to be frozen when the photon interacts with the parton. We indeed neglected parton-parton interactions and final-state interactions. This is also known as the impulse approximation. However, final-state interactions will necessarily take place due to confinement – free partons are never observed. The effect of hadronisation can however be neglected if it is carried out over a larger space-time distance ($\sim 1/\Lambda_{QCD}$) than the hard photon interaction ($\sim 1/Q$). Taking the limit of high Q^2 is thus required. Both requirements, large ν and Q^2 are *de facto* satisfied when taking the "Bjorken limit" – ν and Q^2 infinite keeping x fixed.

It is also fair to say that it is not trivial to justify that we can neglect parton-parton interactions. After all, strong interactions are ... strong. In the late sixties, it was not clear at all how to derive the absence of parton-parton interactions, which would ruin the scaling. One way was to suppose that strong interactions were no longer strong at high energy, that is at short distances.

In his Nobel lecture, D. Gross said [17]:

> *the vanishing of the effective coupling at short distances, later called asymptotic freedom, was necessary to explain scaling [...] One might suspect that this is the only way to get point like behavior at short distances*

VII. RUNNING OF THE COUPLING CONSTANT AND ASYMPTOTIC FREEDOM IN QCD

A. The running of the (QED) coupling constant

In QED, as expected for a relativistic theory, e^+e^- pair fluctuations can constantly appear and disappear. The number of particles in a system is not fixed. This is for instance responsible for the light-by-light scattering, even though QED is an Abelian theory. This also has the consequence that the (bare) charge, e_0 as defined in the QED Lagrangian is screened by these pair fluctuations and is never observed (see Fig.7).

The charge value, e (or equivalently $\alpha \equiv e^2/(4\pi)$), which we actually measure depends on the distance from which we probe it. It depends on the scale of the reaction used for the measurement. Seen from far away, or at low energy, we indeed measure what is known to be in quantum mechanics the fine structure constant, $\alpha = 1/137$. When one gets closer, or when one probes the charge with a higher energy, the value of α changes. The screening due to the pair fluctuation decreases and the effective charge seen grows. Actually, it never ceases to increase to reach infinity (see Fig. 7). The bare charge in the Lagrangian would be infinite but since we can never see it, it does not really matter.

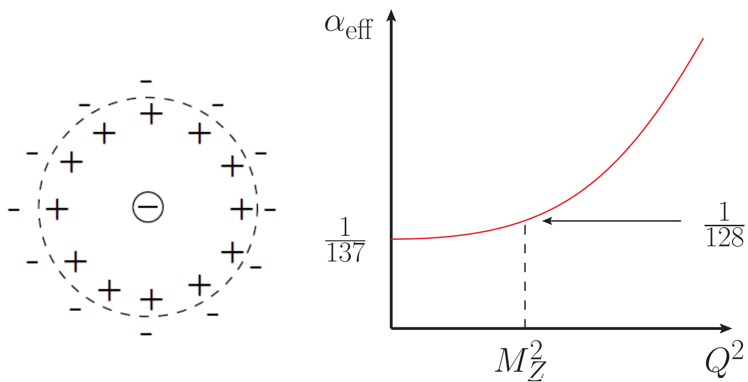


FIG. 7. Cartoon illustration of how the vacuum polarisation, via the alignment of particle-antiparticle fluctuation dipoles towards the electric charge, can induce a modification of the effective charge which becomes a function of the distance from which one probes it. At $Q^2 = 0$, infinitely remote, the charge is screened; α is observed to be $1/137$. At infinite Q^2, infinitely close, the charge is unscreened, and infinite (the Landau pole).

In QED, α increases with the scale, that is Q^2, s or any relevant momentum transfer. Different behaviours can however be obtained in other theories. Let us imagine that we probe an electric charge via a scattering process; formally, we have to deal with the expansion depicted in Fig. 8 in term of α (or e). Contributions with additional pair fluctuations (the loop in the graphs) are successively suppressed by powers of α and these can be resummed.

Thinking in terms of a geometric series (Fig. 9), we can convince ourselves than the charge at a given scale Q^2 can be expressed in terms of the initial charge multiplied by a factor involving the effect of a single loop. The latter exhibits a logarithm of the scale ratio Q^2/μ^2 – reminiscent of the infinities arising in the loop integral[4]. In QED, one expects α to increase with Q^2. Intuitively, the (infinite bare) charge is less and less screened by the e^+e^- pairs which normally align themselves towards the charge (Fig. 7 (left)).

---

[4] This is the leading logarithmic approximation. In general, contributions involving logarithm of logarithm, and so on, can arise.

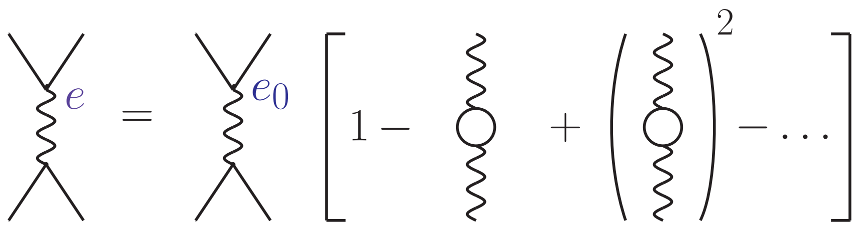
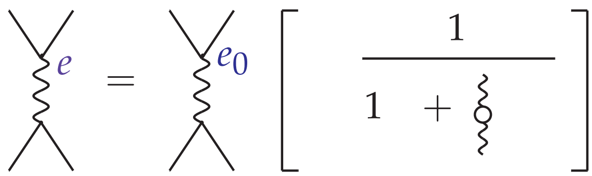


FIG. 8. Cartoon illustrating how the dressed charge can be obtained from the product of the bare charge times and the dressed propagator of the photon which can fluctuate in fermion loops.



$$\rightarrow \alpha(Q^2) = \frac{\alpha(\mu^2)}{1-\frac{\alpha(\mu^2)}{3\pi}\log(\frac{Q^2}{\mu^2})} (\alpha = \frac{e^2}{4\pi})$$

FIG. 9. Cartoon illustrating how a class of perturbative corrections can be resummed in the well-known closed formula for the running of α (for one fermion flavour).

B. The running of α_s and the asymptotic freedom

Before discussing the properties of the coupling of the strong interactions with the distance in the framework of QCD, it is certainly expedient to recall some basic facts about this theory. QCD is a non-Abelian gauge (also referred to as a Yang-Mills) theory, with a $SU(3)$ symmetry for its (colour) charge and whose gauge fields –the gluons– couple to spin 1/2 (Dirac) fermions –the quarks–. The gluons can be in 8 colours and the quarks, in 3. Contrary to what happens in QED, the gluons self-interact.

In the previous section, we have seen that, in QED, the coefficient of the logarithm appearing in the running of the coupling is $-\frac{\alpha(\mu^2)}{3\pi}$. Instead of referring to this coefficient, it is actually more convenient, and common, to refer to the so-called β function, the logarithmic derivative in Q^2/μ^2 of the coupling. It can be computed order by order in power of α itself:

$$\beta(\alpha(Q)) \equiv \frac{d\alpha(Q)}{d\log(Q^2/\mu^2)} = \alpha^2(Q)b_1 + \alpha^3(Q)b_2 + \dots \tag{12}$$

In QED, we have $\beta_{\rm QED}(\alpha(Q)) = \frac{1}{3\pi}\alpha^2(Q)$. This is precisely the sign of the β function in a theory which governs the behaviour at low and large scales. In QED, the growth essentially comes from the lepton loop contributions (the screening).

In QCD, Gross, Wilczek and Politzer showed in 1973 that the $\beta_{\rm QCD}$ function was indeed different because of the presence of loops of the (self-interacting) gluons. They obtained at the leading order [21, 33]

$$b_1 = \frac{-33 + 2n_F}{12\pi}, \tag{13}$$

which has the very important property to be negative (unless $n_F > 16$). The (negative) factor "-33" results from a sum of positive contributions from transverse gluons and negative contributions from Coulomb gluons.

Introducing Λ_{QCD} as the (infrared) scale where α blows up, we can rewrite the coupling at leading order for any scale Q^2 in a form which is more instructive:

$$\alpha(Q^2) = \frac{12\pi}{(33-2n_F)\log(\frac{Q^2}{\Lambda^2_{\mathrm{QCD}}})}, \tag{14}$$

Indeed, for the region where $Q^2 \gg \Lambda^2_{\mathrm{QCD}}$, one sees that $\alpha_s(Q^2) \ll 1$: it is referred to as the perturbative domain, where the expansion in powers of α_s of a given observable is normally justified. For $Q^2 \sim \Lambda^2_{QCD}$, $\alpha_s(Q^2) \gtrsim 1$; this is the non-perturbative region, where the strong interactions are strong !

Overall, at short distances (large Q), the strong interactions are not as strong: this is what we call the asymptotic freedom – this property is in fact shared by the pure gauge Yang-Mills theories. This in turn justifies the idea that the partons in the proton are mostly behaving freely over a distance $\frac{1}{Q} \ll \frac{1}{\Lambda_{\mathrm{QCD}}}$. Feynman's parton models thus makes sense in the Bjorken limit.

VIII. THE RATIO R: $\frac{\sigma(e^+e^- \to \mathrm{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$

> *[... H]istorians have often ignored the developments associated with electron-positron colliders in the decade from 1960 to 1970, and this is my modest effort to contribute to this history.*

thus wrote C. Bernardini in 2004 in a review on AdA, the "The First Electron-Positron Collider" [34]. We partly share his impression, especially because the accounts of the experimental investigations on R, especially in QCD textbooks, are too often loaded with historical short-cuts. This is mostly explainable by the complexity of the situation at that time, which is incompatible with a wishful pedagogical account on this very important, but not that simple, quantity R.

Let us travel in time to the beginning of the year 1974 and recapitulate what we have learnt so far:

- hadrons can be classified in terms of an $SU(3)$ symmetry compatible with the existence of 3 quarks;
- deep-inelastic scattering on proton has revealed the property of scaling which can be easily explained if there are point like particles in the proton, most likely the aforementioned quarks;
- QCD has been proposed as the theory of strong interactions; it is a $SU(3)$ Yang-Mills theory, *i.e.* a non-Abelian gauge theory with 8 (self-interacting) gluons
- a new quantum number, the colour, has been introduced with QCD; it solves the problem of the fermion statistic in the Δ^{++} case and the $\pi^0 \to \gamma\gamma$ width;
- the scaling, which was after all not expected for interacting theories, could hold for QCD thanks to the asymptotic freedom, discovered in 1973;
- but, so far, there is still no experimental evidence of quarks alone;
- a fourth quark seems to be needed in order to cancel the anomaly in weak decays, but no evidence of it is available;

Around 1970, the measurements of the total cross section of hadron production at e^+e^- colliders (ACO at Orsay, VEPP at Novosibirsk and ADONE at Frascati) $\sigma(e^+e^- \to \text{hadrons})$ seem clearly above the theoretical expections. The cross section was indeed expected to drop sharply for collision energies significantly higher than the ρ-meson mass and to be, in any case, lower than that of $\sigma(e^+e^- \to \mu^+\mu^-)$, the reference for point like particle creation[5]. Fig. 10 presents a compilation of the results available in early 1974 from the ADONE colliders.

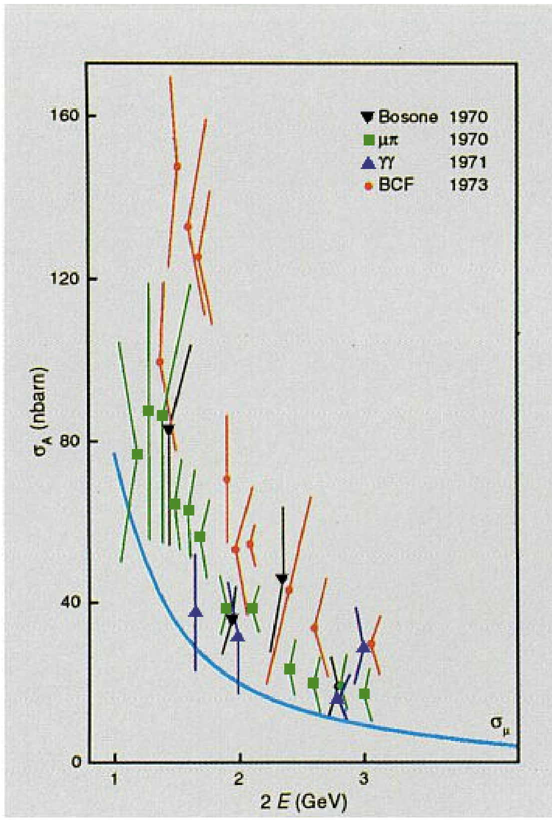


FIG. 10. Compilation of experimental results for the total cross section of hadron production from the Frascati collider ADONE (from http://www.lnf.infn.it/acceleratori/adone/) compared to $\sigma(e^+e^- \to \mu^+\mu^-)$ (blue curve).

In Feynman's parton model, the hadron cross section relative to the μ-pair cross section was simply given by [35, 36]

$$R = \sum_q \frac{e_q^2}{e^2} \tag{15}$$

where e_q^2 is the charge of the quarks and the sum runs over the quarks which can be produced at a given $\sqrt{s}$. One actually expects steps versus the centre-of-mass of the collisions, each time one crosses a new quark-pair production threshold, up to the effects of resonances. In between these thresholds, R is expected to be a constant. In other words, the parton model explains the slow s fall-off of $\sigma(e^+e^- \to \text{hadrons})$, similar to that of $\sigma(e^+e^- \to \mu^+\mu^-)$.

[5] For instance, $\sigma(e^+e^- \to \pi^+\pi^-)$ is proportional to the square the pion form factor at $Q^2 = s$ and is thus expected to have a much stronger s-fall-off than $\sigma(e^+e^- \to \mu^+\mu^-)$.

As regards the step normalisation, one expects:

$$\begin{aligned} &\text{for 3 quarks, } R = \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 = 2/3, \\ &\text{for 4 quarks, } R = \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 + \left(-\frac{1}{3}\right)^2 + \left(\frac{2}{3}\right)^2 = 10/9 \end{aligned} \tag{16}$$

if the fourth quark has a charge 2/3 as expected from the Glashow-Iliopoulos-Maini mechanism [37] – a symmetry between leptons and quarks – to explain the smallness of some weak decays and as proposed earlier by Bjorken and Glashow in 1964 [38]. From Fig. 10, it is clear that $R > 1$ even for the largest energies. It would thus be incompatible with the existence of 3 quarks ($R = 2/3$) and it would barely be compatible with the existence of 4 quarks ($R = 10/9$) below 1.5 GeV.

However, in QCD with coloured quarks, the situation is getting slightly better since $R = 3 \times \sum_q \frac{e_q^2}{e^2}$. One expects $R = 2$ ($R = 10/3$) in the presence of the 3 (4) quarks. This is another success of QCD which however was slightly blurred by the results coming from SLAC (SPEAR) and Harvard (CEA). In July 1974 at the ICHEP London conference, Richter presented [39] the experimental situation as in Fig. 11. The sign of a new quark – as the charm quark first proposed in 1964 and expected from the GIM mechanism– would have been another plateau at $R \simeq 3.3$. Clearly, the trend was for a linear increase of R with s in striking contradiction with the parton model with 3 or 4 quarks. The confusion is complete in the summer of 1974 until . . . November.

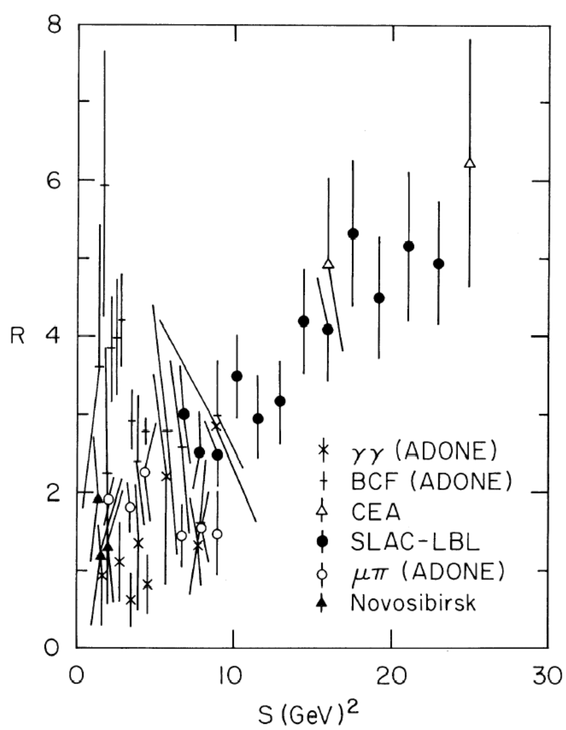


FIG. 11. Experimental status of R as of July 1974 (from [39]).

IX. THE DISCOVERY OF HEAVY QUARKS: QUARKS BECOME REAL !

A. The November revolution (in 1974)

Out of the crowd of particles, the J/ψ (see Fig. 12) directly stands out with its peculiar name composed of two symbols : "J" from S. Ting [40] in comparison to the electric current j_μ – most likely also since the sound [dīng] is represented by a Chinese character looking like a "J"–, and "ψ" because of the typical pattern of its decay with a $\pi^+\pi^-$ at SPEAR [41].

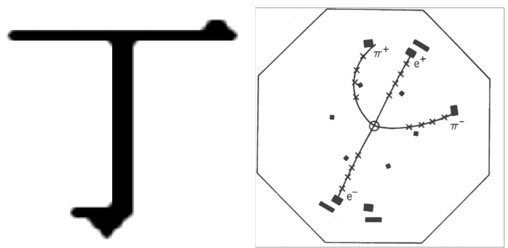


FIG. 12. Left: one of the Chinese characters for the sound [dīng]; Right: typical Mark-I (SPEAR) event display for $\psi' \to J/\psi\pi^+\pi^- \to e^+e^-\pi^+\pi^-$ decay (from [42]).

A tentative explanation why the particle-physics community agreed to keep both symbols for this particle is due to its importance in particle physics: its discovery is now referred to as the November revolution and it was the first tangible sign that the Gell-Mann's and Zweig's quarks were real particles and that they could be identified to the point like partons of Feynman and Bjorken, which had been uncovered by the scaling in DIS experiments. The importance of the discovery was also likely amplified by the confusion reigning during the summer 1974.

The discovery of the J/ψ meson has been attributed to the simultaneous observations of a sharp resonance at $\sqrt{s} = 3.1$ GeV in two different experimental set-ups (Fig. 13). This is rather unique. It was indeed seen at the SLAC-SPEAR e^+e^- collider by the group of B. Richter and at a BNL proton-nucleus fixed-target experiment by the group of S. Ting. The observation was then confirmed at ADONE where the energy of the machine was pushed beyond its design value. The discovery was complete when the first excited state of the J/ψ, the $\psi(2S)$ was also seen at SLAC [44].

In 1976, Ting and Richter were awarded the Nobel prize for this simultaneous discovery. Indeed, considering the state of comprehension of particle physics at that time, this discovery was a shock for many. During the two years between the discovery and the award of the Nobel prize, an intense research activity in this domain took place. Were these new states undoubtly the sign of a new generation of quarks ? Was the quark model to be finally trusted?

It was promptly established that the quantum numbers of the J/ψ were the same as those of the photon, i.e. 1^{--}. It was then clear that the J/ψ and ψ' did have direct hadronic decays. The study of multiplicity in pion decays indicated that ψ decays were restricted by a specific selection rule, called G-parity conservation, known to hold only for hadrons. Consequently, J/ψ and ψ' entered the family of hadrons with isospin 0 and G-parity -1 Particles with charge conjugation C different from -1, i.e. with quantum numbers different from those of the photon, were found later.

It became quite rapidly obvious that the J/ψ was the lowest-mass $c\bar{c}$ system with the same quantum numbers as photons – explaining why it was produced so greatly compared to some other members of its system. These $c\bar{c}$ bound states were named "charmonium", firstly by Appelquist, De Rújula, Politzer and Glashow [45] in analogy with positronium, which has a similar bound-state level structure.

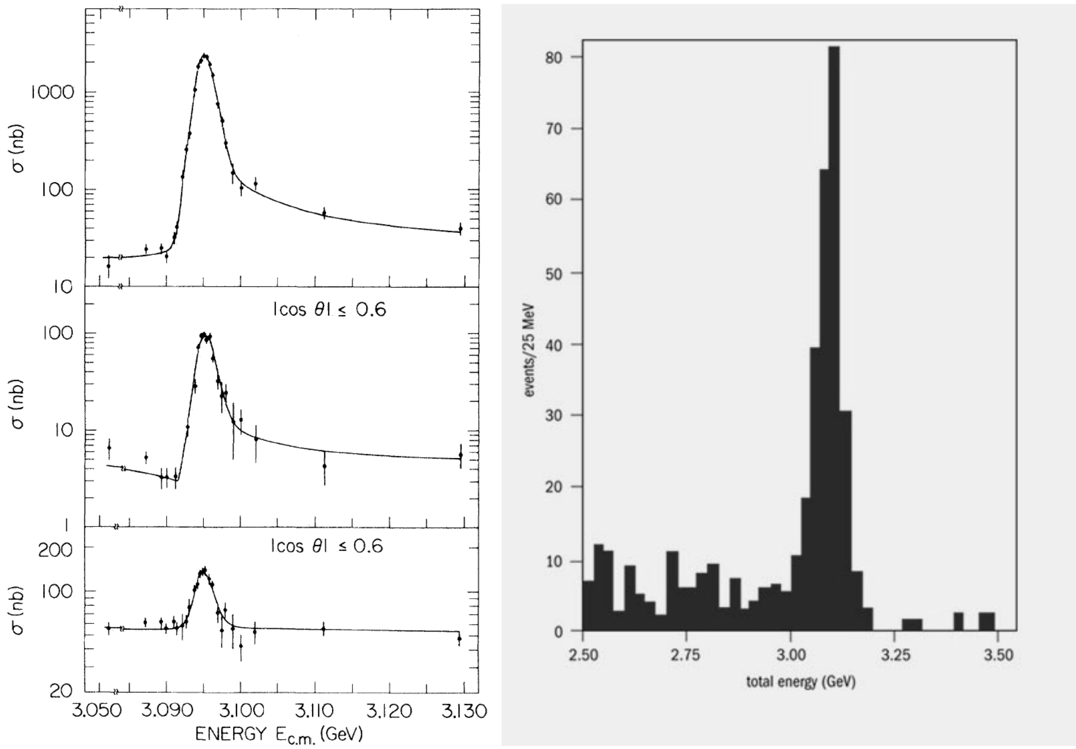


FIG. 13. Left: various pair production cross sections as functions of the centre-of-momentum energy (top: hadrons; middle: $\pi^+\pi^-$, $\mu^+\mu^-$ and K^+K^-; bottom: e^+e^-) [41]. Right: energy distribution of the produced electron-positron pairs for Ting's experiment (from [43] with data from [40]).

All would then be easily described as analogous of a non-relativistic positronium using a QCD potential with coulombic and confinement parts. In comparison to the electromagnetic potential, $V_{QED}(x) = -\alpha/r$, a potential for the strong interactions between quark was proposed $V_{QCD}(x) = -4/3\,\alpha_s/r + kr$, where the factor 4/3 was justified since in QCD more than one gluon can act in $q \to qg$. The ad-hoc linear term was introduced to account for the non-observation of free quarks. It is referred to as the confinement term. In order to reproduce quantitatively the charmonium bound-state level structure the factor k, known as the string tension was found to be of the order of $k = 1$ GeV fm^{-1} and the coupling of the strong interactions α_s was found to be close to 0.2. This is actually the first application of the asymptotic freedom in QCD: the strong coupling was small at "high" energy, here at the charm mass.

All the pieces of the puzzle were then put together; this charm quark was exactly what was theoretically needed by the GIM mechanism [37], in order to cancel the anomaly in weak decays (e.g. $K \to \mu^+\mu^-$). A symmetry between leptons and quarks should exist. The Standard Model of particle physics was being built.

X. 2-JET EVENTS IN e^+e^- ANNIHILATION: "SEEING" THE QUARKS

We have seen that the measurement of the hadron-production cross section in e^+e^- provided a confirmation that hadrons were made of point like quarks with 3 colours. With the improvements of detector techniques, the increase of the luminosity and of the energy, investigations on the production of 2 jets of particles became possible. This provided a handle on the partonic process $e^+e^- \to q\bar{q}$ (see Fig. 14 (a)). In particular, direct information on the spin of the quarks could be obtained from the angular dependence of the jets as if one would look at the quark momentum directly.

By momentum conservation, e^+e^- annihilation would produce a $q\bar{q}$ pair with opposite momenta (see Fig. 14 (b)). However, strong interactions confine quarks, which end up always being bound. We say that they hadronise and we expect to observe sprays (or jets) of hadrons along the original direction of the quarks.

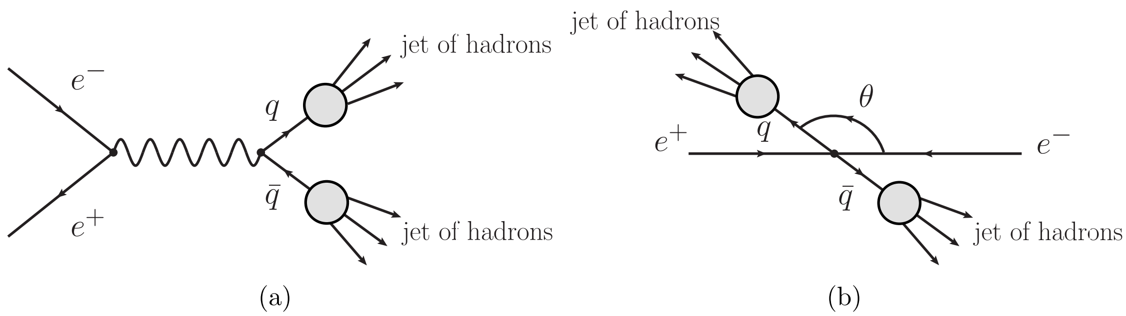


FIG. 14. (a) Feynman graph for the reaction $e^+e^- \to 2$ jets (the soft gluon exchanges between the jets are not represented). (b) Definition of the polar angle for 2-jet production in the e^+e^- centre-of-momentum frame.

In October 1975, the "first evidence for a jet structure in hadron production by e^+e^- annihilation" is found at SLAC [46]. Defining the sphericity as $S = 3(\sum_i p^2_{\perp,i})_{\text{min.}}/(2\sum_i p_i^2)$, it was seen that the 2-jet events showed a S distribution (Fig. 15) very close to what was expect in a "jet" model rather than in "phase space" model.

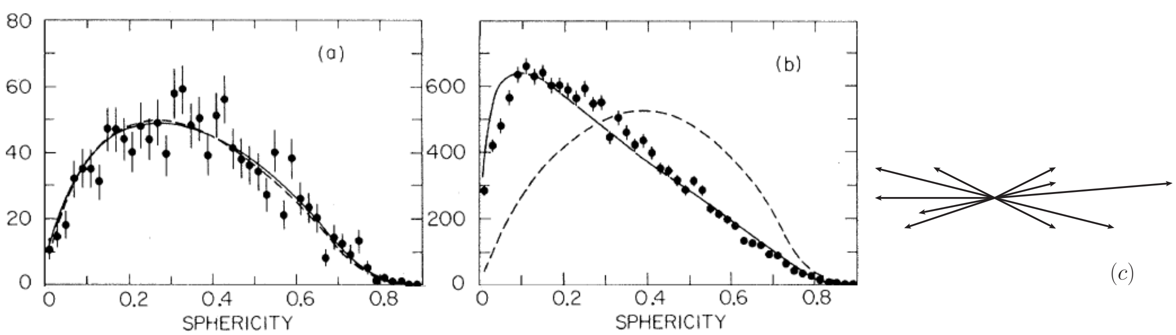


FIG. 15. Sphericity distribution at (a) W=3.0 GeV and (b) at 7.4 GeV observed by MARK-I compared to Monte Carlo simulation based on jet (solid lines) and phase space (dashed lines) models (Adapted from [46]). (c) Sketch of an event with a low sphericity, showing 2 back-to-back so-called "jets".

It indicates that the angular dependence of the jets "remembers" that of the quarks and confirms that they are spin 1/2 particles. Jets provide a way to "see" the quarks. The abstract of [46] reads:

> *We have found evidence for jet structure in* $\sigma(e^+e^- \to \text{hadrons})$ *at centre-of-mass energies of 6.2 and 7.4 GeV. At 7.4 GeV the jet-axis angular distribution integrated over azimuthal angle was determined to be proportional to* $1 + (0.78 \pm 0.12)\cos^2\theta$.

Similarly to μ pair production, the angular distributions for the pair production of spin-1/2 quarks reads:

$$\frac{d\sigma(e^+e^- \to q\bar{q})}{d\cos\theta} \propto 1 + \cos^2\theta \tag{17}$$

On the contrary, for the production of a pair of hypothetical spin-0 quarks, one would have

$$\frac{d\sigma(e^+e^- \to q\bar{q})}{d\cos\theta} \propto 1 - \cos^2\theta \tag{18}$$

This value of the angular distribution was a compelling confirmation that quarks are spin 1/2 particles, following the verification of the Callan-Gross relation.

XI. 3-JET EVENTS IN e^+e^- ANNIHILATION: "SEEING" THE GLUONS

In the same way as electrically charged particles can radiate photons in QED, quarks are expected to radiate gluons in QCD with a rate proportional to the strong coupling. The gluons should then produce a jet of hadrons, just as the quarks hadronise into a jet. 3-jet events should naturally come from a genuine $e^+e^- \to q\bar{q}g$ process, but nothing forbids the reaction $e^+e^- \to q\bar{q}$ to also produce 3 sprays of hadrons, which would mimic a $q\bar{q}g$ final state. The analysis of 3-jet events requires the use of specific observables describing the topology of the event, which are then used in statistical analyses quantifying the agreement between the data and theoretical expectations within one or another model/assumption.

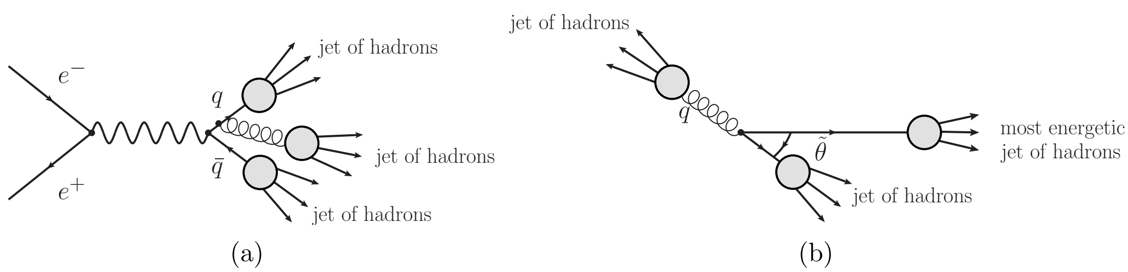


FIG. 16. (a) A typical Feynman graph for the reaction $e^+e^- \to 3$ jets (the soft gluon exchanges between the jets are not represented). (b) Definition of the angle $\tilde{\theta}$ for 3-jet production in the centre-of-momentum frame of the two least energetic jets.

However, since the gluon can only be radiated by one of the quarks which are produced back-to-back, momentum conservation results in simple features of the event, for instance the planarity of the 3 jets. The first evidence for 3-jet events (and e.g. for the planarity) was found in 1979 by 4 collaborations (TASSO [47], PLUTO [48], JADE [49] and MARK-J [50]) working on the PETRA accelerator at DESY. This is now referred to as the first evidence for gluon Bremsstrahlung. Further tensorial quantities inspired by the introduction of the sphericity by Bjorken and Brodsky [51] (see above) were also used by the JADE collaboration for instance and allowed to quantify the observation, which quickly became a discovery.

It was also proposed by Ellis and Karliner to look for information on the gluon spin by observing a specific angular distribution, referred to now as the Ellis-Karliner angle[6]

---

[6] $\tilde{\theta}$ is defined, in the centre-of-momentum frame of the two least energetic jets, as the angle between the direction of these jets and that of the third jet (see Fig. 16 (b)).

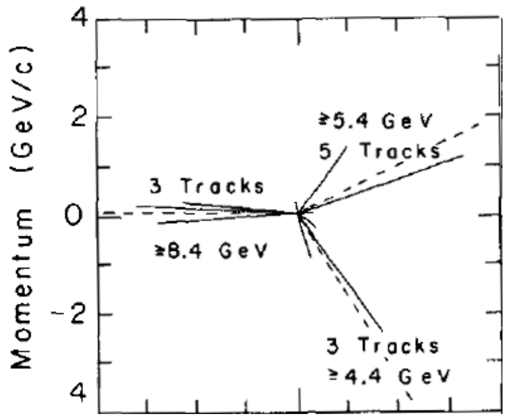


FIG. 17. Momentum space representation of a 3-jet event in the TASSO experiment [47]. The particle momenta are projected on the event plane (see [47] for exact definitions).

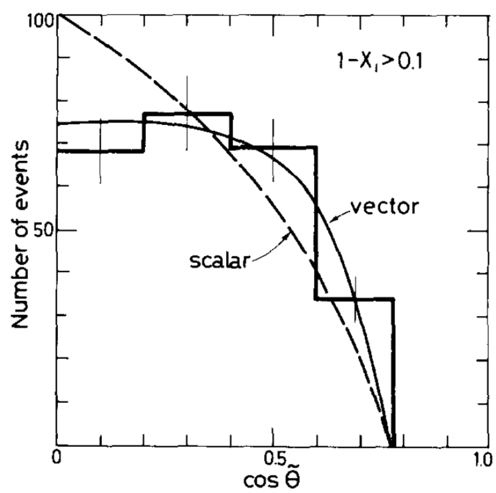


FIG. 18. Comparison between the event distribution seen by the TASSO experiment [53] as function of the cosine of the Ellis-Karliner angle (see text). The solid line is for QCD (spin-1 gluon) and the dashed one for scalar (spin 0) gluons. Both are normalised to the event yield.

$\tilde{\theta}$ [52]. This required the use of Monte Carlo simulations and models to predict the angular distribution in different scenari (spin-1 or scalar gluon, ...)

The confirmation that the angular distribution of the jet likely initiated[7] by the gluon was close to the one expected for spin-1 gluon arrived quickly [53]. It re-confirmed QCD as the theory of strong interactions.

XII. MEASUREMENT OF α_s

As discussed above, QCD exhibits the property of asymptotic freedom: α_s decreases with energy. This variation is nevertheless rather mild, since logarithmic. It took some time

[7] Among the 3 jets, only statistical tools could tell which one comes from the gluon. Later, it was confirmed that gluon jets were more spread because of the stronger colour coupling of the gluon.

to obtain measurements with sufficient lever arm in energy (more precisely, in momentum transfer) to pin down this "running" as we call it.

Before looking at it, the community wanted to simply measure the size of the coupling. To understand how, let us go back to the ratio R discussed in section VIII. Now that we are about to be convinced that QCD is the correct theory of strong interactions and that we are aware of the asymptotic freedom of QCD, we can compute the contribution of $e^+e^- \to q\bar{q}g$ to R, which gives [54, 55]

$$R = 3\sum_q e_q^2 \left(1 + \frac{\alpha_s(s)}{\pi}\right). \tag{19}$$

A measure of R precise enough (in a region where there is no resonance) should give us a measurement of $\alpha_s(s)$. In the late seventies, the situation is nonetheless not as ideal as presented in textbooks. In his SLAC Ph.D. thesis on hadron production by e^+e^- annihilation [56], which he defended in October 1979, J.L. Siegrist explains that the extraction of $\alpha_s(s)$ from R in the region covered by SPEAR would provide

$$\alpha_s(s = 36 \text{ GeV}^2) = 0.83 \pm 0.32 \text{ and } \alpha_s(s = 9 \text{ GeV}^2) = 1.2 \pm 0.8. \tag{20}$$

Clearly, the effect of a number of resonances in the SPEAR domain "may preclude straightforward interpretation of the above result for α_s".

However, as early as in December 1979, a value of $\alpha_s = 0.17 \pm 0.04$ was obtained by the JADE collaboration [49] using their 3-jet analysis. They compared Monte Carlo predictions and their measurement for the rate of the most planar events (those which are most likely to come from gluon Bremsstrahlung).

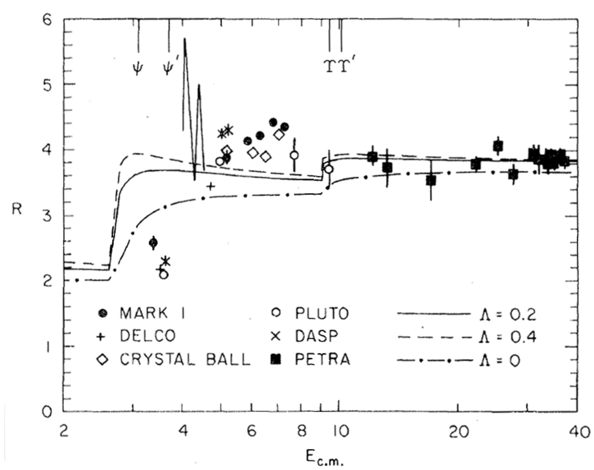


FIG. 19. Compilation of various experimental results on R in 1982 along QCD predictions with different value of $\alpha_s(s)$ (different choices of Λ). From [60].

It is only later, with the results from PETRA, that R, at high s, allowed for a sensible extraction of α_s. It is clear on Fig. 19 that data below the Υ[8] threshold are "polluted" by

[8] The Υ is the heavier sister of the J/ψ, made of a $b\bar{b}$ pair. Its discovery in 1977 [57] at Fermilab provided a decisive indication of a third generation of particles. As for the charmonium, excited states were soon discovered [59] and it was quickly accepted that a fifth quark had been uncovered. The immediate consequence – following the Standard Model proposed in 1974 – was the existence of a third generation of quark and leptons, compatible with the anomalous production of leptons at SLAC observed by M. Perl *et al.* in 1975 – the τ lepton discovery [58].

the presence of many resonances. At larger $\sqrt{s}$ up to 40 GeV, one observes that the curve with $\Lambda_{(\mathrm{QCD})} = 0$, *i.e.* $\alpha_s = 0$, is systematically below the data, while the other choices show a good agreement. If one tries to extract α_s using Eq. 20 with $R(\sqrt{s} \sim 40$ GeV$) = 3.9$, one would obtain $\alpha_s = 0.2$.

In addition, it is fair to say that the running of $\alpha_s(s)$ – *i.e.* the slight decrease of the curves in between steps – is still not obvious in the data (Fig. 19). With the progress in the definitions, simulations and studies of jets, the study of the 3-jet fraction in e^+e^- annihilation,

$$R_3 = \frac{\sigma(e^+e^- \to 3\text{ jets})}{\sigma(e^+e^- \to \text{hadrons})}, \tag{21}$$

became a competitive way to study the running of $\alpha_s(s)$, especially with the advent of the LEP colliders with its 4 detectors (OPAL, DELPHI, L3 and ALEPH) running first up to $\sqrt{s} = 100$ GeV. The first significant observation of the running of $\alpha_s(s)$ was done thanks to the data by the AMY collaboration [61] at Tristan in late 1989. It ruled out a constant $\alpha_s(s)$ with a χ^2_{dof} of 3.8 when trying to describe the world data including theirs (Fig. 20.).

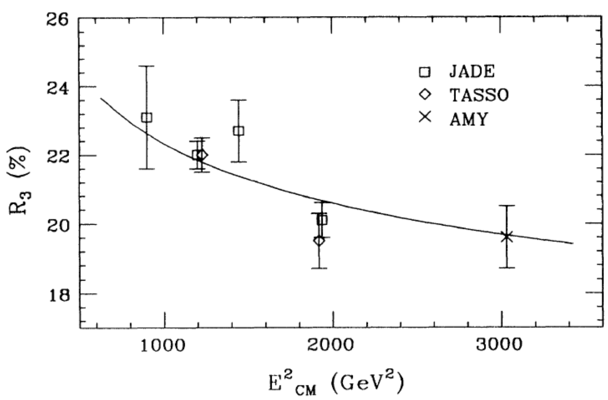


FIG. 20. The 3-jet fraction R_3 as function of the centre-of-mass energy square from different e^+e^- colliders. The decrease is indicative of the running of α_s. From [61].

XIII. A WORD ON CONFINEMENT

Now that we have a solid theory candidate for strong interactions, it is time to go back to the issue mentioned earlier; quarks had never been seen in isolation. Remember that Nambu postulated that they were too heavy to be seen in the sixties –future accelerators would eventually allow one to see them– and that only their (strongly) bound states –the hadrons– were light enough to be produced and seen. Remember also how Gell-Mann was neither much convinced that quarks really existed.

QCD is a non-Abelian gauge theory, where the gauge bosons self interact. We have seen that it exhibits the very nice property of asymptotic freedom at short distances, but also an increase of the coupling at low energies, sometimes referred to as a the infrared slavery. However, we should realise that, as soon as the coupling is becoming large, the perturbative method used to derive the running of the coupling is no longer reliable. Despite this drawback, it has been postulated by many, notoriously S. Weinberg, that infrared slavery would be the explanation of the quark confinement and the impossibility of observing quarks alone.

The usual picture in terms of potential energy between, for instance, one quark and one anti-quark is that the lines of force between the two colour charges are squeezed by the gluon self-interaction within a flux tube or a string. This string acquires a tension – the parameter k which we encountered in the potential describing the quarkonium system. Overall, one expects the QCD potential between two coloured charges to behave at large distance r as

$$V(r) = kr. \tag{22}$$

Now, if we tried to separate both charges sufficiently far away, we would store enough energy to create another $q\bar{q}$ pair for instance. Instead of ending up with separate charges, we would have 2 pions. This is minimally what would happen in $e^+e^- \to q\bar{q} \to \pi^+\pi^-$. Quarks are confined.

All this explanation of confinement supposes that gluons actually self interact. This property is sufficient to get asymptotic freedom, but it is not necessary. A direct observation of the non-Abelian character of strong interactions is therefore welcome.

XIV. EVIDENCE FOR GLUON SELF INTERACTION

The first observation of 4-jet events was done by the JADE experiment based at DESY-PETRA in 1982 [62]. If one enumerates the possible Feynman graphs which could result in 4-jet events, one finds an interesting graph, typical of non-Abelian theories, with a vertex with self-coupling for the gluons (see Fig. 21 (d)).

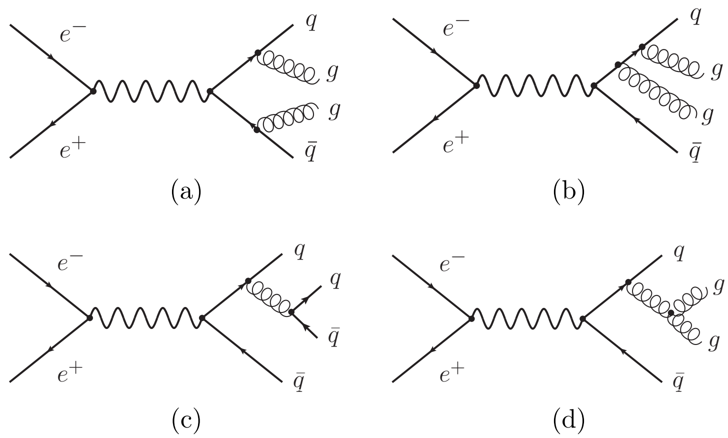
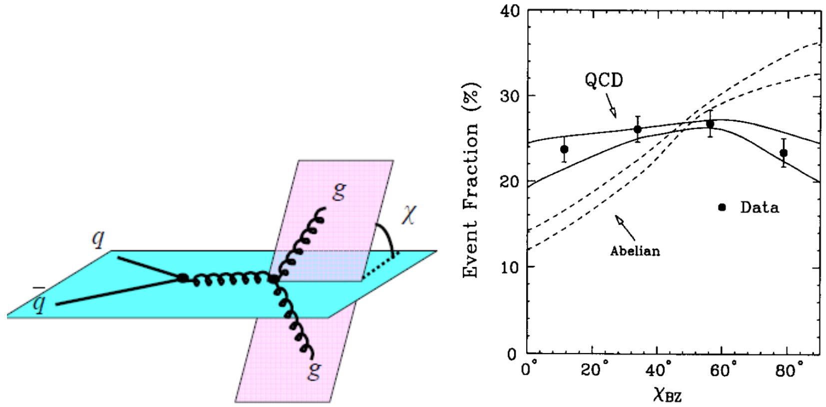


FIG. 21. Representative Feynman graphs for the production of 4 jets in e^+e^- annihilation. The graph (d) is sensitive to gluon self-coupling.

Since such a coupling has a proper Lorentz structure, different from the others in QCD, it produces a specific angular distribution of the jets thanks to which one can get a handle on the gluon self-coupling. On average, it can be checked from Monte Carlo simulations that both jets carrying the least energy out of the four are from the gluons. From their directions, one can define the angle, χ, proposed by Bengtsson and Zerwas [63] between the supposed gluon jets plane and the supposed quark jets plane (Fig. 22 (a)).

However, it took a couple of years, after the first observation of 4-jet events by JADE, before it was found that their distribution in χ was in better agreement with that expected from QCD than from a hypothetical Abelian theory of strong interactions. The first significant evidence was provided by the AMY collaboration [61] with a fit with $\chi^2_{dof} = 0.3$ for QCD and $\chi^2_{dof} = 6.5$ for the Abelian version of QCD. This was further confirmed the LEP experiments, *e.g.* the L3 collaboration in July 1990 (Fig. 22 (b)) [64].



(a) (b)

FIG. 22. (a) Illustration of the angle χ (provided that the 4-jet event is from a genuine $e^+e^- \to q\bar{q}gg$ process and that the two jets carrying the least energy are actually the gluon jets) (b) Comparison between the L3 data and two theoretical bands from Monte Carlo analyses in QCD (solid lines) and in an Abelian version of QCD (dashed lines). From [64].

XV. CONCLUSION

Throughout this lecture note, we have tried to convince you that Quantum Chromodynamics is the best theory we have to account for Strong Interactions. The notion of quarks was introduced in the early sixties; it became tangible a bit later with the results of deep-inelastic scattering and with the discovery of the first heavy-quark bound state. Nearly at the same time, in the early seventies, QCD was introduced and was shown to be strong at low energies and weaker at high energy – the asymptotic freedom was found. Soon after, the quarks could be analysed through the hadron jets they produced once they are created; their spin, 1/2, could be checked. The same happened for the gluons slightly later, followed by evidences that gluons do self-interact, as expected in QCD.

We have seen that the hadrons – the particles which "feel" the strong interactions– are made of (coloured) quarks and gluons. For now, we know that there are 6 quarks: u,d,s (the "light" quark) and c, b, t (the "heavy" quarks). All but the t can form hadronic bound states, which are all well understood within QCD.

Asymptotic freedom of QCD allows us to carry out, using what is called perturbative QCD, systematic computations of scattering and decay processes provided that they are occurring over small distances – these systematic computations rely on Feynman graphs. This has been employed since nearly thirty years to describe reactions at proton-proton, electron-proton and electron-positron colliders. Until now, perturbative QCD has been been able to account for all the existing measurements in the high-energy range. The low-energy and non-perturbative domain is now the realm of effective theories and lattice QCD method. The aim of the latter is to obtain, from the full QCD Lagrangian and by using involved numerical methods, computations of fundamental properties of hadrons, such as their mass, magnetic moment, charge distribution, decay width, etc. With the steady increase of computer power and improvements in the computing algorithms, encouraging results are collected and precision increase are constant.

We would like to complete the picture by adding that QCD also fits very well into the Standard Model; actually it is very close in spirit to the electroweak theory – eventually they could even unify at high energies . . . The only real difficulty remaining with QCD is the lack of an *ab initio* explanation of confinement.

FURTHER READING

- Related Nobel lectures:
  - R. E. Taylor *Deep Inelastic Scattering:, the Early Days*, Rev. Mod. Phys. **63** (1991) 573.
  - H.W. Kendall, *Deep Inelastic Scattering: Experiments on the Proton and the Observation of Scaling*, Rev. Mod. Phys. **63** (1991) 597.
  - J. I. Friedman, *Deep Inelastic Scattering: Comparisons with the Quark Model* Rev. Mod. Phys. **63** (1991) 615.
  - G. 't Hooft, *Nobel lecture: A confrontation with infinity*, Rev. Mod. Phys. **72** (2000) 333
  - M. J. G. Veltman, *Nobel lecture: From weak interactions to gravitation*, Rev. Mod. Phys. **72** (2000) 341;
  - D.J. Gross *The discovery of asymptotic freedom and the emergence of QCD*, Rev. Mod. Phys. **77** (2005) 837;
  - H. D. Politzer, *The dilemma of attribution*, Rev. Mod. Phys. **77** (2005) 851.
  - F. Wilczek, *Asymptotic freedom: From paradox to paradigm*, Rev. Mod. Phys. **77** (2005) 857;
- Other historical accounts, reviews, ...
  - P. Soding, *On the discovery of the gluon*, Eur. Phys. J. H **35** (2010) 3.
  - A. Ali and G. Kramer, *Jets and QCD: A Historical Review of the Discovery of the Quark and Gluon Jets and its Impact on QCD*, Eur. Phys. J. H **36** (2011) 245
  - P. Soding and G. Wolf, *Experimental evidence on QCD*, Ann. Rev. Nucl. Part. Sci. **31** (1981) 231.
- Particle physics, Quantum Field Theory and QCD Textbooks
  - R. Brock *et al.* [CTEQ Collaboration], *Handbook of perturbative QCD: Version 1.0*, Rev. Mod. Phys. **67** (1995) 157.
  - F. Halzen and A. D. Martin, *Quarks And Leptons: An Introductory Course In Modern Particle Physics*, New York, Usa: Wiley (1984) 396p
  - G. F. Sterman, *An Introduction to quantum field theory*, Cambridge, UK: Univ. Pr. (1993) 572 p
  - M. E. Peskin and D. V. Schroeder, *An Introduction to quantum field theory*, Reading, USA: Addison-Wesley (1995) 842 p
  - V. D. Barger and R. J. N. Phillips, *Collider Physics*," Redwood city, USA: Addison-Wesley (1987) 592 P. (Frontiers in Physics, 71)

[1] K. Nakamura *et al.* [Particle Data Group Collaboration], *Review of particle physics*, J. Phys. G **37**, 075021 (2010).

[2] R. Frisch,O. Stern, *Über die magnetische Ablenkung von Wasserstoffmoleklen und das magnetische Moment des Protons. I*, Z. Phys. 85 (1933) 4

[3] L. W. Alvarez and F. Bloch, *A Quantitative Determination of the Neutron Moment in Absolute Nuclear Magnetons*, Phys. Rev. **57** (1940) 111.

[4] G. Zweig, *Memories of Murray and the Quark Model*, Int. J. Mod. Phys. A **25** (2010) 3863

[5] M. Gell-Mann, *Symmetries of baryons and mesons*, Phys. Rev. **125** (1962) 1067.

[6] S. Okubo, *Note on unitary symmetry in strong interactions*, Prog. Theor. Phys. **27**, 949 (1962).

[7] M. Gell-Mann, *The Eightfold Way: A Theory of strong interaction symmetry*, CTSL-20, 1961.

[8] Y. Ne'eman, *Derivation of strong interactions from a gauge invariance*, Nucl. Phys. **26** (1961) 222.

[9] S. Okubo, *Note on Unitary Symmetry in Strong Interaction. II Excited States of Baryons*, Prog. Theor. Phys. **28** (1962) 24.

[10] M. Gell-Mann, *A Schematic Model of Baryons and Mesons*, Phys. Lett. **8** (1964) 214.

[11] G. Zweig, *An* SU_3 *Model for Strong Interaction Symmetry and its Breaking*, CERN Report 8419/TH.401; G. Zweig, *An* SU_3 *Model for Strong Interaction Symmetry and its Breaking II* CERN Report 8419/TH.412, [Symmetries in Elementary Particle Physics (Academic Press, New York, 1965) p.192]

[12] M. Gell-Mann, *The Symmetry Group of Vector and Axial Vector Currents*, Physics 1, 63 (1964).

[13] M. Gell-Mann, *Current Topics in Particle Physics*, Proc. ICHEP 1967, Berkeley, University of California Press, USA.

[14] M. Gell-Mann, *Elementary Particles ?*, Proceedings of the Royal Institution, 41, no. 189 (1966).

[15] O. W. Greenberg, *Spin and Unitary Spin Independence in a Paraquark Model of Baryons and Mesons*, Phys. Rev. Lett. **13** (1964) 598.

[16] M. Y. Han and Y. Nambu, *Three Triplet Model with Double SU(3) Symmetry*, Phys. Rev. **139** (1965) 1006.

[17] D. Gross, *The discovery of asymptotic freedom and the emergence of QCD*, Rev. Mod. Phys, 77 (2005) 837

[18] Bardeen, Fritzsch, Gell-Mann, *Light cone current algebra,* π^0 *decay, and* e^+e^- *annihilation.*, Procs: Scale and Conformal Symmetry in Hadron Physics. Wiley. 1973

[19] H. Fritzsch, M. Gell-Mann and H. Leutwyler, *Advantages of the Color Octet Gluon Picture*, Phys. Lett. B **47** (1973) 365.

[20] H. Fritzsch and M. Gell-Mann, *Current algebra: Quarks and what else ?*, Procs: ICHEP 1972, Chicago, USA, p.135 (J. D. Jackson, A. Roberts, eds.)) eConf C **720906V2** (1972) 135 [hep-ph/0208010].

[21] D. J. Gross and F. Wilczek, *Ultraviolet Behavior of Nonabelian Gauge Theories*, Phys. Rev. Lett. **30** (1973) 1343.

[22] J. I. Friedman, *Deep inelastic scattering: Comparisons with the quark model*, Rev. Mod. Phys. **63** (1991) 615.

[23] M. Breidenbach, *et al.*, *Observed Behavior of Highly Inelastic electron-Proton Scattering*, Phys. Rev. Lett. **23** (1969) 935.

[24] H. W. Kendall, *Deep inelastic scattering: Experiments on the proton and the observation*, Rev. Mod. Phys. **63** (1991) 597.

[25] J. D. Bjorken and E. A. Paschos, *Inelastic Electron Proton and gamma Proton Scattering, and the Structure of the Nucleon*, Phys. Rev. **185** (1969) 1975.

[26] J. I. Friedman and H. W. Kendall, *Deep inelastic electron scattering*, Ann. Rev. Nucl. Part. Sci. **22** (1972) 203.
[27] R. P. Feynman, *The Behavior Of Hadron Collisions At Extreme Energies*, Procs: 3rd International Conference on High Energy Collisions, Stony Brook, N.Y., 5-6 Sep 1969, pp 237-258.
[28] R. P. Feynman, *Very high-energy collisions of hadrons*, Phys. Rev. Lett. **23** (1969) 1415.
[29] C. G. Callan, Jr. and D. J. Gross, *High-energy electroproduction and the constitution of the electric current*, Phys. Rev. Lett. **22** (1969) 156.
[30] D. Griffiths, *Introduction to elementary particles*, Weinheim, Germany: Wiley-VCH (2008) 454 p.
[31] T. Eichten, *et al.Measurement Of The Neutrino - Nucleon Anti-neutrino - Nucleon Total Cross-sections*, Phys. Lett. B **46** (1973) 274.
[32] A. Bodek, *et al.*, *Experimental Studies of the Neutron and Proton Electromagnetic Structure Functions*, Phys. Rev. D **20** (1979) 1471.
[33] H. D. Politzer, *Reliable Perturbative Results for Strong Interactions ?*, Phys. Rev. Lett. **30** (1973) 1346.
[34] C. Bernardini, *AdA:The first electron-positron collider*, Phys. Perspect. **6** (2004) 156.
[35] J. D. Bjorken, *Applications of the Chiral U(6) x (6) Algebra of Current Densities*, Phys. Rev. **148** (1966) 1467.
[36] N. Cabibbo, G. Parisi and M. Testa, *Hadron Production in e^+e^- Collisions*, Lett. Nuovo Cim. **4S1** (1970) 35 [Lett. Nuovo Cim. **4** (1970) 35].
[37] S. L. Glashow, J. Iliopoulos and L. Maiani, *Weak Interactions with Lepton-Hadron Symmetry*, Phys. Rev. D **2** (1970) 1285.
[38] J. D. Bjorken and S. L. Glashow, *Elementary Particles and SU(4)*, Phys. Lett. **11**, 255 (1964).
[39] B. Richter, *e^+ e^- $\rightarrow$ Hadrons* , ICHEP 1974, London, England, July 1-10, 1974. SLAC-PUB-1478.
[40] J. J. Aubert *et al.* [E598 Collaboration], *Experimental Observation of a Heavy Particle J*, Phys. Rev. Lett. **33** (1974) 1404.
[41] J. E. Augustin *et al.* [SLAC-SP-017 Collaboration], *Discovery of a Narrow Resonance in e^+e^- Annihilation* , Phys. Rev. Lett. **33** (1974) 1406.
[42] H. L. Lynch, *et al.*, *Recent Results for e^+e^- Annihilation at SPEAR*, Procs: 2nd Orbis Scientiae, Coral Gables, Florida, Jan 20-25, 1975.
[43] F. Close, *A November revolution: the birth of a new particle*, CERN Courier, **44**, no. 10, (2004) 25
[44] G. S. Abrams, *et al.*, *The Discovery of a Second Narrow Resonance in e^+e^- Annihilation*, Phys. Rev. Lett. **33** (1974) 1453.
[45] T. Appelquist, A. De Rujula, H. D. Politzer and S. L. Glashow, *Charmonium Spectroscopy*, Phys. Rev. Lett. **34** (1975) 365.
[46] G. Hanson, *et al. Evidence for Jet Structure in Hadron Production by e^+e^- Annihilation*, Phys. Rev. Lett. **35** (1975) 1609.
[47] R. Brandelik *et al.* [TASSO Collaboration], *Evidence for Planar Events in e^+e^- Annihilation at High-Energies*, Phys. Lett. B **86** (1979) 243.
[48] C. Berger *et al.* [PLUTO Collaboration], *Evidence for Gluon Bremsstrahlung in e^+e^- Annihilations at High-Energies*, Phys. Lett. B **86** (1979) 418.
[49] W. Bartel *et al.* [JADE Collaboration], *Observation of Planar Three Jet Events in e^+e^- Annihilation and Evidence for Gluon Bremsstrahlung*, Phys. Lett. B **91** (1980) 142.
[50] D. P. Barber, *et al.*, *Discovery of Three Jet Events and a Test of Quantum Chromodynamics at PETRA Energies*, Phys. Rev. Lett. **43** (1979) 830.
[51] J. D. Bjorken and S. J. Brodsky, *Statistical Model for electron-Positron Annihilation Into Hadrons*, Phys. Rev. D **1** (1970) 1416.
[52] J. R. Ellis and I. Karliner, *Measuring the Spin of the Gluon in e^+e^- Annihilation*, Nucl. Phys. B **148** (1979) 141.

[53] R. Brandelik *et al.* [TASSO Collaboration], *Evidence for a Spin One Gluon in Three Jet Events*, Phys. Lett. B **97** (1980) 453.

[54] T. Appelquist and H. Georgi, *e^+e^- annihilation in gauge theories of strong interactions*, Phys. Rev. D **8** (1973) 4000.

[55] A. Zee, *Electron positron annihilation in stagnant field theories*, Phys. Rev. D **8** (1973) 4038.

[56] J. Siegrist, *Hadron Production By e^+e^- Annihilation At Center-Of-Mass Energies Between 2.6-GeV And 7.8-GeV*, PhD Thesis, Oct 1979. 163 pp.

[57] S. W. Herb, *et al.*, *Observation of a Dimuon Resonance at 9.5-GeV in 400-GeV Proton-Nucleus Collisions*, Phys. Rev. Lett. **39** (1977) 252.

[58] M. L. Perl, *et al.*, *Evidence for Anomalous Lepton Production in e^+e^- Annihilation*, Phys. Rev. Lett. **35** (1975) 1489.

[59] W. R. Innes, *et al.*, *Observation of structure in the Υ region*, Phys. Rev. Lett. **39** (1977) 1240 [Erratum-ibid. **39** (1977) 1640].

[60] J. Siegrist, *et al.*, *Hadron Production by e^+e^- Annihilation at Center-Of-Mass Energies Between 2.6-GeV and 7.8-GeV. Part 1. Total Cross-Section, Multiplicities and Inclusive Momentum Distributions*, Phys. Rev. D **26** (1982) 969.

[61] I. H. Park *et al.* [AMY Collaboration], *Experimental Evidence For The Nonabelian Nature Of Qcd*, Phys. Rev. Lett. **62** (1989) 1713.

[62] W. Bartel *et al.* [JADE Collaboration], *OBSERVATION OF FOUR - JET STRUCTURE IN e^+e^- ANNIHILATION AT s**(1/2) = 33-GeV*, Phys. Lett. B **115** (1982) 338.

[63] M. Bengtsson and P. M. Zerwas, *Four Jet Events in e^+e^- Annihilation: Testing the Three Gluon Vertex*, Phys. Lett. B **208** (1988) 306.

[64] B. Adeva *et al.* [L3 Collaboration], *A Test of QCD based on four jet events from Z0 decays*, Phys. Lett. B **248** (1990) 227.