Phase transitions from nuclei to stars

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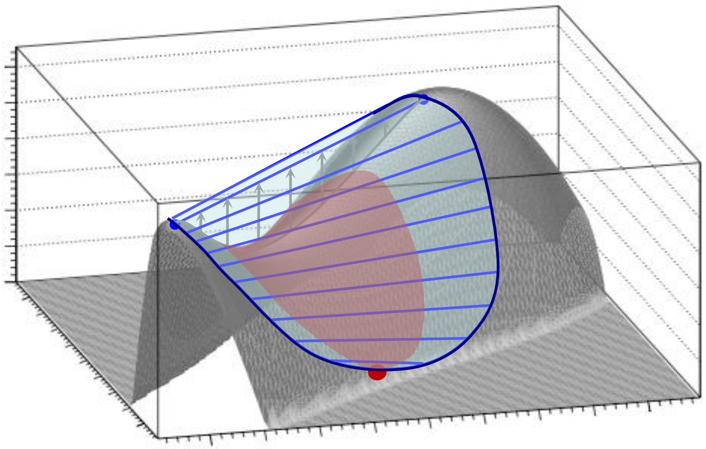


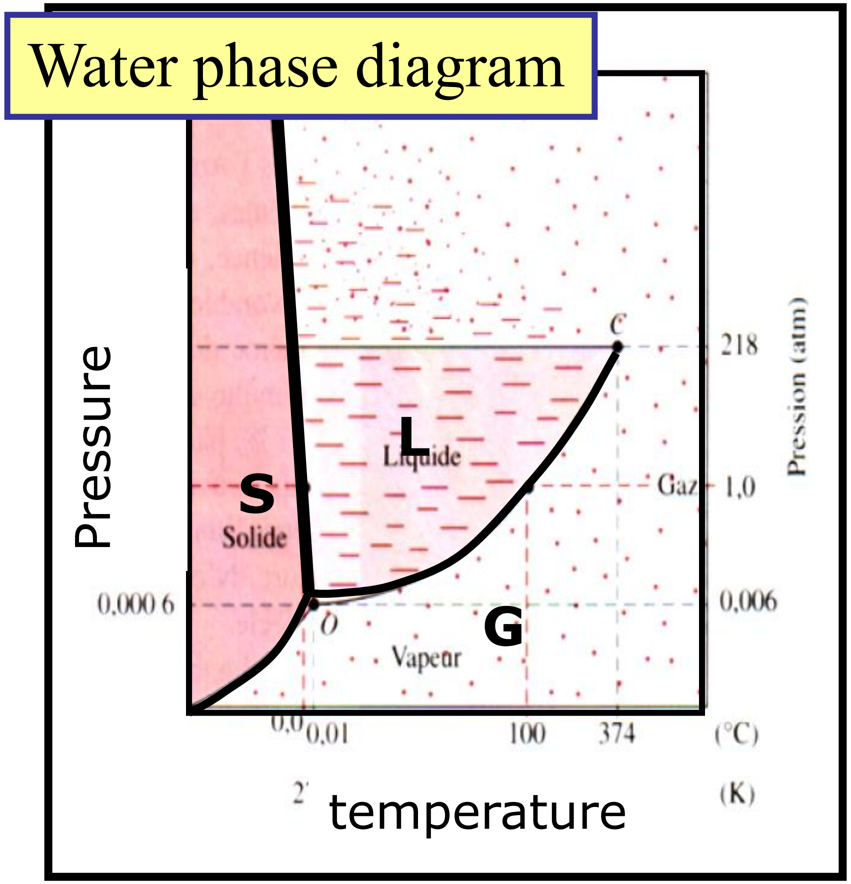
Plan

1. Thermal and Quantum phase transitions
   - Concepts and definitions
   - Classical theory: Landau
   - Thermal versus Quantum
2. Phase transitions in nuclear physics
   - Symmetry breaking and shape transitions
   - Superfluidity and pairing transition
   - Liquid-Gas and multifragmentation
3. Phase transitions in finite nuclei
   - Finite size effects: transition rounding
   - Yang-Lee zeroes
   - Bimodality
4. Supernova and neutron star matter
   - Phase transitions in stellar matter
   - Frustration and dishomogeneous phases

1 - Thermal and Quantum phase transitions

What is a phase transition?

- A small variation of a control parameter induces a dramatic qualitative modification of the system properties.



Microscopic description of phase transitions

- **Macrostate:** **ensemble of all microstates $|\psi\rangle$ satisfying some given constraints**
- **Controlled Variables :** **extensive (*Observables*) and intensive (*Lagrange*)**
- **Entropy:** *(Shannon)*

  minimal biais : max(Ent)

$$\Psi = \sum_n p^{(n)}_{\vec{\alpha},\vec{B}} \left|\psi^{(n)}\right\rangle$$

$$\hat{D}_{\vec{\alpha},\vec{B}} = \sum_n \left|\psi^{(n)}\right\rangle p^{(n)}_{\vec{\alpha},\vec{B}} \left\langle\psi^{(n)}\right|$$

$$B_j^{(n)} = \left\langle\psi^{(n)}\right|\hat{B}_j\left|\psi^{(n)}\right\rangle \quad j=1,\ldots,m$$

$$\alpha_\ell = \alpha_\ell\left(\left\langle\hat{A}_\ell\right\rangle\right) \quad \ell=1,\ldots,r$$

$$S = -Tr\hat{D}\log\hat{D}$$
$$= \max$$

An equilibrium is the statistical ensemble of microstates which maximizes the statistical entropy under a given set of constraints

$$\hat{D}_{\vec{\alpha},\vec{B}} = Z^{-1} \sum_{B_j^{(n)}=B_j} \left|\psi^{(n)}\right\rangle e^{-\sum_\ell \alpha_\ell A_\ell^{(n)}} \left\langle\psi^{(n)}\right|$$

- **Partition sum :**
  sum of all the physical partitions of the system

- **Equations of state:**
  relation between extensive and intensive variables

$$Z(\vec{\alpha}, <\hat{\vec{B}}>) = \underset{<\hat{\vec{B}}>=cte}{Tr} \; e^{-\sum_\ell \alpha_\ell \hat{A}_\ell}$$

$$<\hat{A}_\ell> = -\partial_{\alpha_\ell} \log Z$$

$$\beta_\ell = \partial_{\langle \hat{B}_\ell \rangle} S$$

If Log Z is an analytic function, then observables <A> vary continuously with control parameters α (ex:**V**=nRT/**P**)

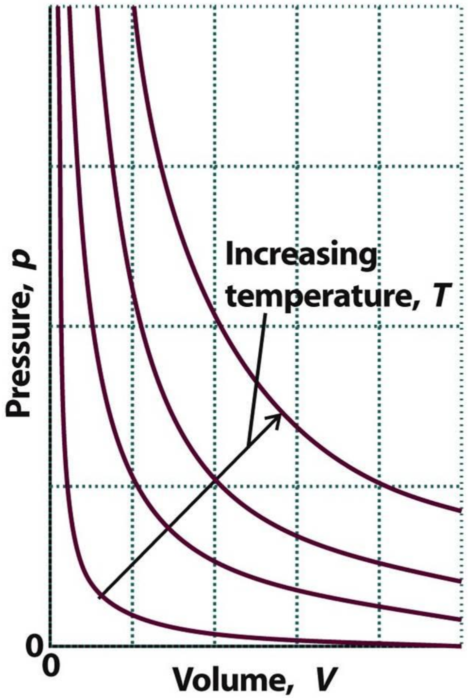


Figure 1-4
Atkins Physical Chemistry, Eighth Edition

What is a phase transition?

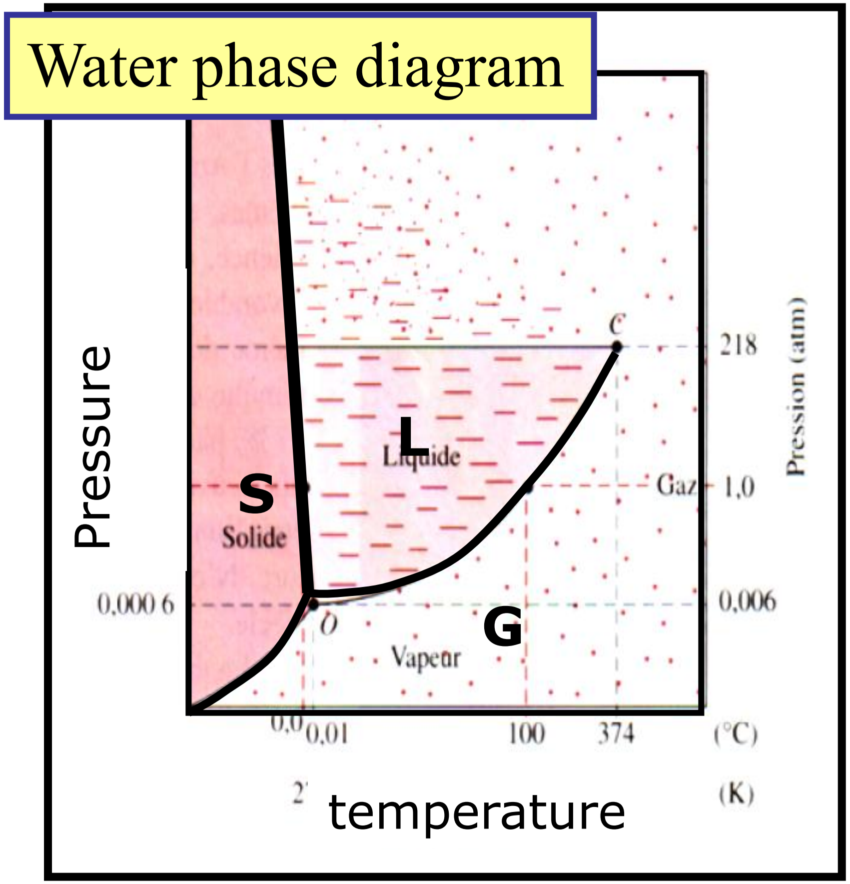
- A small variation of a control parameter **(α)** induces a dramatic qualitative modification of the system properties **(<A>)**

⇒ « accident » in an Equation of State

$$< A >= -\partial_{\alpha} \log Z$$

⇒ Non-analyticity of the partition sum

⇒ <A> order parameter of the transition



Definition of a phase transition (thermo limit)

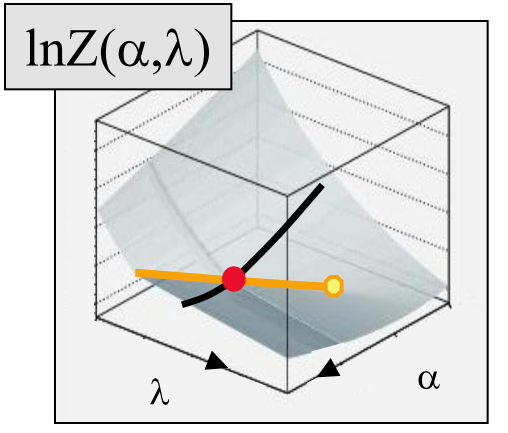
- Non-analyticity of the partition sum

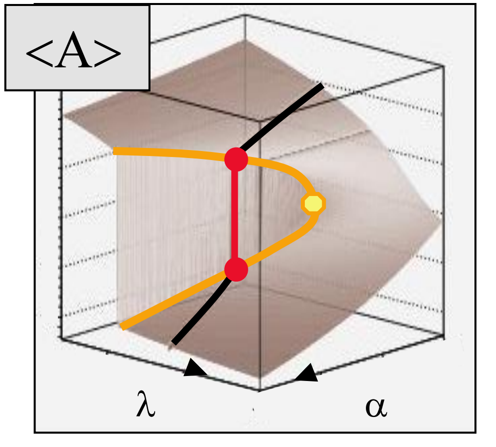
$$N \to \infty \qquad Z = \sum_{(n)} e^{-\sum_\ell \alpha_\ell A_\ell^{(n)}}$$

- Order of the transition: discontinuity (or divergence) in $\partial_\alpha^n \log Z$
- **First order**: order parameter jumps $< A >= -\partial_\alpha \log Z$
- **Second order**: <A> continuous but divergent fluctuations

$$\sigma_A^2 = \partial_\alpha^2 \log Z$$

- Why do discontinuities occur? what is the physics behind these jumps?





Landau theory of phase transitions

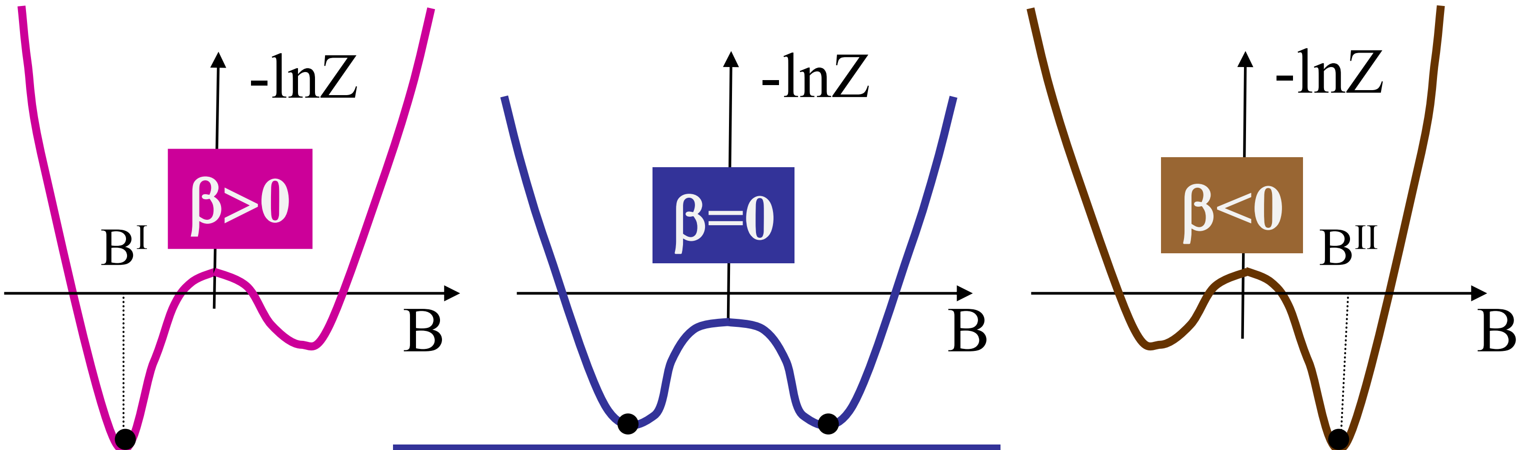
Lev Landau, 1936

- Constrained entropy=thermo potential
- Series developement around the transition point B–B^{II} => B = 0
- two minima of equal depth => a line of first order transition

$$-\log Z_{\beta\lambda} = -S + \beta B + \lambda L = -S_c^{\lambda}(B) \quad = \min$$

$$\log Z_{\beta\lambda} = -\beta B + \frac{1}{2} a_\lambda B^2 + \frac{1}{3} b_\lambda B^3 + \frac{1}{4} c_\lambda B^4 + \dots$$

$$a = a_0 + a_1 \lambda \rightarrow B = \begin{cases} B^I & \beta < 0 \\ B^{II} & \beta > 0 \end{cases}$$



First order transition:
two minima in a generalized potential energy surface

Landau theory : second order

Lev Landau, 1936

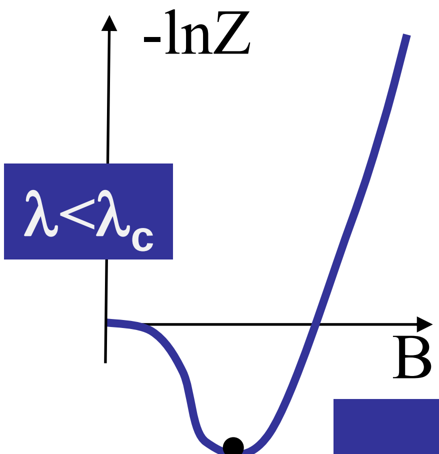
- Constrained entropy =thermo potential
- Series developement around the transition point B–B^{II} => B = 0
- Symmetry B⇔–B

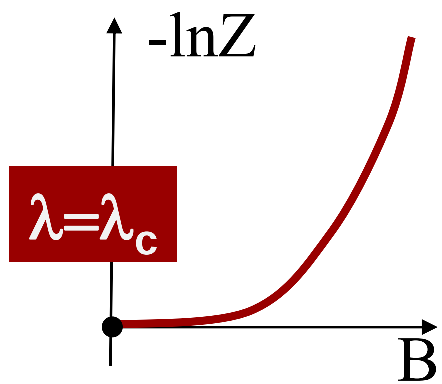
=> a (isolated) second order transition

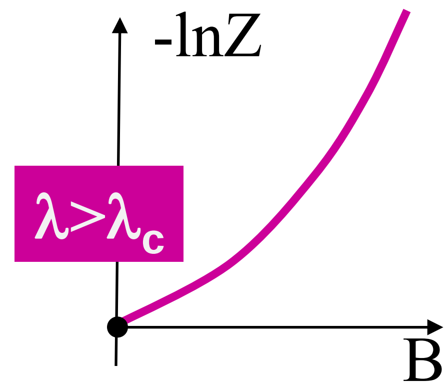
$$-\log Z_{\beta\lambda} = -S + \beta B + \lambda L = -S_c^{\lambda}(B) \quad = \min$$

$$\log Z_{\beta\lambda} = \cancel{-\beta B} + \frac{1}{2} a_\lambda B^2 + \cancel{\frac{1}{3} b_\lambda B^3} + \frac{1}{4} c_\lambda B^4 + \dots$$

$$a = a_0(\lambda - \lambda_c) \rightarrow B = \pm\sqrt{\frac{a_0}{c}}(\lambda_c - \lambda)^{1/2}$$







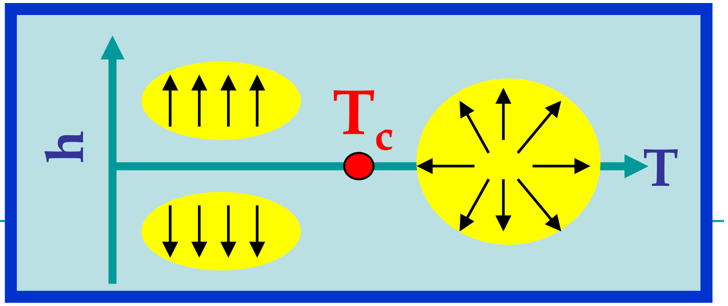
Second order transition:
Spontaneous symmetry breaking

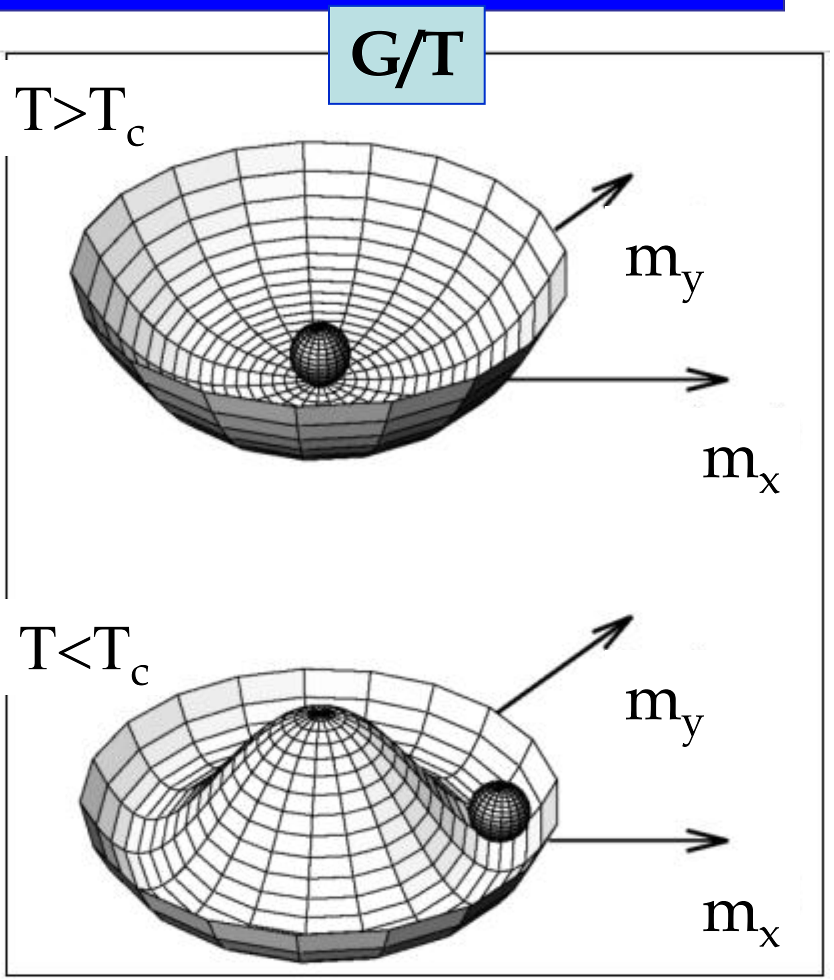
Example of Thermal PT

- **Standard Ferromagnetic material (eg:Fe) with**
  h=magnetic field =0
  β=inverse temperature

$$-\ln Z = -\underset{h=0}{Tr}\, e^{-\beta \hat{H}} = G/T = \min$$

=> ferro/para THERMAL transition at the Curie temperature



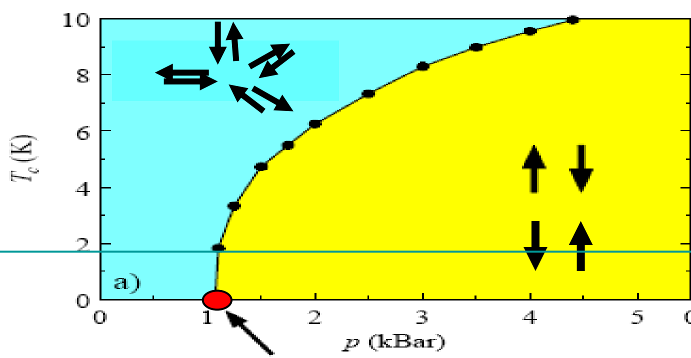


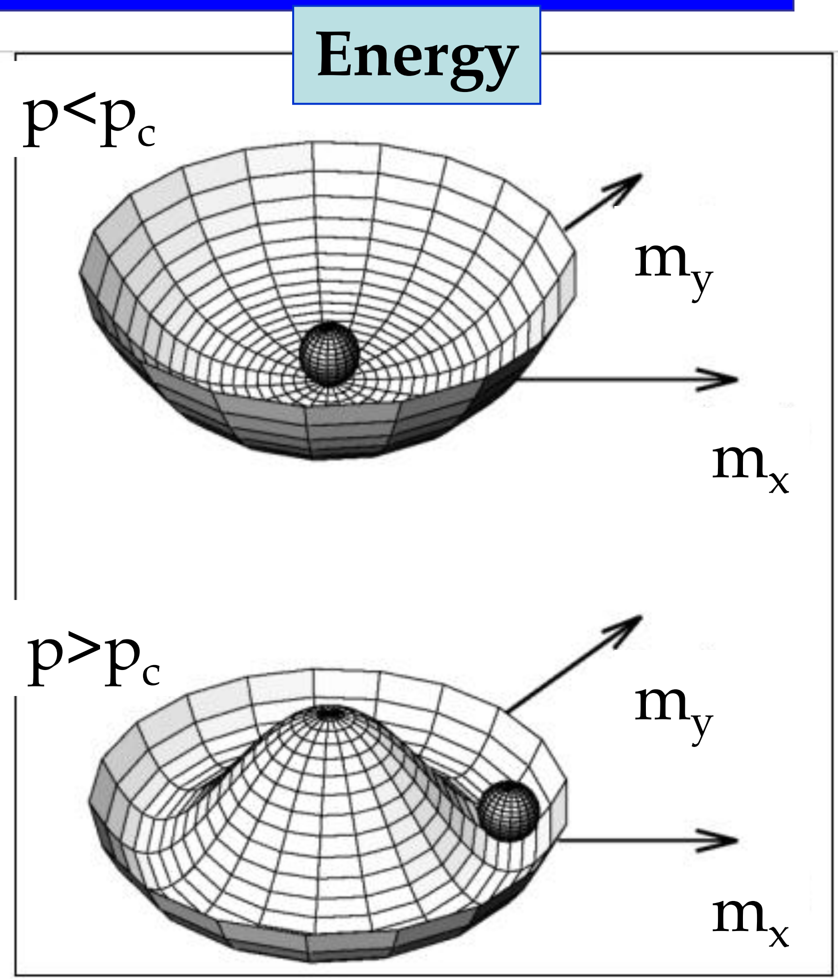
Example of Quantum PT

- **Composite insulator ($TlCuCl_3$) with**
  p=pressure
  β=inverse temperature

$$-\ln Z_{T=0} = -\lim_{\beta\to\infty} \frac{1}{\beta} \ln \underset{p}{Tr}\, e^{-\beta \hat{H}} = E = \min$$

=> (anti)**ferro/para QUANTUM transition at the critical pressure**





Sachdev, Subir (2011). *Quantum Phase Transitions*

What is the specificity of a quantum phase transition ?

- Thermal: T>0 => classical physics
- Quantum: T=0

BUT

- From the microscopic viewpoint, T is a Lagrange among others
- T>0 classical **if and only if** T>>e*, and quantum mechanics obviously needed both for ground and excited nuclear states

⇒**No principle difference between quantum and thermal PT in the microscopic world !**

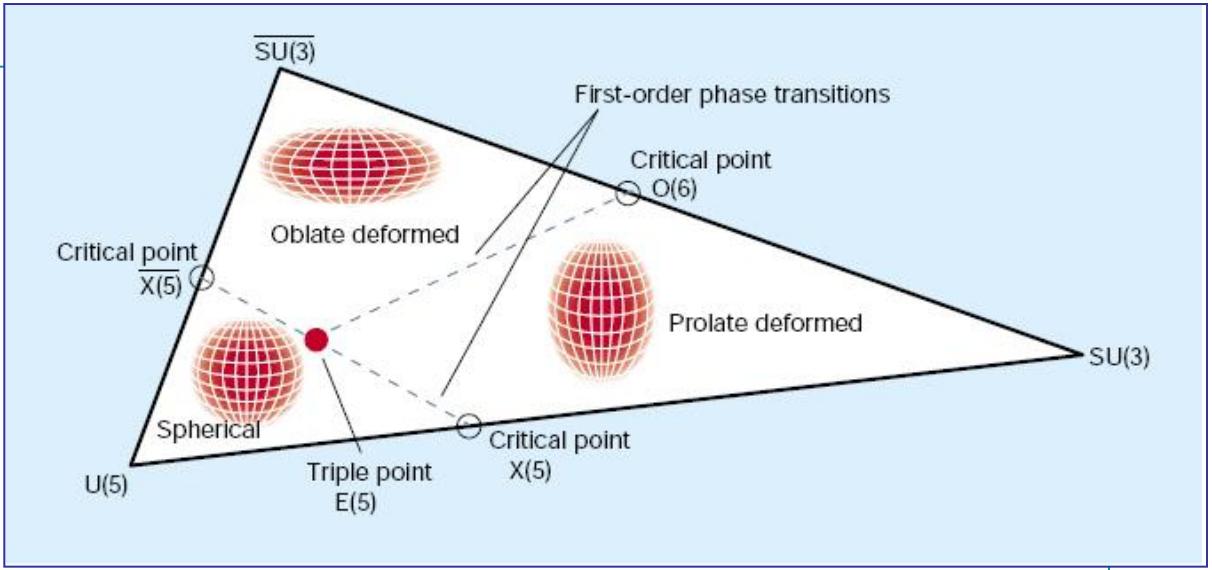
⇒**But the absence of thermodynamic limit is an issue**

2- Phase transitions in nuclear physics

- Shape transitions
- Pairing transition
- Liquid-gas transition

Shape transitions

Extended Casten triangle in IBM

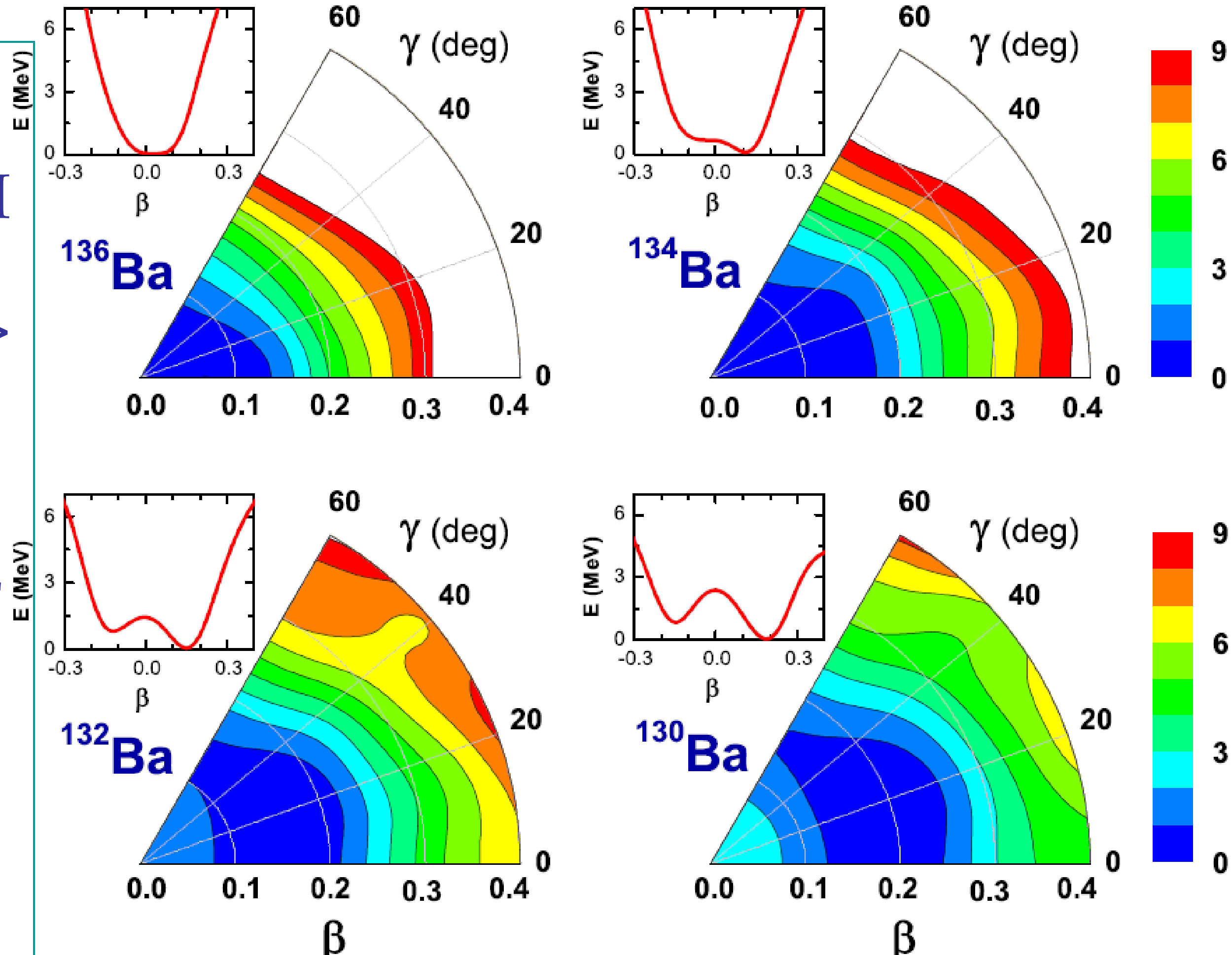


- 1st and 2nd order QPT
- Couplings as intensive control parameters
- Could describe the shape change along isotopic chains

Jolie, J. *et al.* *Phys. Rev. Lett.* 89, 182502 (2002).

Shape transitions

- N and J projected RMF+BCS+GCM
- 2nd order QPT spherical U(5) => γ–soft 0(6) transition through E(5)
- Neutron number as control parameter
- <Q> as order parameter: no bulk limit



Z.P.Li et al, arXiv:1003.4109v1 [nucl-th]

Shape transitions

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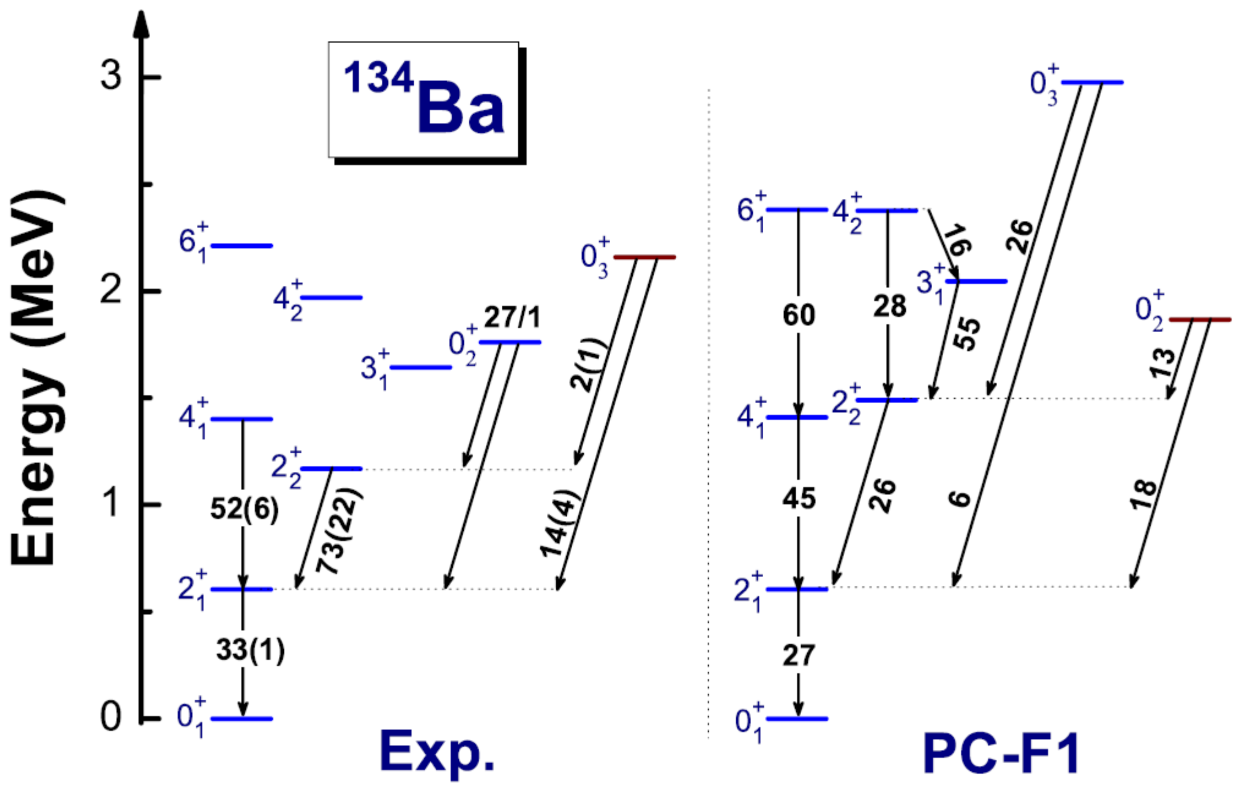
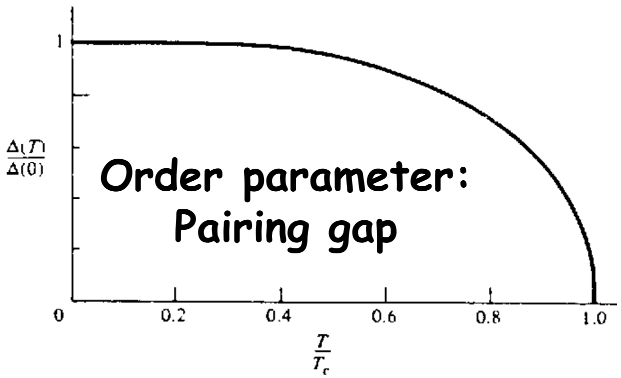


Z.P.Li et al, arXiv:1003.4109v1 [nucl-th]

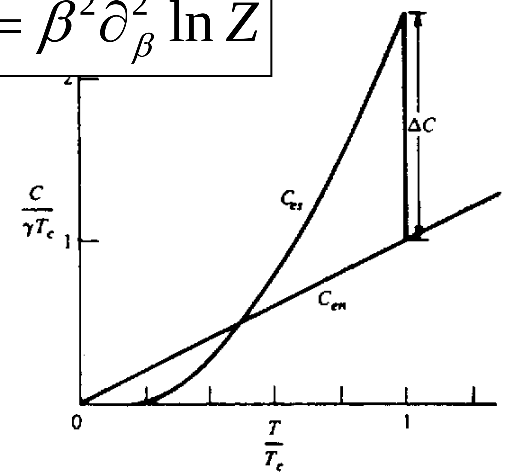
Pairing transition

BCS theory of superconductivity

- predicts a 2nd order TPT
- Could describe vanishing of nuclear pairing correlations with increasing excitation



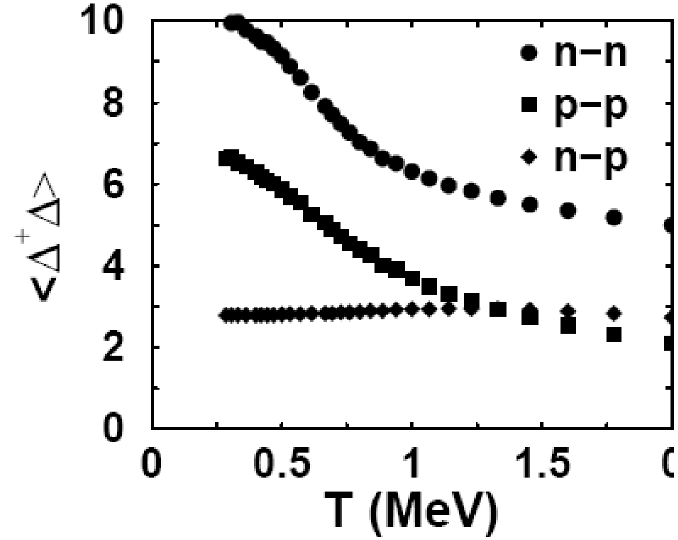
$$C = \beta^2 \partial_\beta^2 \ln Z$$

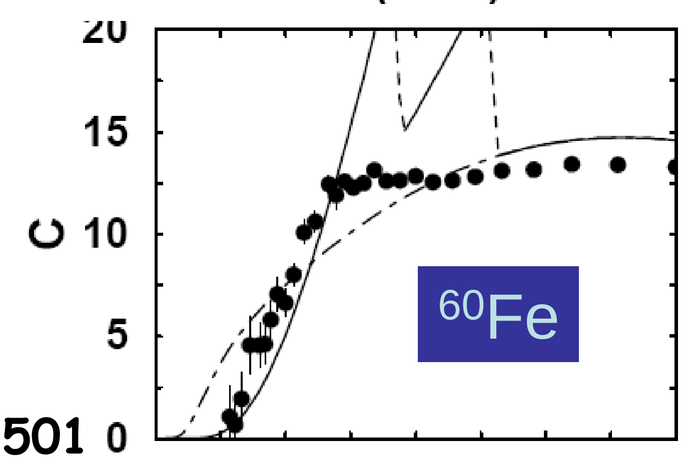


Pairing transition

- Shell model MC for even Fe isotopes

=> Vanishing of pairing correlation as a superfluid-normal fluid TPT smoothed by finite size effects ?





S.Liu,Y.Alhassid (2001) Phys.Rev.Lett.87,022501

Pairing transition

Oslo data

- Shell model MC for even Fe isotopes

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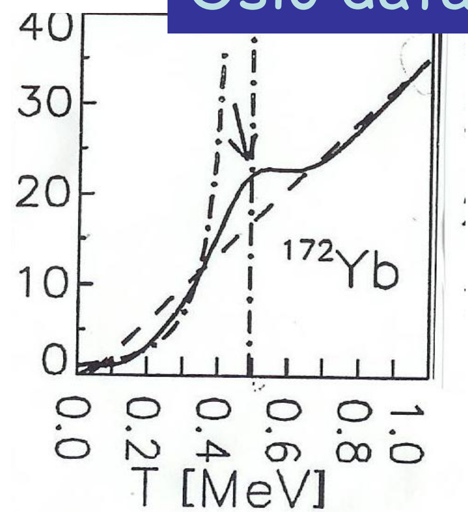
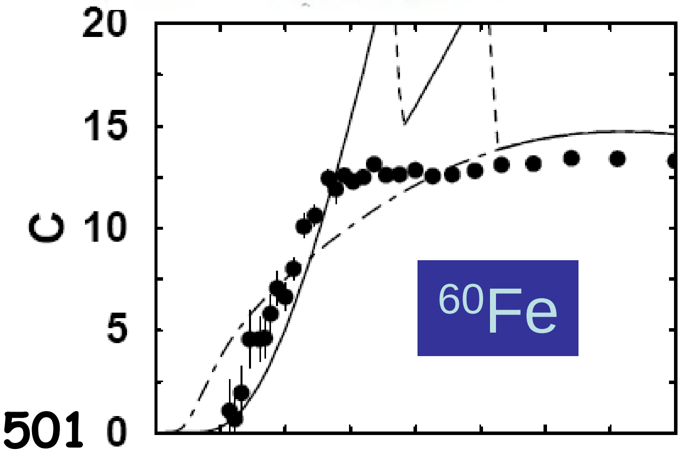


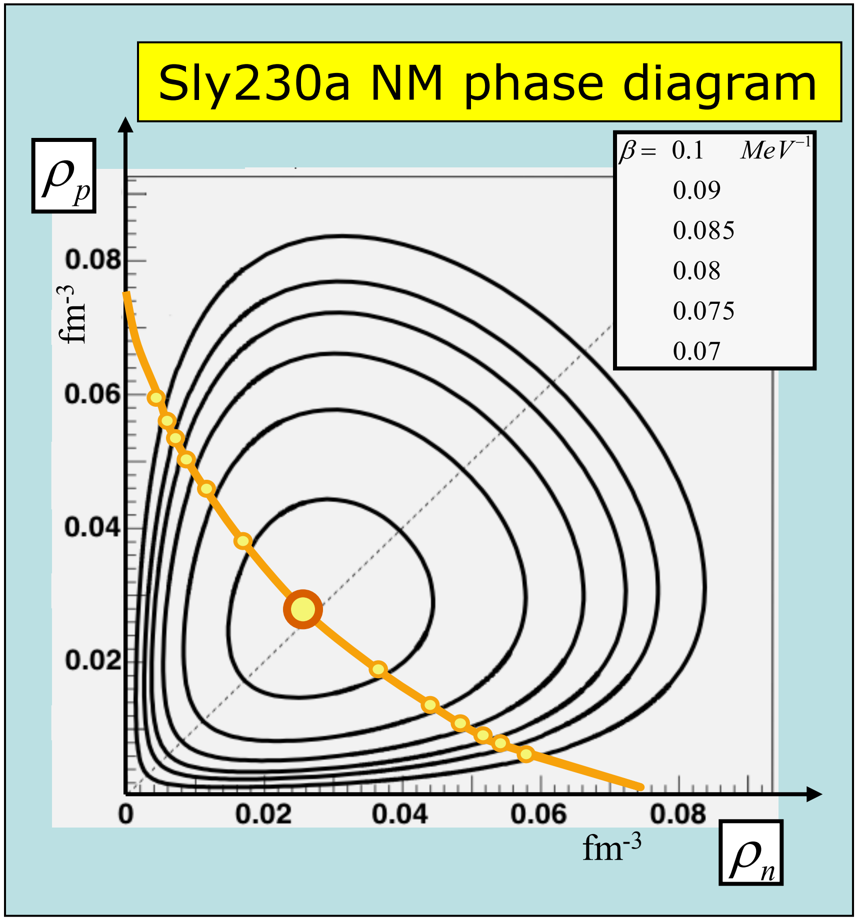


A. Schiller et al., PRC63, 021306(R)(2001).

S.Liu,Y.Alhassid (2001) Phys.Rev.Lett.87,022501

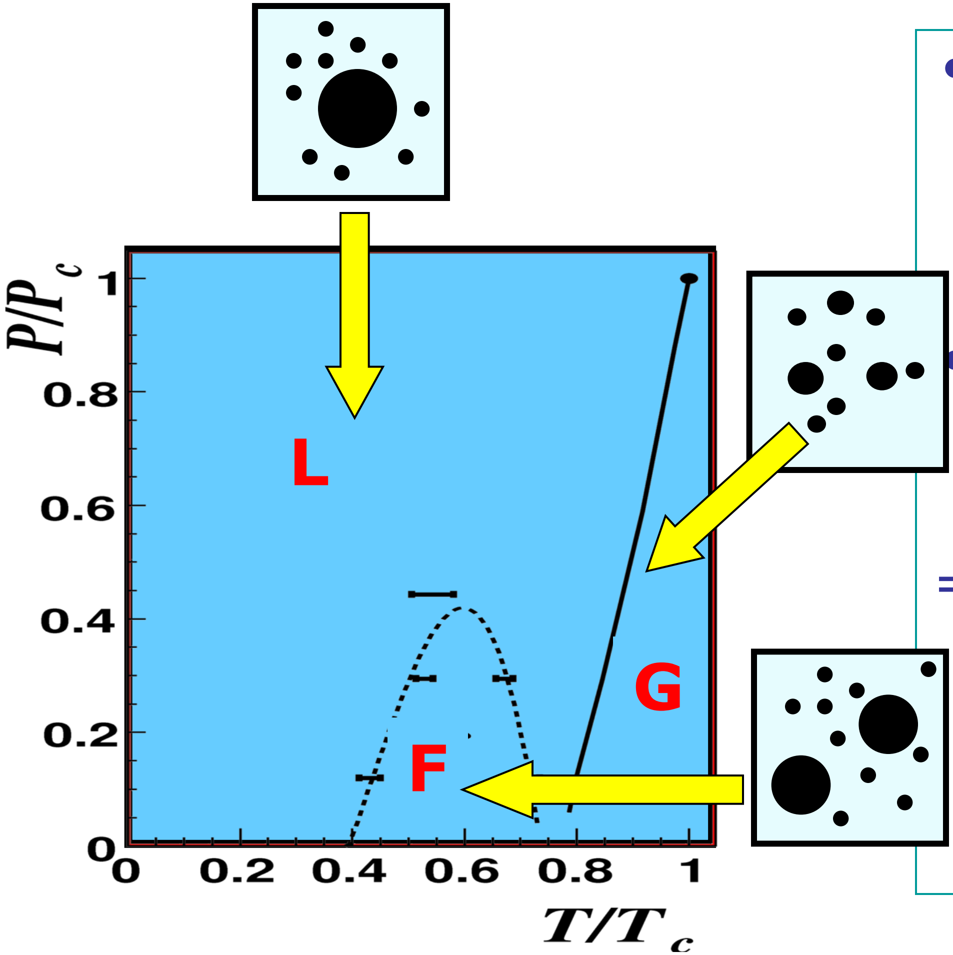
Liquid-Gas transition

- Low density Nuclear Matter belongs to the Liquid-Gas universality class
- First and second order **thermal and quantum phase transitions**
- **Could describe nuclear multifragmentation**



C.Ducoin, P.Chomaz, F.Gulminelli Nucl.Phys.A 771 (2006) 68.

Fragmentation transition

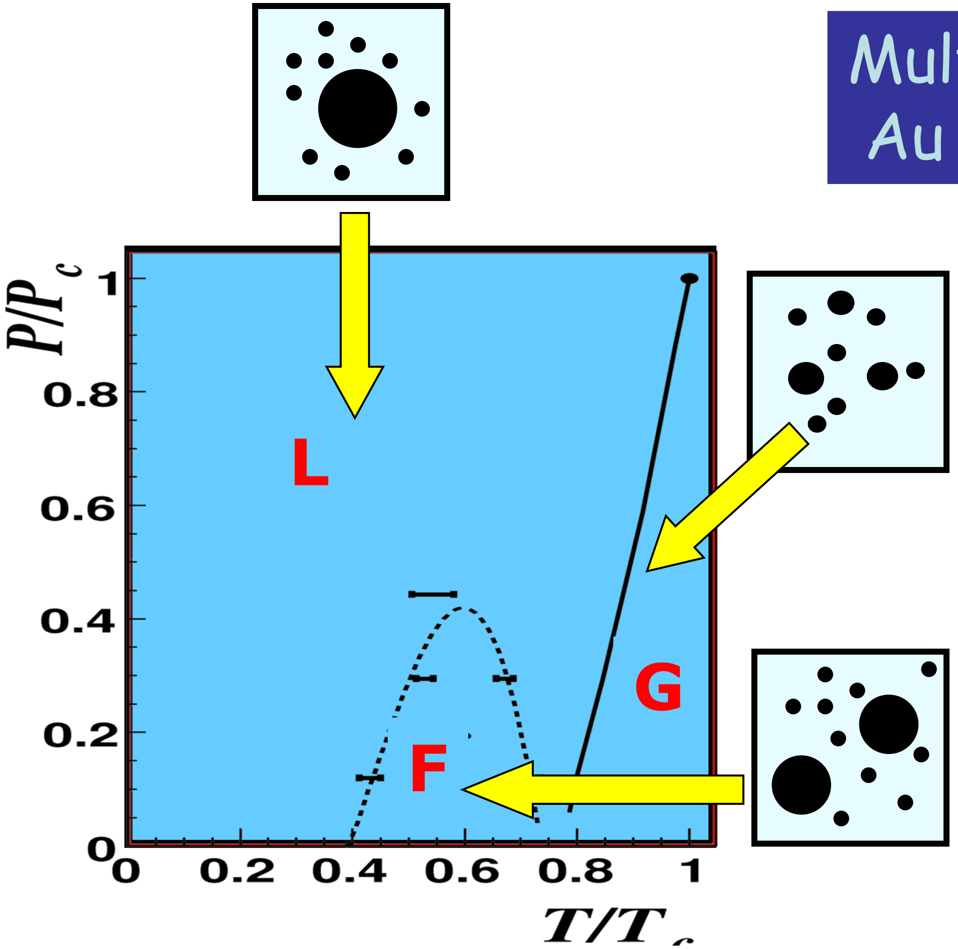


- **Ising model** with short and long range couplings and continuum states
- Size of the heaviest fragment as order parameter

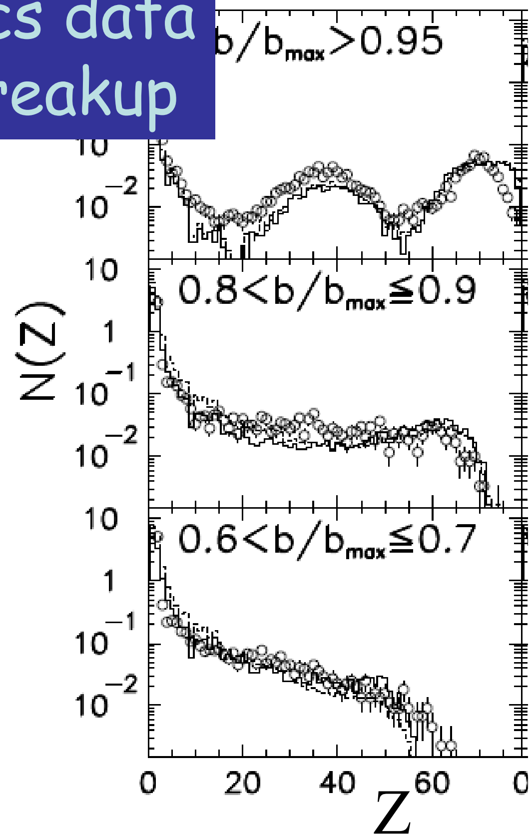
=> Nuclear multifragmentation as the finite system counterpart of LG?

G.Lehaut, F.Gulminelli, O.Lopez,PRL102, 142503 (2009) and PRE 81 (2010) 051104

Fragmentation transition



Multics data
Au breakup



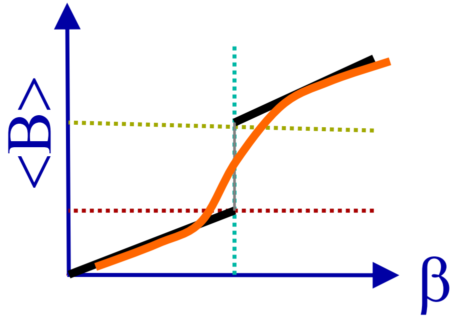
M.D'Agostino et al., Nucl.Phys. A650 (1999) 329

3- Phase transitions in finite nuclei ?

Phase transitions in finite systems

- Z analytic: transition rounded

$$Z(\beta) = \sum_{n=1}^{N} e^{-\sum_\ell \beta_\ell B_\ell^{(n)}}$$

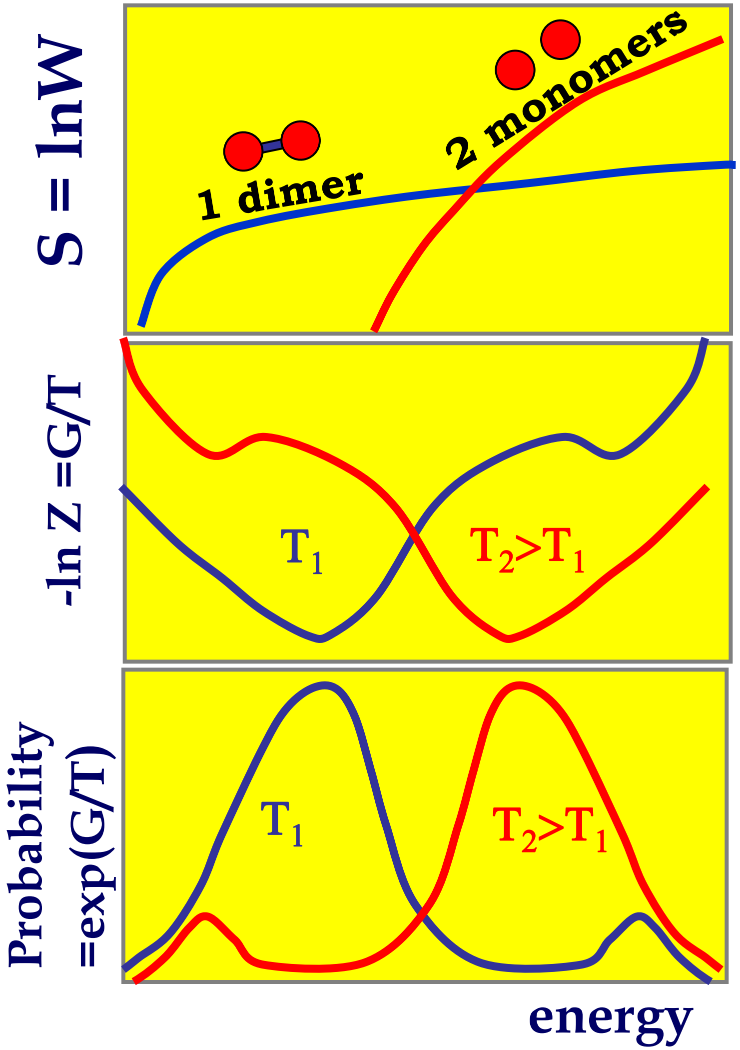


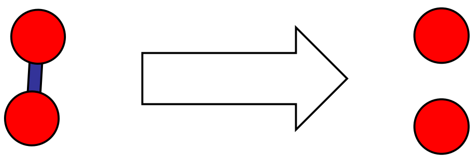
PROBLEM:

- how to distinguish a PT from a cross-over?
- how to distinguish a PT from a channel opening?

Phase transition or channel opening?

F.Gulminelli, Ann.Phys.Fr.29(2004)6



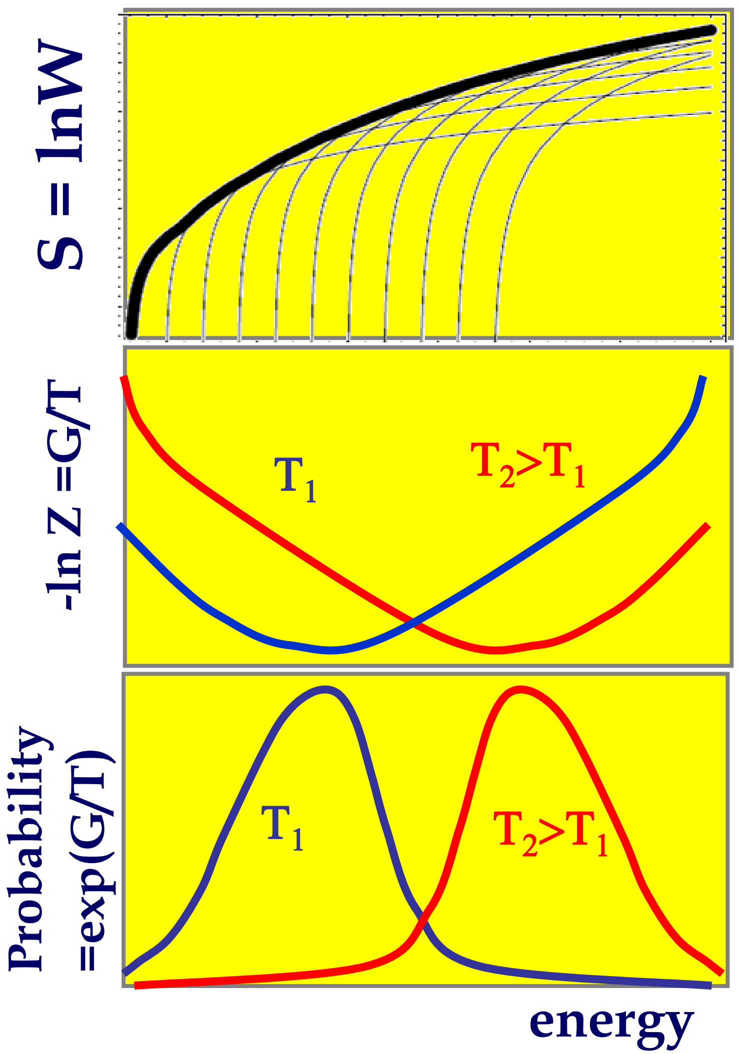


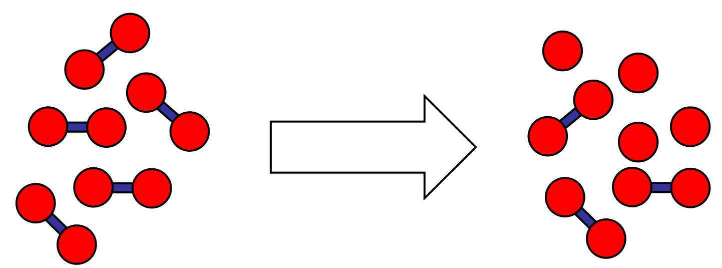
$$W_{monomer}(e) = \frac{1}{3!}\left(\frac{\pi^{3/2}}{h^3}V\left(2me\right)^{3/2}\right)^2 \theta(e)$$

$$W_{dimer}(e) = \frac{\pi^{3/2}V}{(3/2)!h^3}\left(4m\left(e+\varepsilon\right)\right)^{3/2}\theta\left(e+\varepsilon\right)$$

Phase transition or channel opening?

F.Gulminelli, Ann.Phys.Fr.29(2004)6



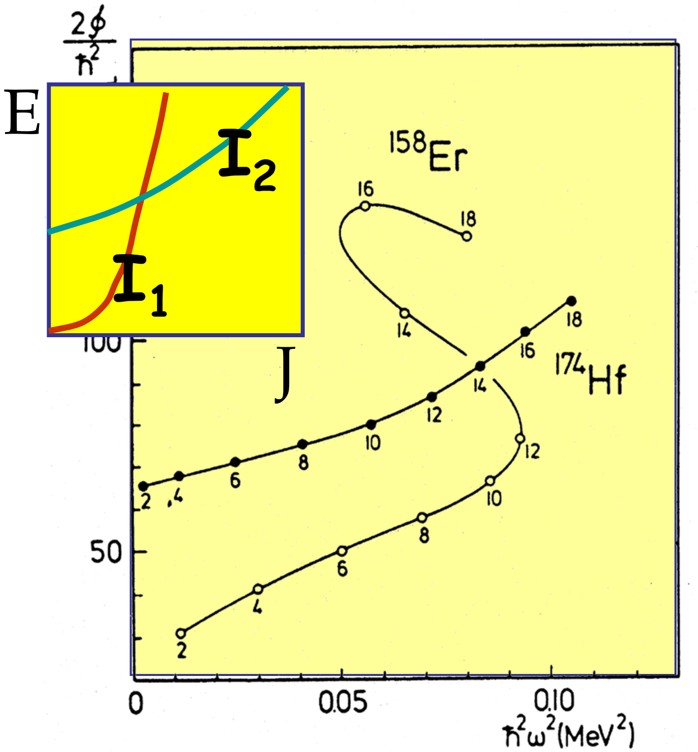


$$W_{monomer}(e) = \frac{1}{3!}\left(\frac{\pi^{3/2}}{h^3} V (2me)^{3/2}\right)^2 \theta(e)$$

$$W_{dimer}(e) = \frac{\pi^{3/2} V}{(3/2)!\, h^3} \left(4m(e+\varepsilon)\right)^{3/2} \theta(e+\varepsilon)$$

Single bond breaking is NOT
a phase transition!
(e.g. isomerization)

Example: the rare earth region

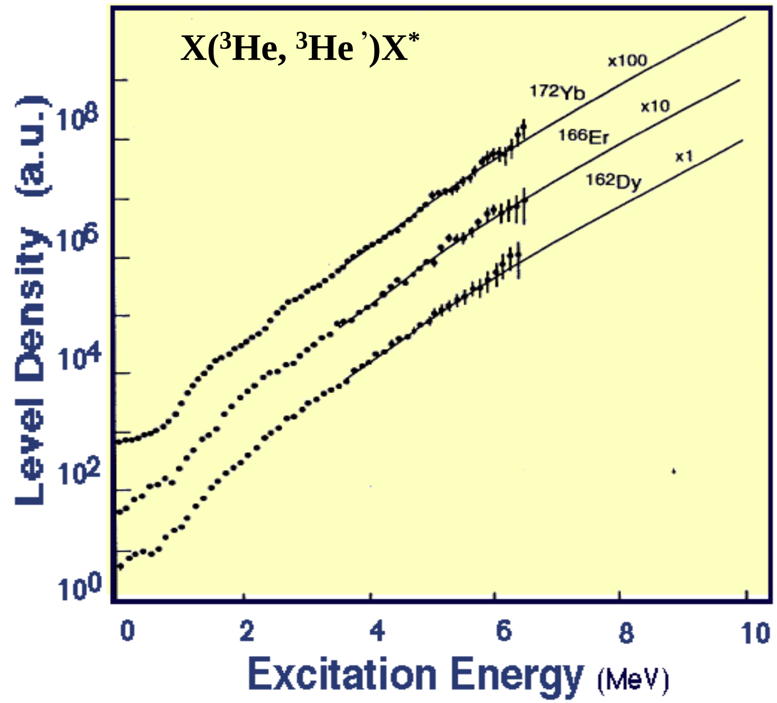


Backbending at high spin:

Crossing between two rotational bands with different moment of inertia

- single pair rotational alignement

Stephens, Simon NPA183(1972)



Level density (Oslo group)

- Breaking of single Cooper pairs

A.Schiller et al.,PRC63,021306(R)(2001).

The Yang-Lee theorem

Phase transition: thermo potential $-\log Z(\beta)$ $\left(Z(\beta) = \sum_{n=1}^{N} e^{-\sum_\ell \beta_\ell B_\ell^{(n)}} \right)$

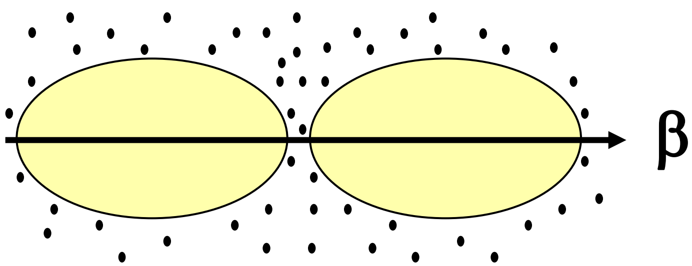
non analytic for $N \to \infty$

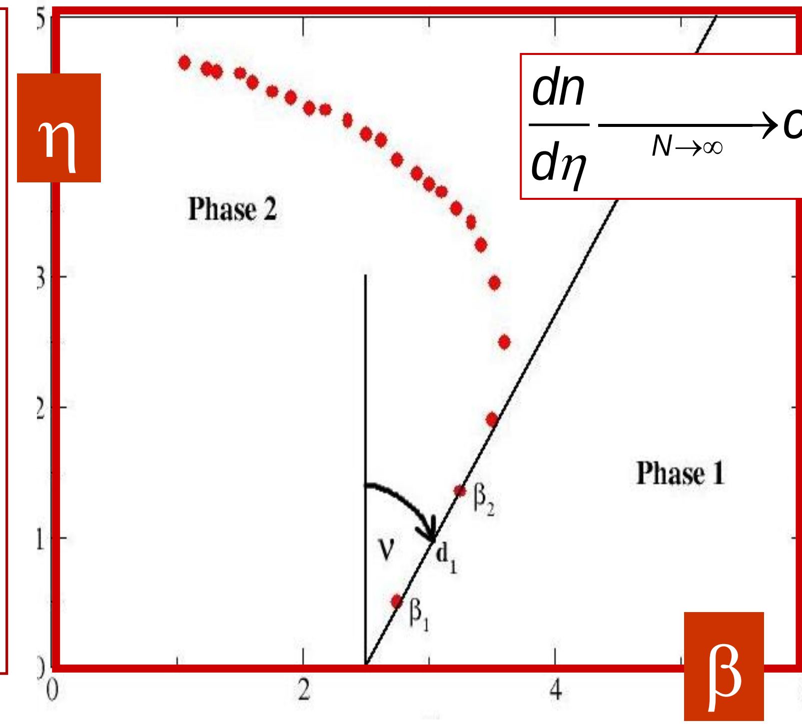
Origin of non-analyticities

$\gamma = \beta + i\eta \quad Z(\gamma) = 0$

$\Re : Z(\gamma) \neq 0$ **analytic**

$\log Z / N \quad N \to \infty$





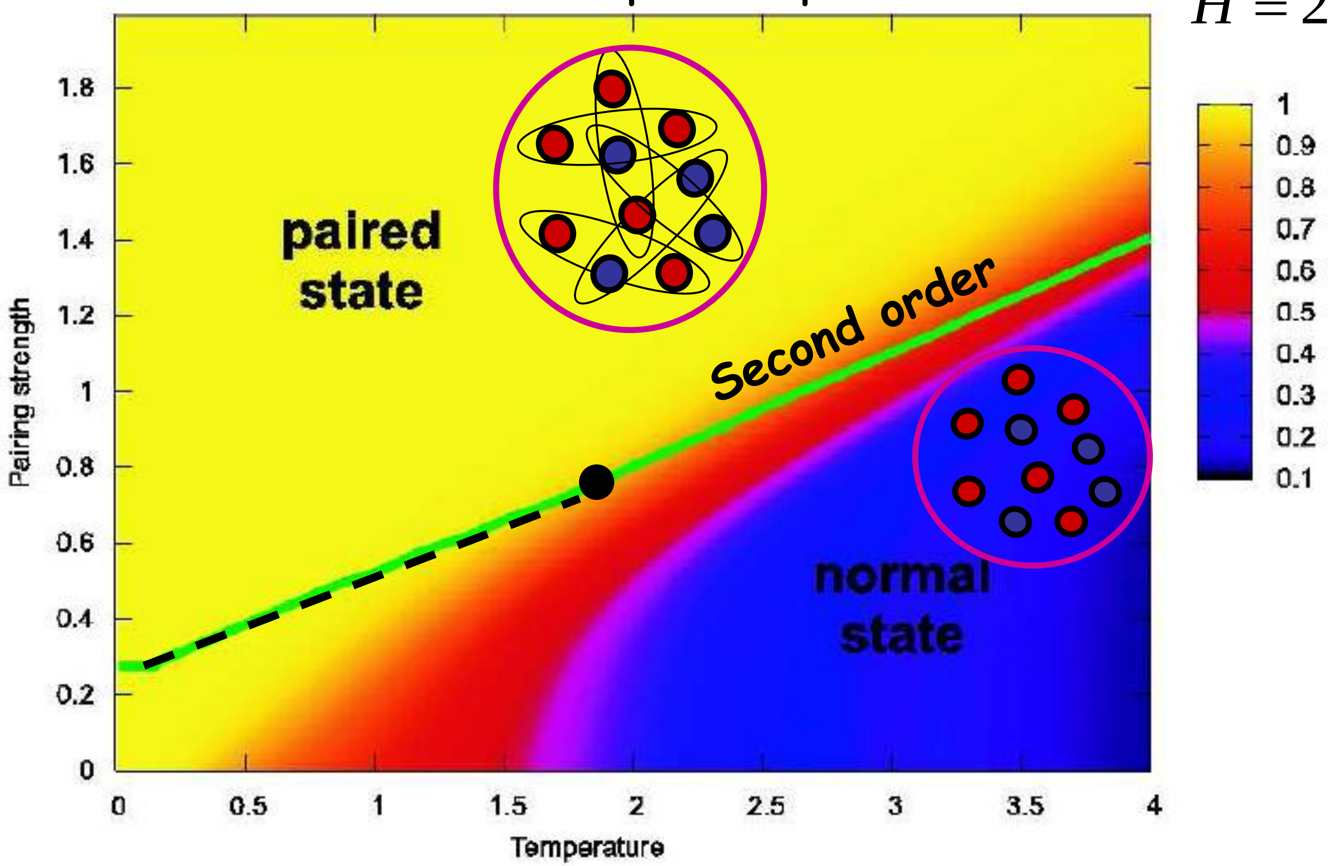
$\frac{dn}{d\eta} \xrightarrow[N\to\infty]{} cN$

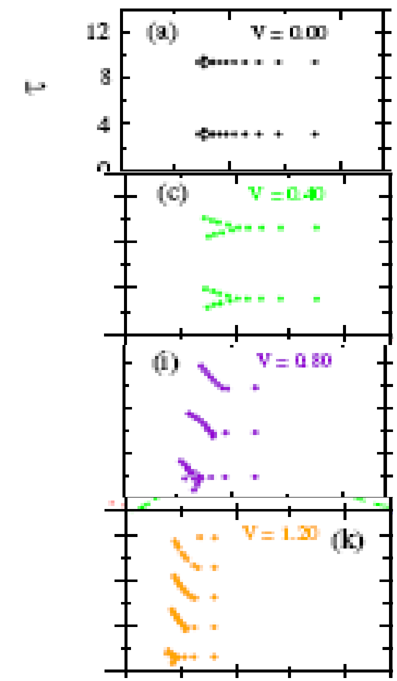
C.N. Yang, T.D.Lee Phys.Rev.87(1952)410
P.Borrmann, O.Mulken, J.Harting, Phys.Rev.Lett 84 (2000)3511

Application to the pairing transition

B=fraction of paired particles

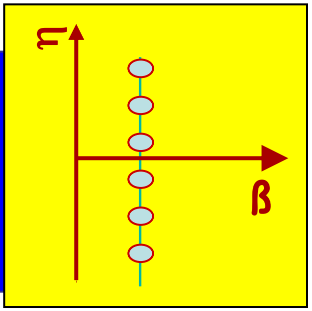
$$\hat{H} = 2\sum_{i>0} \varepsilon_i \hat{n}_i - \sum_{i,j>0} G_{ij} \hat{p}_i^{+} \hat{p}_j$$





T.Sumaryada, A.Volya Phys.Rev.C76:024319,2007

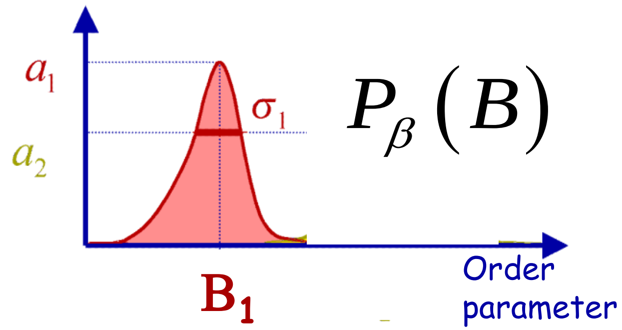
Yang-Lee zeroes (first order) and bimodalities



Partition sum and probability distribution

$$Z_\gamma = Z_\beta \int dB P_\beta(B) e^{-i\eta B}$$

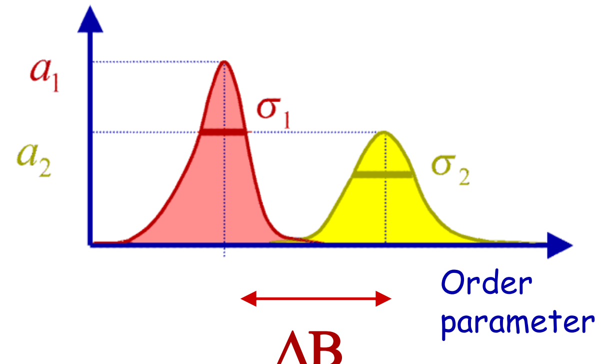
Normal distribution: no zeros



Bimodal distribution $P = P_1 + P_2$: double saddle point approximation

$$\eta_k = \frac{i(2k+1)\pi}{\Delta B}$$

$$\Delta B \xrightarrow{N \to \infty} N \Delta b$$

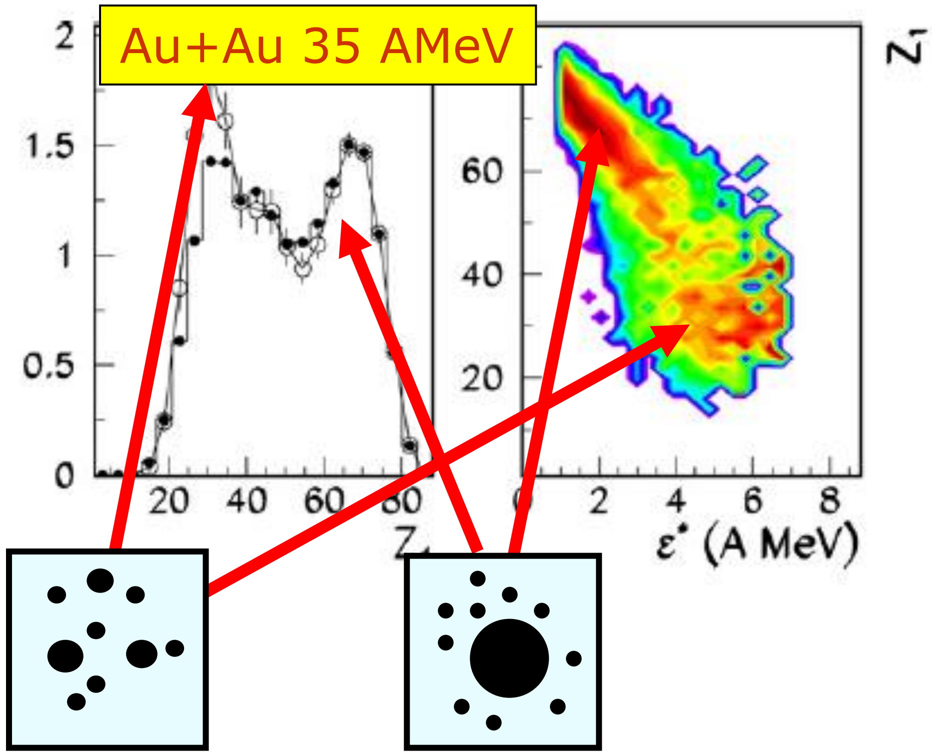


K.C. Lee Phys Rev E 53 (1996) 6558

Ph.Chomaz, F.Gulminelli Physica A 330 (2003) 451.

Application to multifragmentation

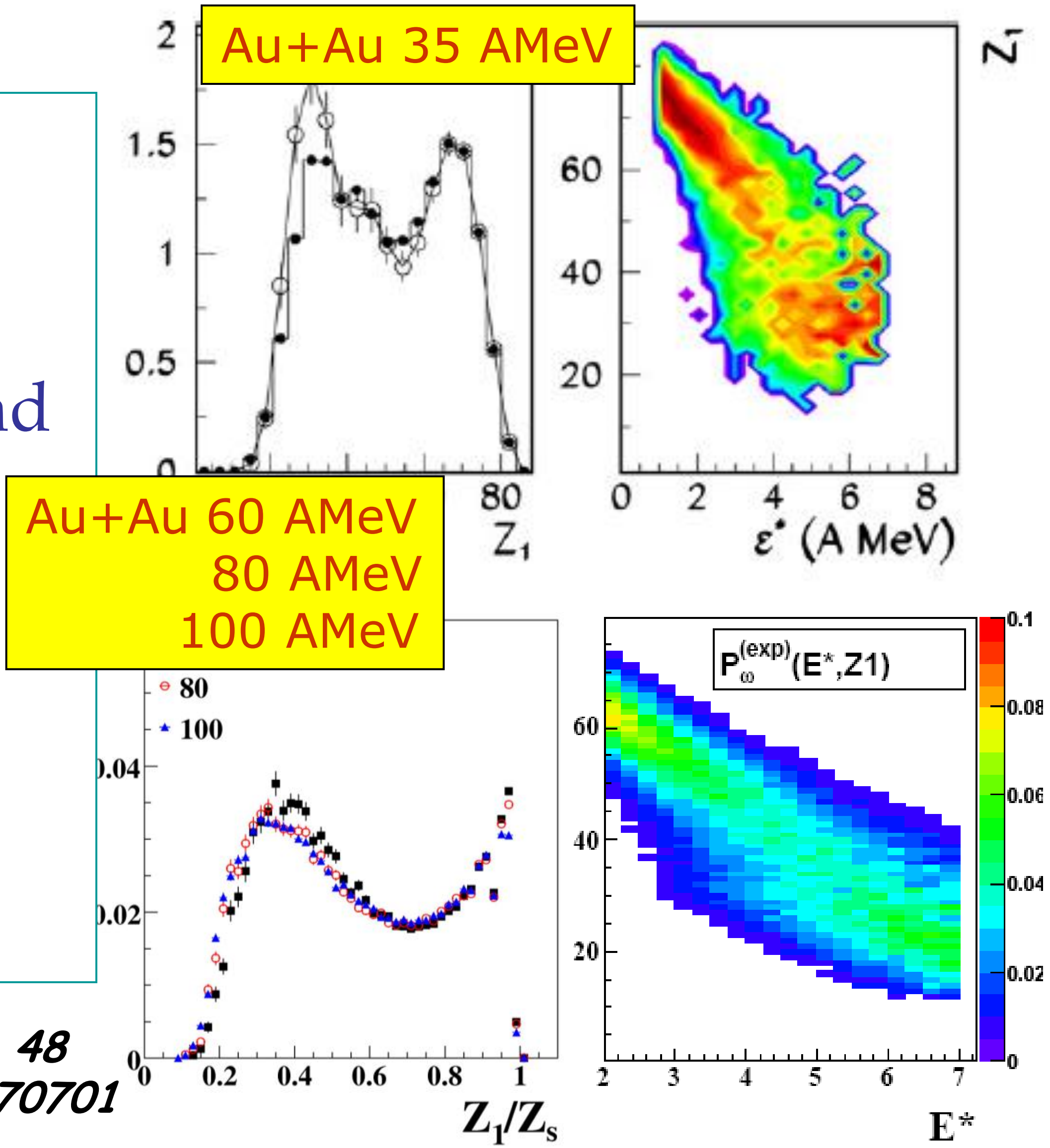
- The heaviest fragment charge distribution is bimodal



Multics Coll., Nucl. Phys. A 807 (2008) 48

Application to multifragmentation

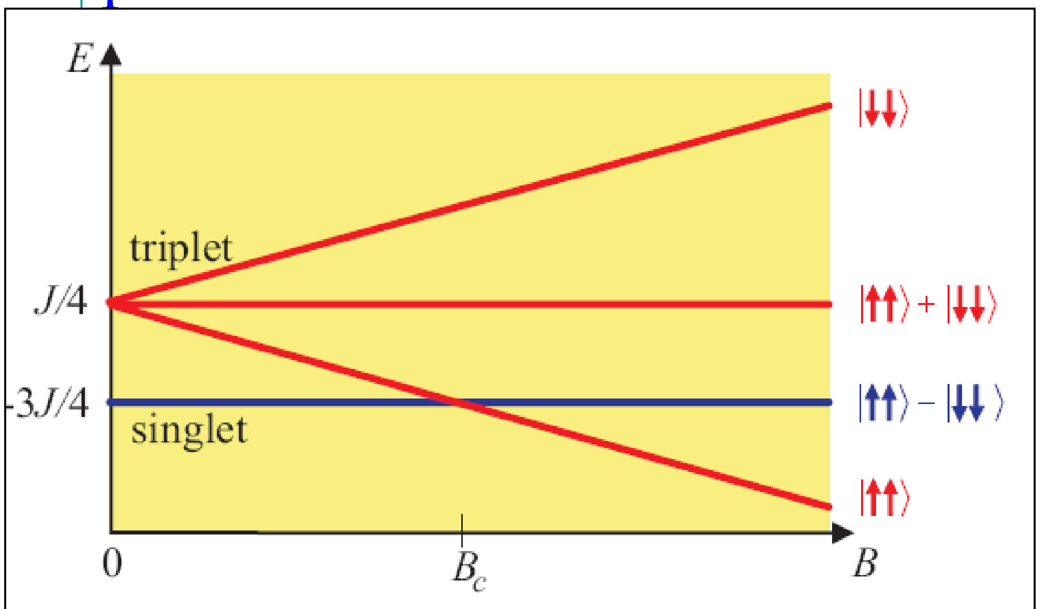
- The heaviest fragment charge distribution is bimodal
- Different experiences lead to compatible results



Multics Coll., Nucl. Phys. A 807 (2008) 48
Indra Coll., Phys.Rev.Lett.103 (2009)270701

Quantum phase transitions

Heisenberg model: Level crossing induced by an external parameter



$$H = \sum_{\langle ij \rangle} J_{ij} \vec{S}_i \cdot \vec{S}_j - \vec{h} \cdot \sum_i \vec{S}_i$$

$$-\ln Z = -\lim_{\beta \to \infty} \frac{1}{\beta} \ln Tr\, e^{-\beta \hat{H}}$$

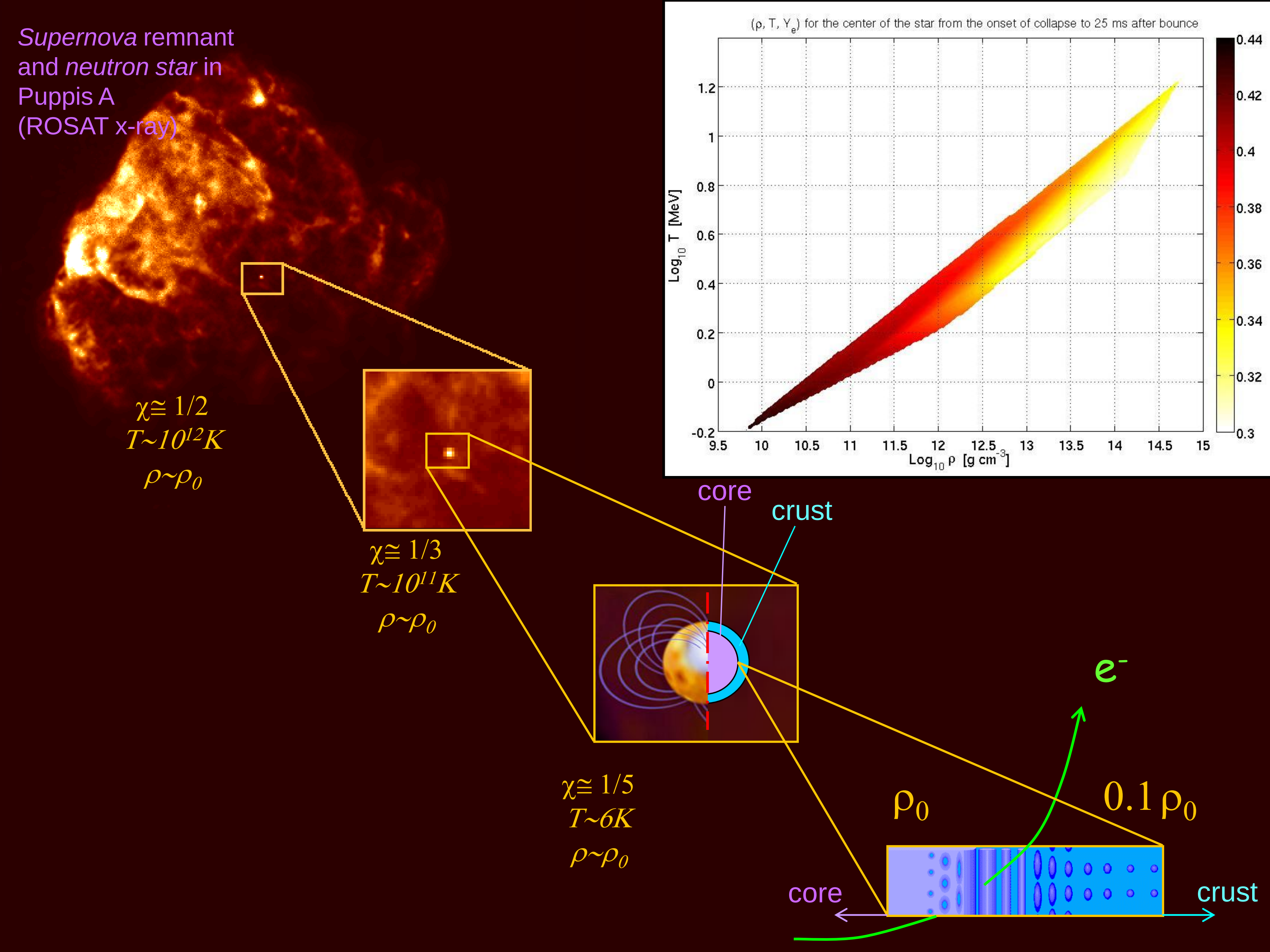
$$= < \hat{H} >_{GS}$$

$$= \begin{cases} -3J/4 & h < J/2 \\ -J/4 - h & h > J/2 \end{cases}$$

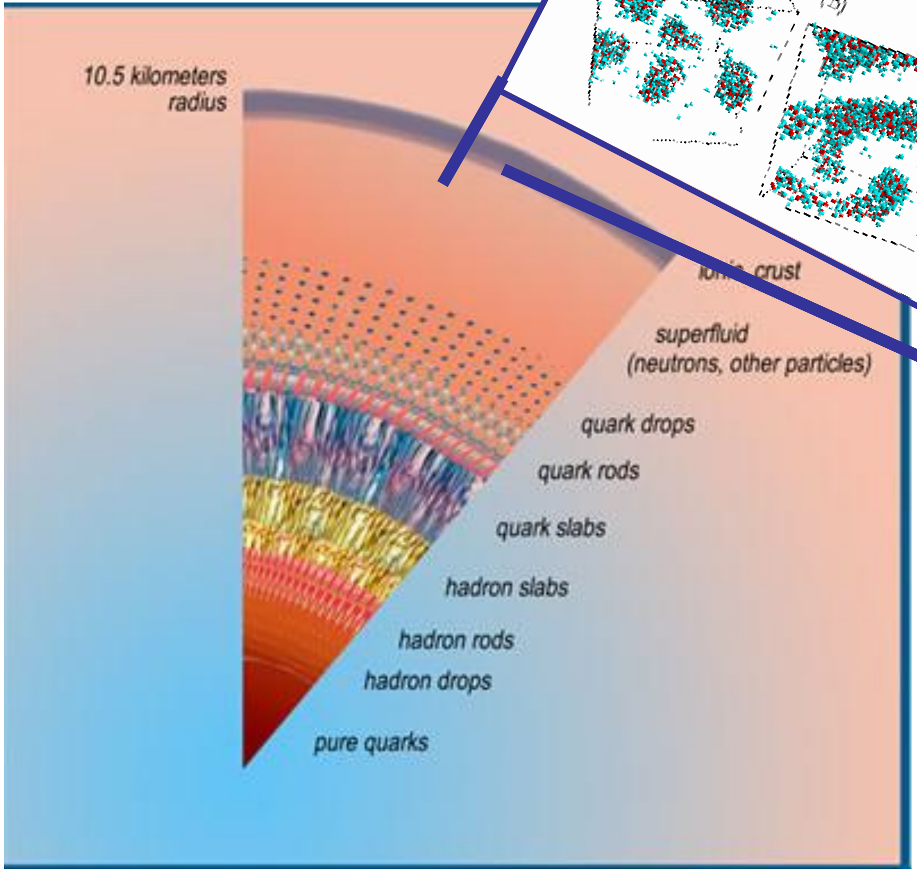
The T->0 limit induces the non-analyticity even without thermo limit!

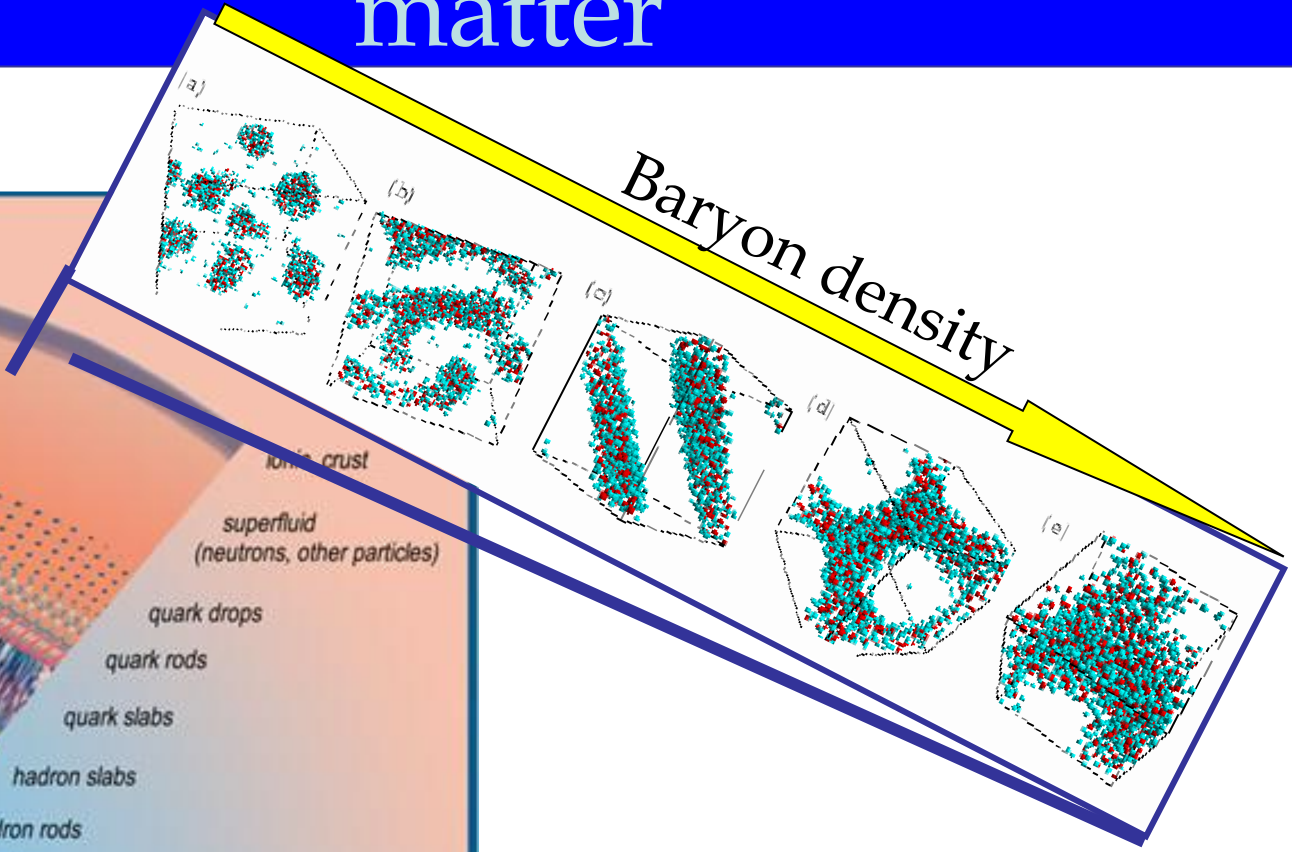
Sachdev, Subir (2011). *Quantum Phase Transitions*

4 - The thermodynamic limit: supernova and neutron star matter


Supernova remnant and neutron star in Puppis A (ROSAT x-ray)
(ρ, T, Ye) for the center of the star from the onset of collapse to 25 ms after bounce
Log10 T [MeV]
Log10 ρ [g cm-3]
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
9.5
10
10.5
11
11.5
12
12.5
13
13.5
14
14.5
15
0.44
0.42
0.4
0.38
0.36
0.34
0.32
0.3
χ≅ 1/2
T~10^12K
ρ~ρ0
χ≅ 1/3
T~10^11K
ρ~ρ0
core
crust
χ≅ 1/5
T~6K
ρ~ρ0
e-
ρ0
0.1 ρ0
core
crust

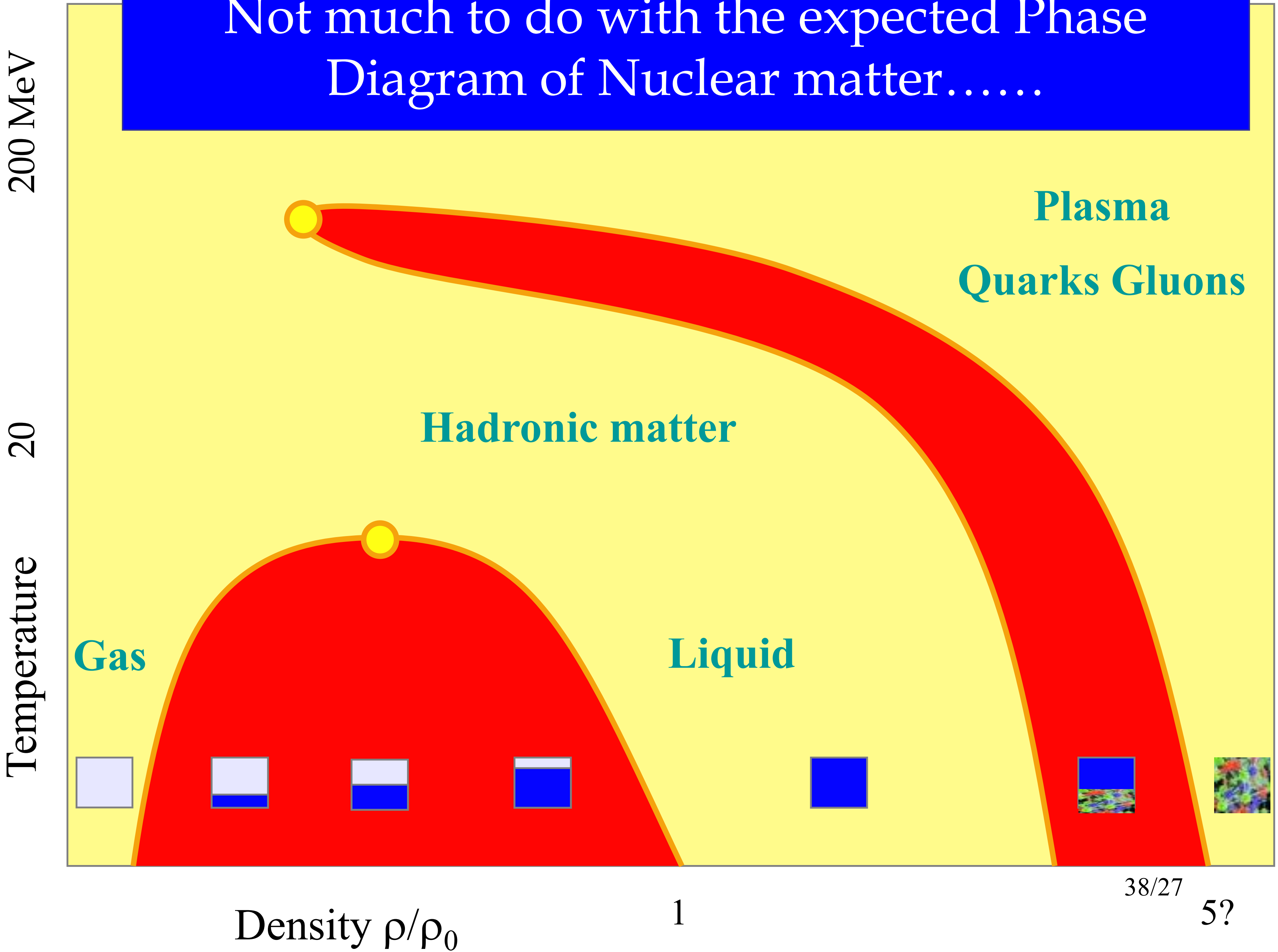
(Quantum) phases in stellar matter





G.Watanabe et al, PRL 2009

Not much to do with the expected Phase Diagram of Nuclear matter……

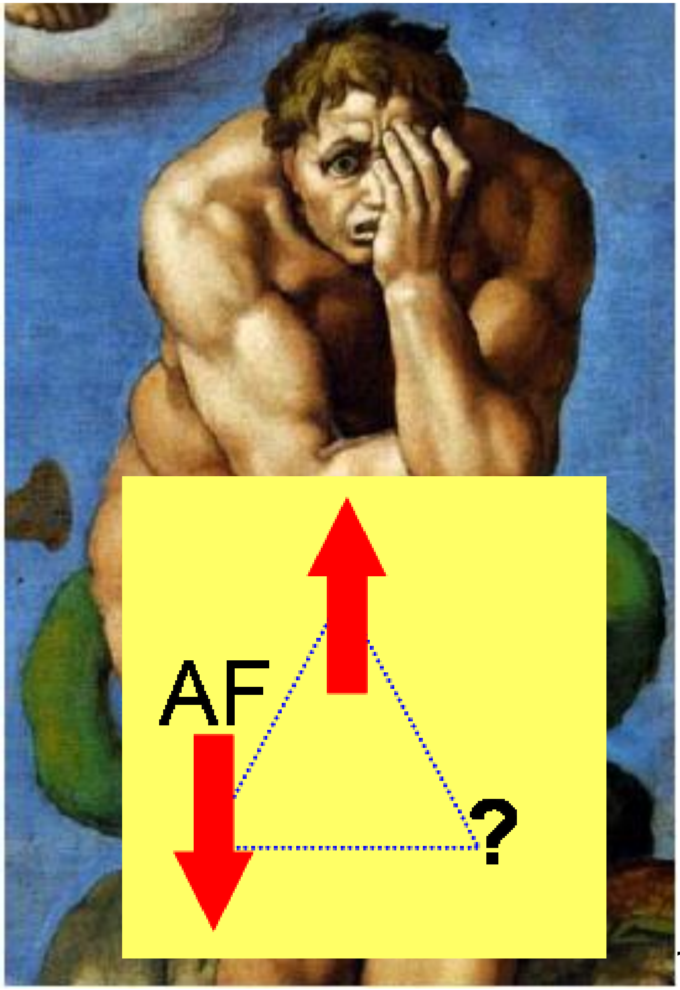


Frustration and dishomogeneous phases

- **Frustration** is a generic phenomenon in physics
- It occurs whenever matter is subject to opposite interactions on comparable length scales
- Global variations of the order parameter (here: density) are replaced by local variations

=>Phase coexistence is quenched

=>dishomogeneous phases arise



Frustration and dishomogeneous phases

A.Raduta, F.G. 2011

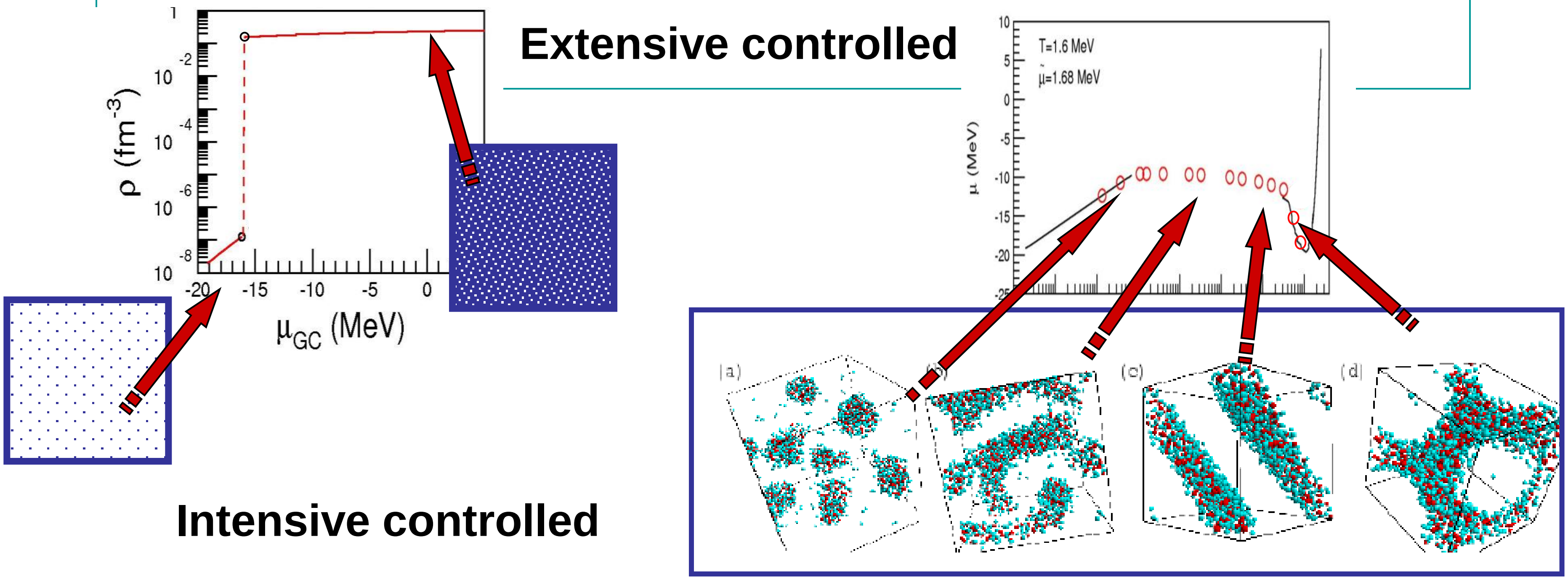
- **Example :** crust structure of a neutron star (matter of neutrons, protons in an homogeneous electron background)

$$E = \alpha_{coul} \left\langle \left(\rho_p(r) - \rho_e \right)^2 N^{2/3}\left(r\right) \right\rangle - \alpha_{nucl}\rho$$

Frustration and dishomogeneous phases

A.Raduta, F.G. 2011

- **Example :** crust structure of a neutron star (matter of neutrons, protons in an homogeneous electron background)



Frustration and dishomogeneous phases

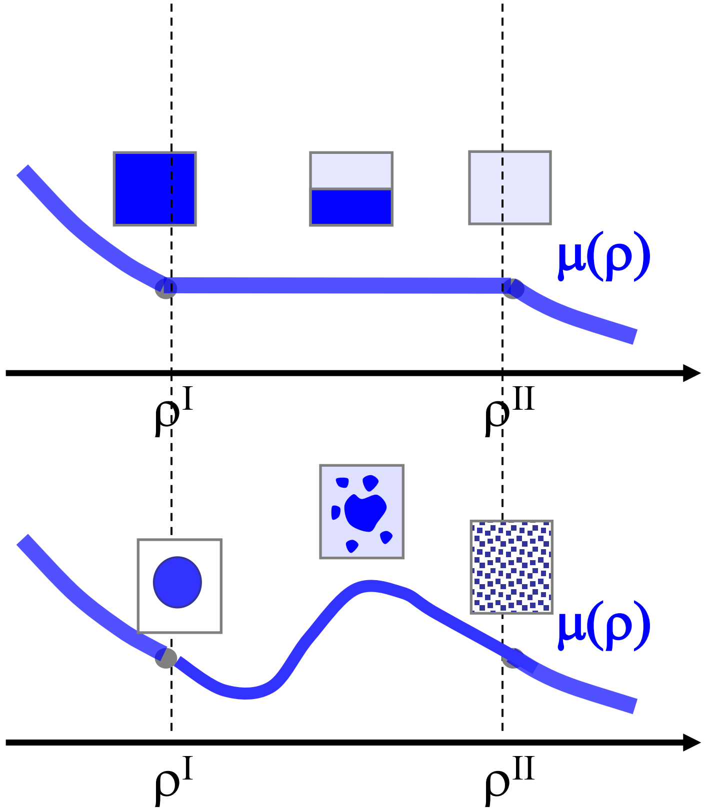
- **Example :** fluid transition in a finite system

$$E(N) = -e_{vol}N + e_{surf}N^{2/3}$$

=>Phase coexistence is quenched

⇒ dishomogeneous phases arise

⇒ Thermodynamic anomalies appear



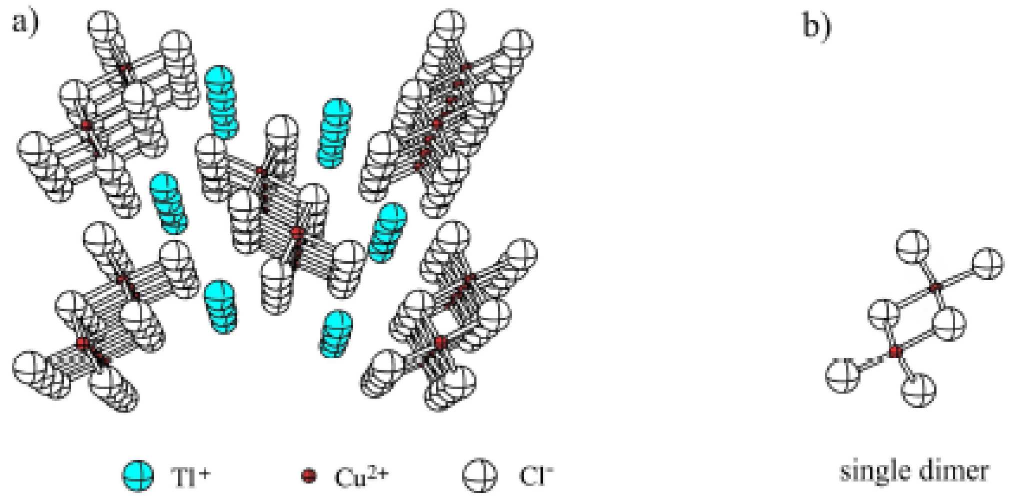
Summary

- **Phase transitions are ubiquitous in nuclear systems: a tremendous *formal, theoretical,* and *experimental* progress in the past ten years**

- No principle difference between thermal and quantum phase transitions
- First order shape transitions well defined in spite of the absence of a thermo limit
- Vanishing of pairing correlation as a unique microscopic probe of the superfluid-normal fluid PT
- Nuclear multifragmentation as a unique microscopic probe of the frustrated liquid-gas PT
- Neutron stars as a unique laboratory of ensemble inequivalence and thermodynamic anomalies

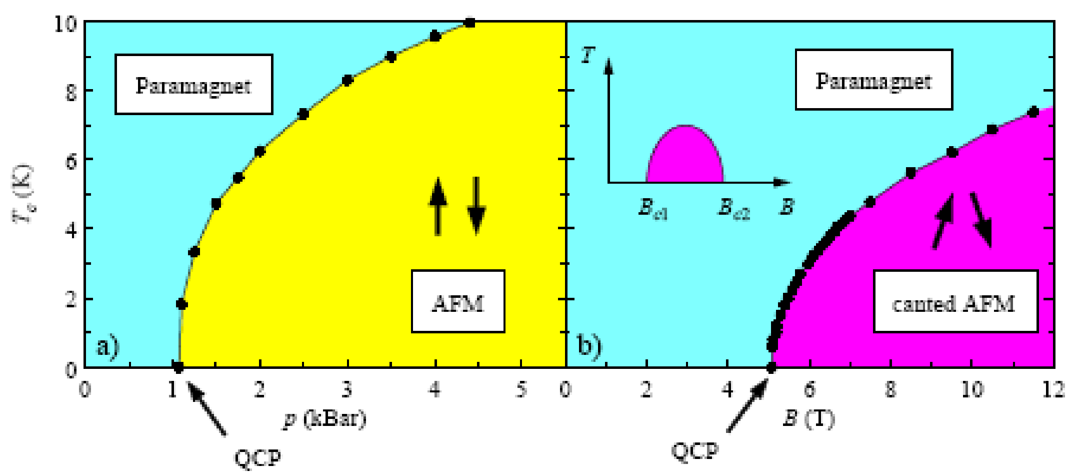
Magnetic quantum critical points of $TlCuCl_3$

- $TlCuCl_3$ is magnetic insulator
- planar Cu_2Cl_6 dimers form infinite double chains
- Cu^{2+} ions carry spin-1/2 moment



antiferromagnetic order can be induced by

- applying pressure
- applying a magnetic field



Courtesy T.Vojta