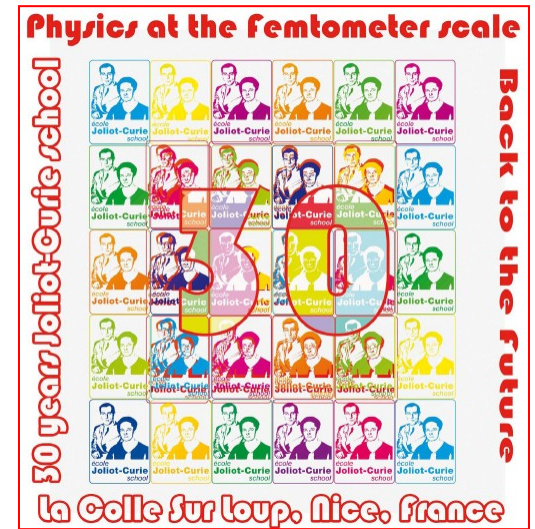


# Probing the nucleon structure

## What have we learnt during the last 30 years?

*Nicole d'Hose, Irfu – CEA Saclay  
La Colle sur Loup, Sept 16, 2011*



# Probing the nucleon structure

## *A brief story of the nucleon*

Proton and neutron are the basic building blocks of *the visible matter of the universe*

Nucleon Identity card (PDG):

Proton

$$Q=+e$$

$$M_p=938.27 \text{ MeV}$$

$$S=\frac{1}{2}$$

$$\mu_p=2.79 \mu_N \quad (\kappa_p=1.79)$$

Neutron

$$Q=0$$

$$M_n=939.57 \text{ MeV}$$

$$S=\frac{1}{2}$$

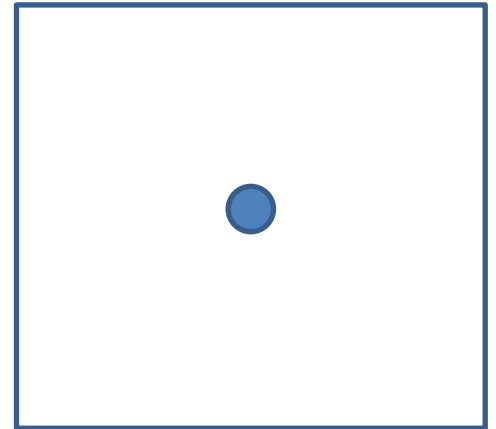
$$\mu_n=-1.91 \mu_N \quad (\kappa_n=-1.91)$$

# Probing the nucleon structure

*A brief story of the nucleon*

Identified in 1919 (proton) and 1932 (neutron)

Until 1933: they were thought as  
a point-like particle, like electron

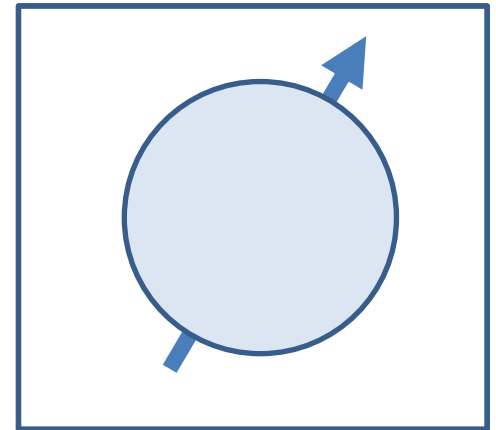


# Probing the nucleon structure

*A brief story of the nucleon*

1933-1960: extended object

1933: *Stern (NP 1943)* measured  
the anomalous magnetic moment  
1st evidence the proton is not point-like



1955: Stanford :  $e^-$  beam of 1 GeV  $\rightarrow$  **elastic scattering**  
*Hofstadter (NP 1961)* measured the charge radius of the proton  $\approx 0.8\text{fm}$

# Probing the nucleon structure

*A brief story of the nucleon*

1960-1980: nucleon composed of **quarks and gluons**

1964: *Gell-Mann and Zweig (NP 1969)* postulated that there are 3 quarks in the proton:  $u u d$   
-> hadron spectroscopy and classification

1969: Stanford:  $e^-$  beam of 20 GeV  $\rightarrow$  **deep inelastic scattering**

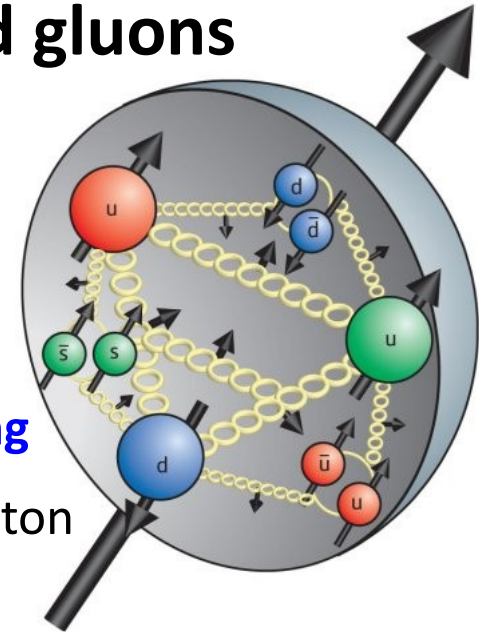
*Friedman, Kendall, Taylor (NP 1990)* found quarks in the proton

*Feynman, Bjorken*      *Gross, Politzer, Wilczek (NP 2004)*

**Quantum ChromoDynamics (QCD)**

✓ **Asymptotic Freedom**

✓ **Confinement**



# Probing the nucleon structure

In 1980:

## We know a lot and we know little

Proton is made of 2 up quarks ( $e=2/3$ ) + 1 down quark ( $e=-1/3$ )  
+ any number of quark-antiquarks  
+ any number of gluons

## Still fundamental questions ?

Origin of mass?

$$M_p \approx 2000 \times M_{e^-}$$

~10% from Higgs interaction

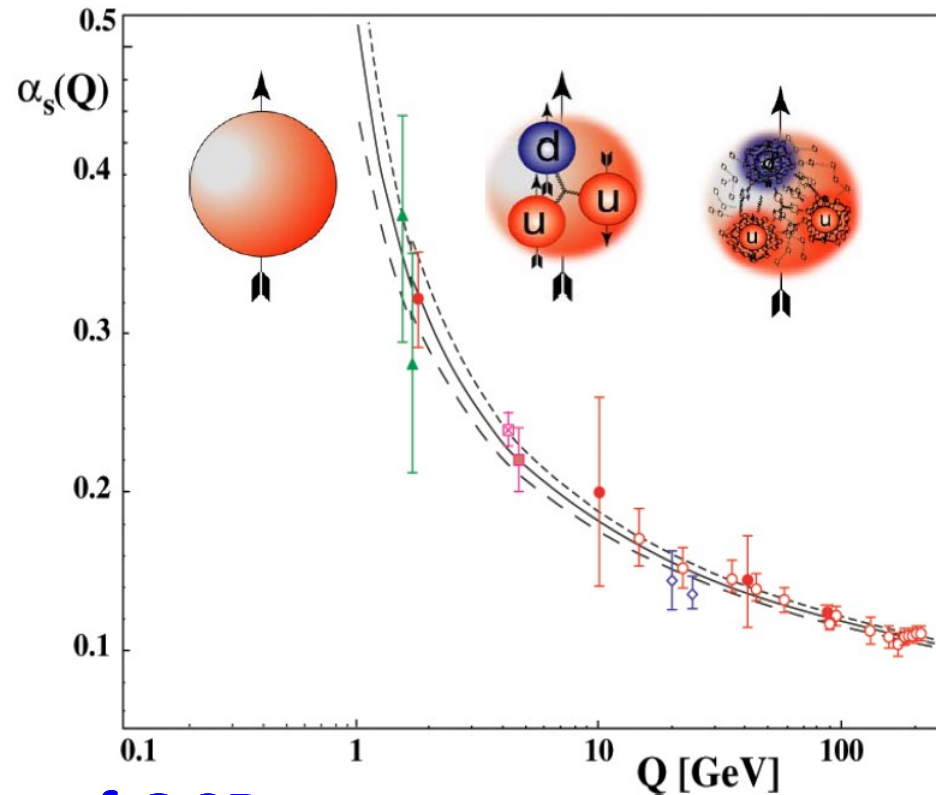
~90% comes from the motion of quarks and gluons

Proton spin crisis 1987: the valence quarks contribute very little to the proton spin

# QCD: still unsolved in non-perturbative region



Gross, Politzer, Wilczek (NP 2004)  
Asymptotic freedom



Non perturbative regime of QCD

One of the top 10 challenges for physics

Nucleon structure provides much insight about how QCD works in the confinement regime

# Understanding the nucleon structure

## Solving QCD

- ✓ Numerically simulation, lattice calculations continue to make advances in techniques and computing power
- ✓ Effective field theories (chiral physics, large  $N_c$ , ...)
- ✓ (Phenomenological models, fits of structure functions...)

## Experimental probes

- ✓ Require clean reaction mechanisms with photons, electrons
- ✓ through low and high-energy scattering off the nucleon



# Probing the nucleon structure

What has been done during the last 30 years?



# Exploration

# Probing the nucleon structure

What has been done during the last 30 years?



Hand-held compass

# Consolidation

# Probing the nucleon structure

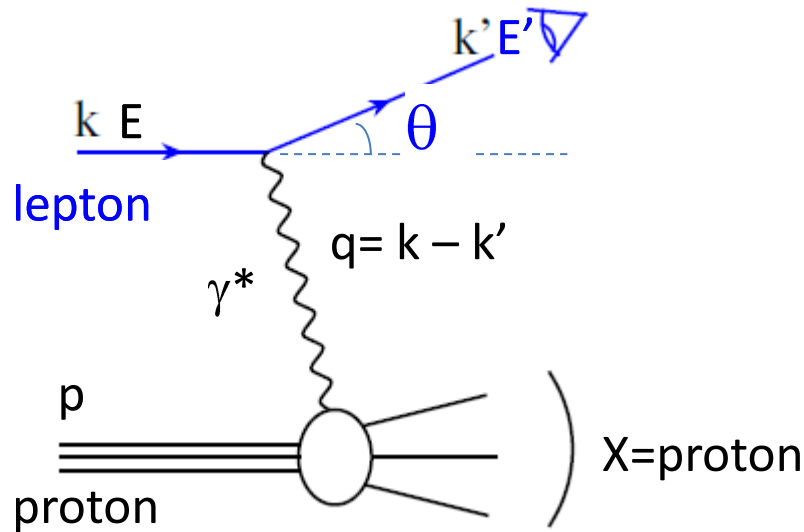
What has been done during the last 30 years?



GPS compass

# Precision

# Probing: Lepton-nucleon scattering



Hypotheses:

- ✓ One photon exchanged
- ✓  $M_{e^-} \ll$

$$Q^2 = -q^2 = 4 E E' \sin^2 \theta/2 > 0$$

$$x_B = \frac{Q^2}{(2p \cdot q)} = \frac{Q^2}{(2M_p(E - E'))}$$

in lab.  
for fixed target

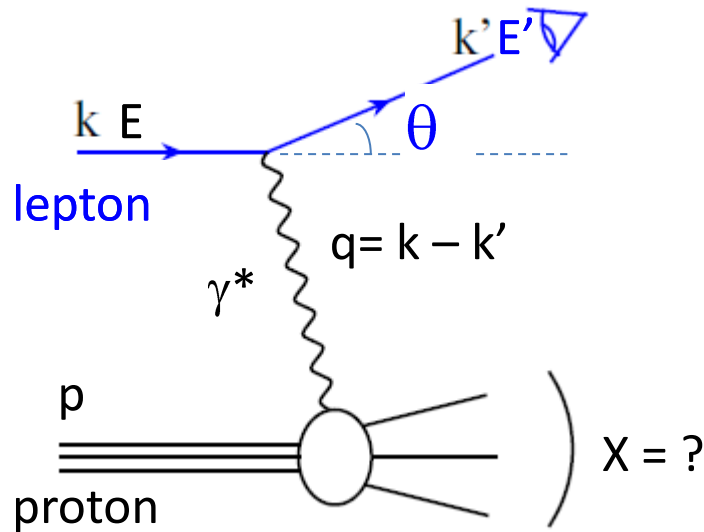
**Elastic Scattering:**  $X = \text{proton}$

$$s = (p+q)^2 = M_p^2 - Q^2 + 2p \cdot q = M_p^2 \Rightarrow x_B = 1$$

at fixed beam energy  $E$ , only one variable  $Q^2$  ( $E'$  and  $\theta$  are not independent)

- Form Factors ( $Q^2$ )  $\Rightarrow$  Consolidation and Exploration at higher  $Q^2$
- Nucleon radius (from  $Q^2 \rightarrow 0$ )  $\Rightarrow$  High Precision, but also need of Consolidation

# Probing: Lepton-nucleon scattering



Hypotheses:

- ✓ One photon exchanged
- ✓  $M_{e^-} \ll$

$$Q^2 = -q^2 = 4 E E' \sin^2 \theta/2 > 0$$

$$x_B = Q^2 / (2p \cdot q) = Q^2 / (2M_p(E - E'))$$

in lab.  
for fixed target

**Deep Inelastic Scattering (DIS):** the proton is broken in many debris  $X$

$$s = (p+q)^2 = M_p^2 - Q^2 + 2p \cdot q > M_p^2 \Rightarrow 0 < x_B < 1$$

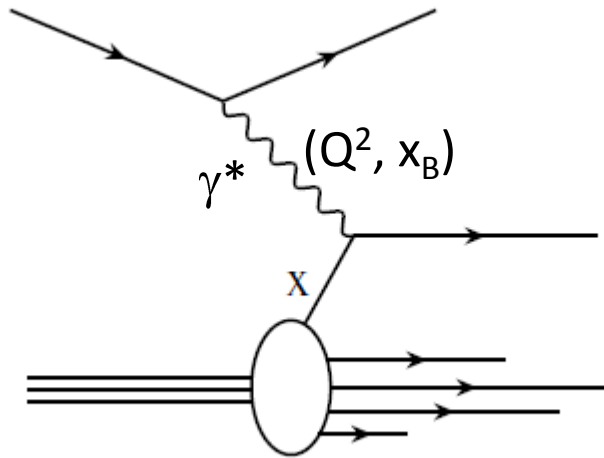
at fixed beam energy  $E$ , 2 variables ( $E'$ ,  $\theta$ ) or ( $Q^2$ ,  $x_B$ )

We learnt that :

$$\sigma_{\text{DIS}}(ep \rightarrow e X) = \sum_q \sigma_{\text{elastic}}(eq \rightarrow eq)$$

incoherent

# Probing: Lepton-nucleon scattering



longitudinal size contracted  
time dilatation  
 $\text{time}_{\text{hadronization}} \gg \text{time}_{\gamma^*q \text{ interaction}}$

## Deep Inelastic Scattering (DIS):

$$\sigma_{\text{DIS}}(ep \rightarrow e X) = \sum_q \sigma_{\text{elastic}}(eq \rightarrow eq)$$

incoherent

## Quark parton model (QPM)

- Point-like, non-interacting partons
- Collinear to the nucleon movement in photon-nucleon collision (longitudinal direction)
- Each parton carries a fraction  $x$  of the nucleon momentum and for the struck parton:  $x = x_B$
- Scaling: observables function of  $x_B$  (at first order)

## Parton Distribution Functions (PDF ( $x$ ))

unpolarized  $\Rightarrow$  High precision

polarized  $\Rightarrow$  Nucleon Spin Crisis Consolidation

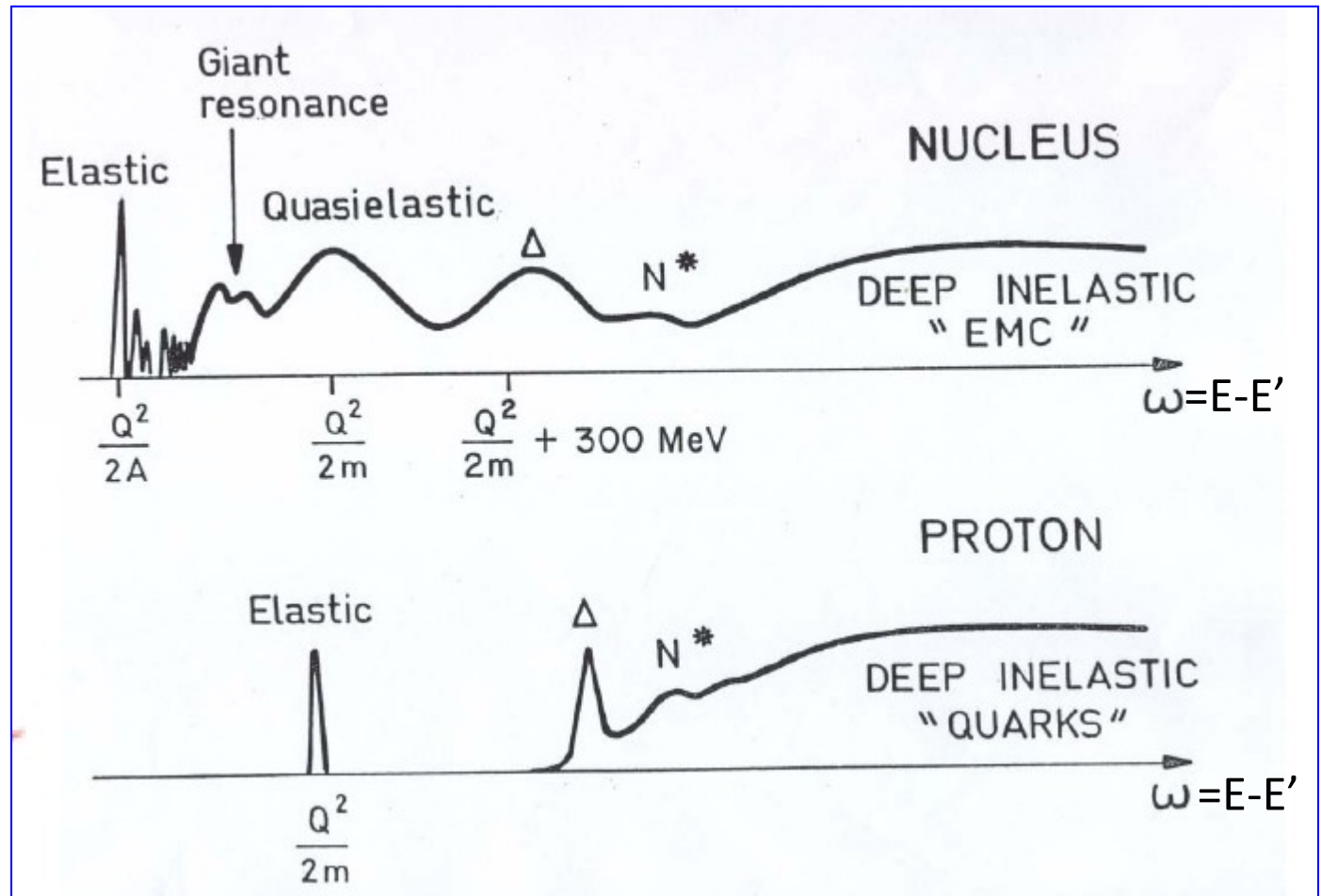
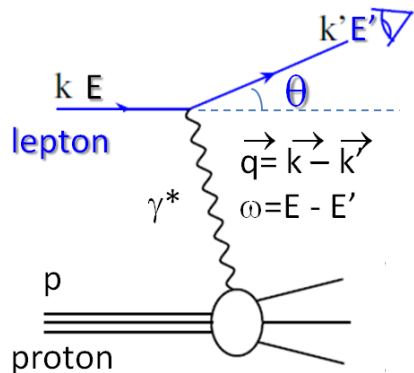
## More on transverse information

momentum: Transverse Momentum Dependent PDF

position: Generalized Parton Distributions (GPD)

$\Rightarrow$  Exploration in 3D

# Lepton nucleon or nucleus scattering

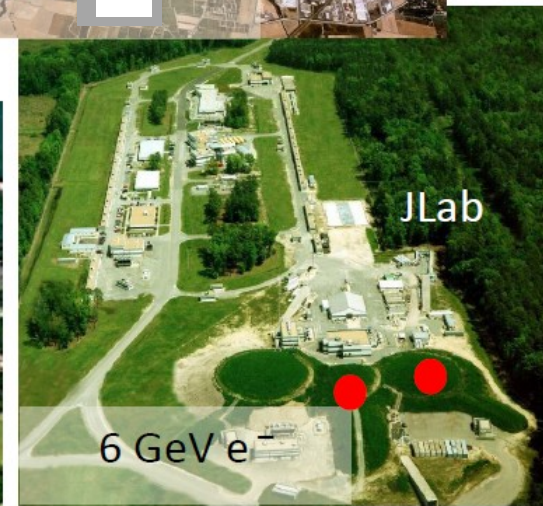
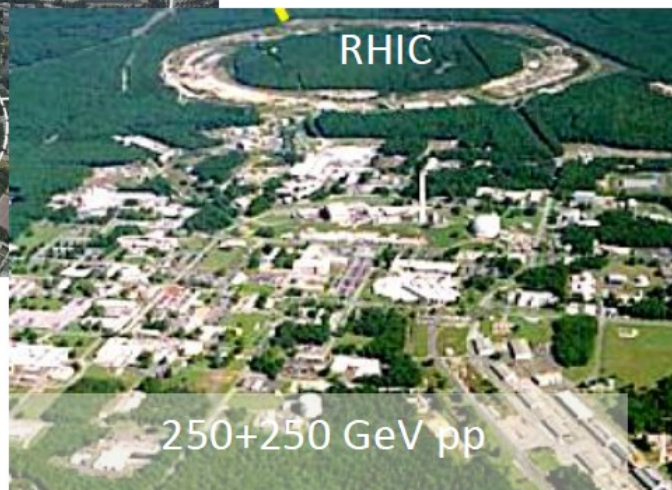
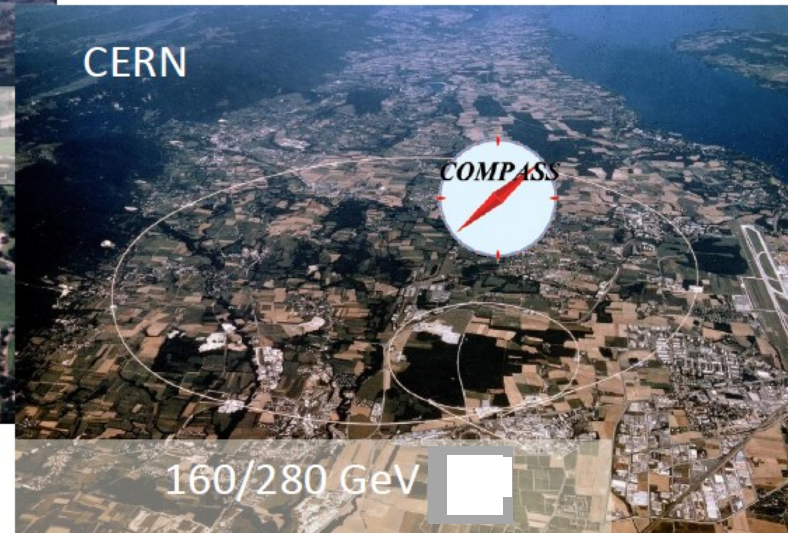


Elastic:  $\Delta x = \hbar c / |\vec{q}|$  exploration of distance of 1 fm with 200 MeV

Deep Inelastic:  $q$  and  $\omega$  varie independently

$\Delta t = \hbar c / \omega$  instantaneous picture to observe free proton

# The main facilities in the world (1980-2015)

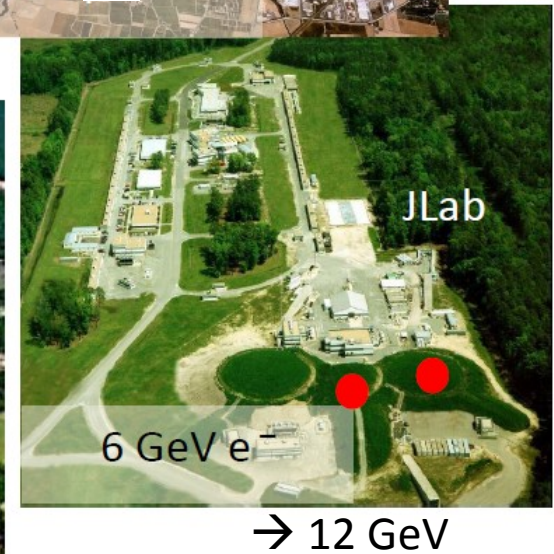
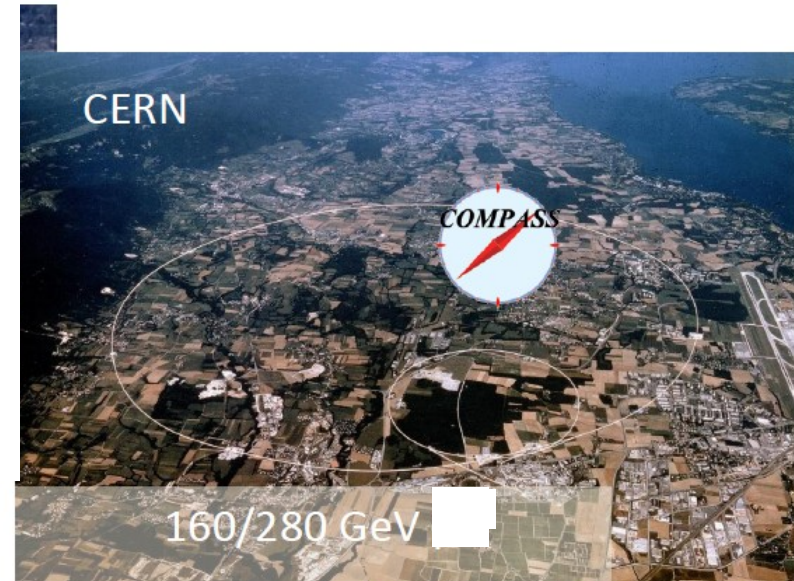


+KEK 6 GeV  $e^\pm$  (Belle)

+ MAMI (Mainz Microtron) 1.5 GeV  $e^-$



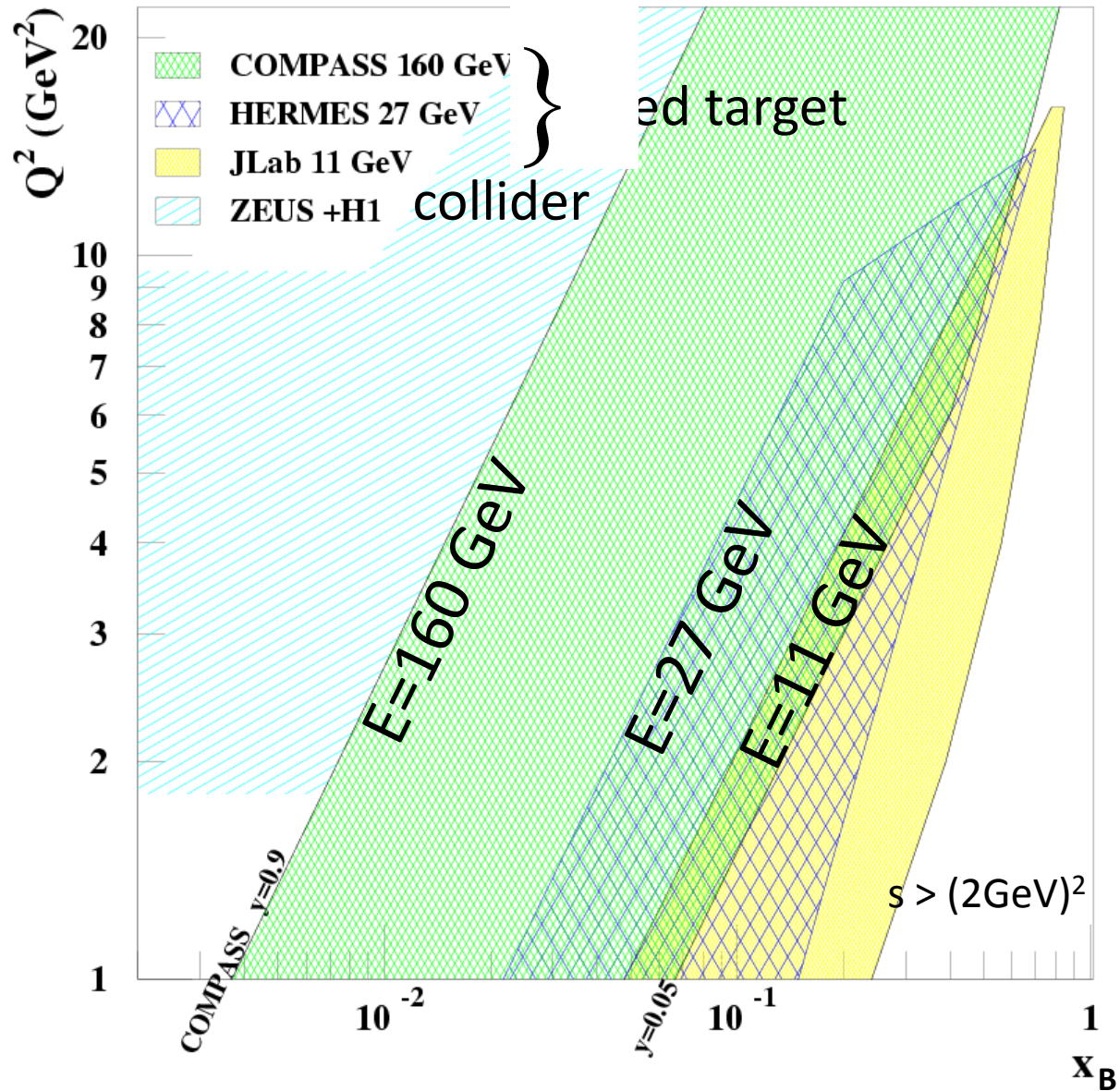
# The main facilities in the world (1980-2015)



+KEK 6 GeV e<sup>±</sup> (Belle)

+ MAMI (Mainz Microtron) 1.5 GeV e<sup>-</sup>

# Kinematic domain ( $Q^2, x_B$ ) for DIS



# Elastic Scattering

- Form Factors ( $Q^2$ )  $\Rightarrow$  Consolidation and Exploration at higher  $Q^2$
- nucleon radius (from  $Q^2 \rightarrow 0$ )  $\Rightarrow$  Very High Precision and Consolidation

# Elastic Scattering

Spin-less point-like electron in a the static Coulomb field given by a charge Z of infinite mass

$$\frac{d\sigma}{d\Omega} = |f(\vec{q})|^2 = \left(\frac{Z\alpha}{2k}\right)^2 \frac{1}{\sin^4 \theta/2} |F(\vec{q})|^2 \quad \boxed{F(\vec{q}) = \int \rho(\vec{r}) e^{i\vec{q}\vec{r}} d^3\vec{r}}$$

$$= \sigma_{Rutherford} |F(\vec{q})|^2$$

Relativistic calculation for a spin 1/2 electron in the EM field given by a charge +e and finite mass

$$\frac{d\sigma}{d\Omega} = \sigma_{Mott} \frac{E'}{E_{recoil}} \times |F(\vec{q})|^2 \quad \text{with} \quad \sigma_{Mott} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)}$$

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{point} \times |F(\vec{q})|^2}$$

The **Form Factor** measures the deviations from a structure less particle

•Point particle :  $\rho(\vec{r}) = \delta(\vec{r}) \quad F(\vec{q}) = 1$

•Spherical distribution:  $\rho(\vec{r}) = \rho(r) \quad F(\vec{q}) = \frac{4\pi}{q} \int_0^\infty \rho(r) \sin(qr) r dr$

$$= 1 - \frac{\vec{q}^2}{6} \langle r^2 \rangle + \frac{\vec{q}^2}{120} \langle r^4 \rangle + \dots \quad \text{Taylor expans.}$$

$$\boxed{\langle r^2 \rangle = -6 \frac{\partial F}{\partial \vec{q}^2} \Big|_{\vec{q}^2=0}}$$

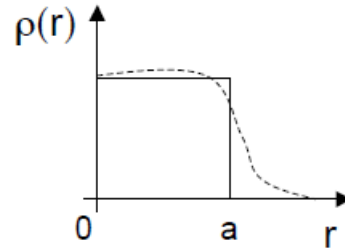
**Quadratic mean charge radius of the nucleon**

# Form Factors for sphere or nucleus

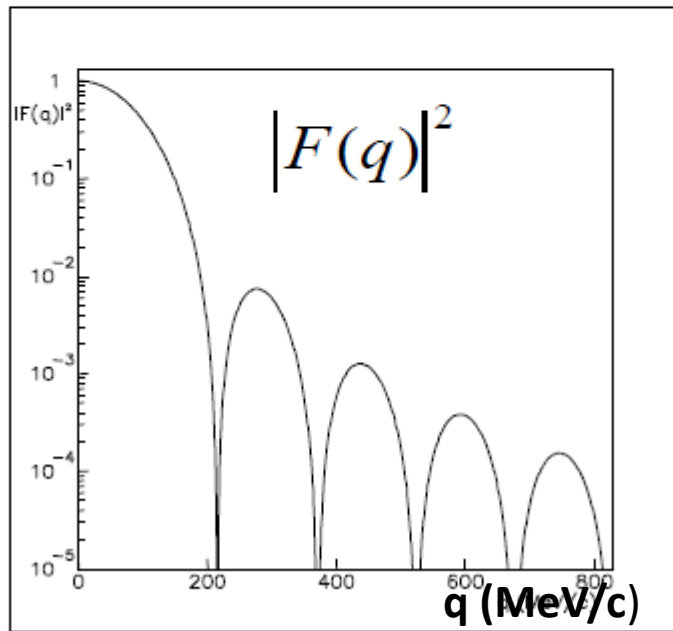
Sphère uniforme :

$$\rho(r) = \rho_0 \text{ pour } r < a$$

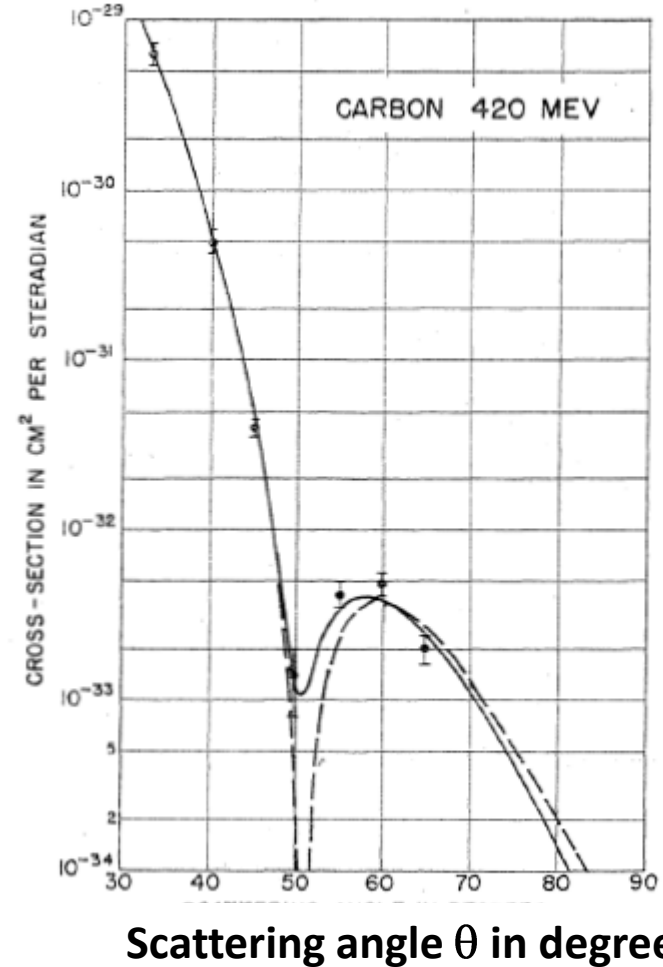
$$0 \text{ pour } r > a$$



$$\rightarrow F(q) = 3(\sin x - x \cos x) / x^3 \quad (x = qa)$$

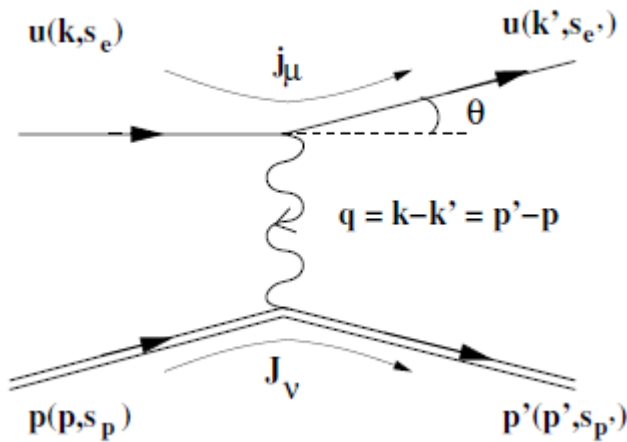


Minima due to the discontinuity of  $\rho(r)$



Real nucleus without discontinuity  
 $\Rightarrow$  Diffractive minimum partly filled

# Scattering on a spin ½ particle: $ep \rightarrow ep$



$$-iM^{(ep)} = -j^\mu \frac{-ig_{\mu\nu}}{q^2} J^\nu$$

$$|\mathcal{M}^{(e,p)}|^2 \Rightarrow \frac{1}{4} \sum_{s_e, s_e', s_p, s_p'} |\mathcal{M}^{(e,p)}|^2$$

General expression for the hadronic current according symmetries of the interaction

$$J_\mu = (-ie) \bar{u}(p') \left[ \underset{\uparrow}{F_1(q^2)} \gamma_\mu + F_2(q^2) \underset{\uparrow}{\frac{i}{2M}} \sigma_{\mu\nu} q^\nu \right] u(p)$$

**Dirac and Pauli Form Factors**

With Gordon identity

$$J_\mu = \frac{-ie}{2Mp} \bar{u}(p') \left[ \underbrace{F_1(q^2)}_{\text{charge } Q} P_\mu + i\sigma_{\mu\nu} q^\nu \underbrace{(F_1(q^2) + F_2(q^2))}_{\text{moment magn. } \mu} \right] u(p)$$

$$Q^2 = -q^2 > 0$$

$$F_1(0) = Q$$

$$F_2(0) = \kappa$$

$$\frac{\mu}{\mu_N} = Q + \kappa$$

# Scattering on a spin ½ particle: $ep \rightarrow ep$

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \sigma_{Mott} \frac{E'}{E} \left[ F_1^2 + \tau F_2^2 + \tau (F_1 + F_2)^2 \tan^2 \frac{\theta}{2} \right]$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \sigma_{Mott} \frac{E'}{E} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right]$$

characterize the magnetic interaction with a particle of spin 1/2

$$\left(\frac{d\sigma}{d\Omega}\right)_{lab} = \sigma_{Mott} \frac{E'}{E} \left[ \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1 + \tau)} \right]$$

$$\epsilon = [1 + 2(\vec{q}^2/Q^2) \tan^2(\theta/2)]^{-1}$$

$$\tau = \frac{Q^2}{4M_p^2}$$



**Electric and Magnetic Sachs Form Factors** (Fourier Transform (TF) of the charge and magnetization distribution in the Breit frame or brick-wall frame)

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$

$$G_E(0) = Q$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

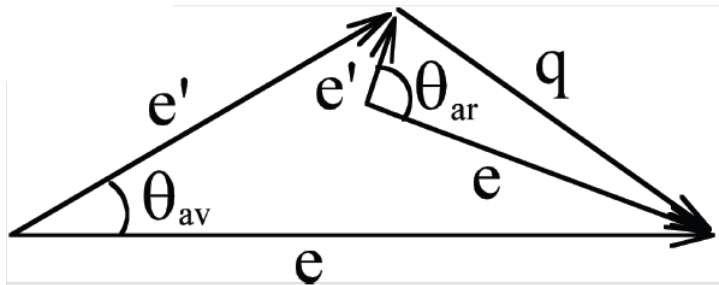
$$G_M(0) = \mu_N$$

# Measurement of Form Factors ( $Q^2$ )

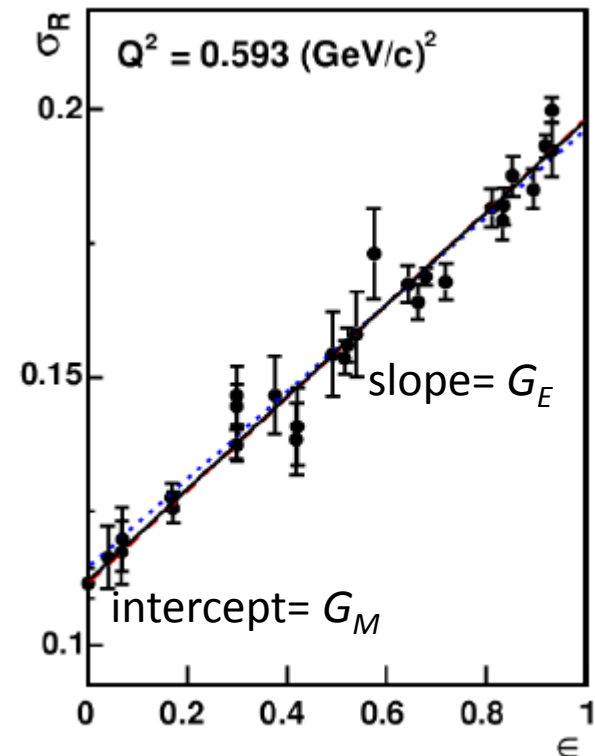
## Rosenbluth separation:

$$\text{We plot } \sigma_R = \epsilon(1 + \tau) \frac{d\sigma}{d\Omega} \frac{E}{E'} \frac{1}{\sigma_{Mott}} = \epsilon G_E^2 + \tau G_M^2$$

Fixed  $Q^2 \rightarrow$  use of different beam energies to get different  $\epsilon$



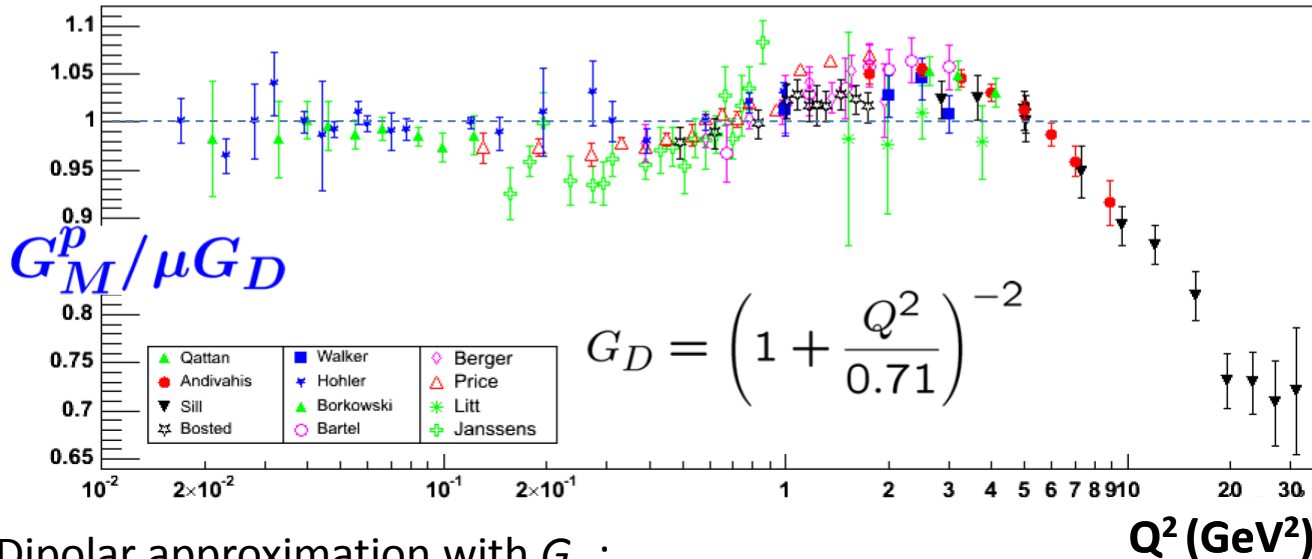
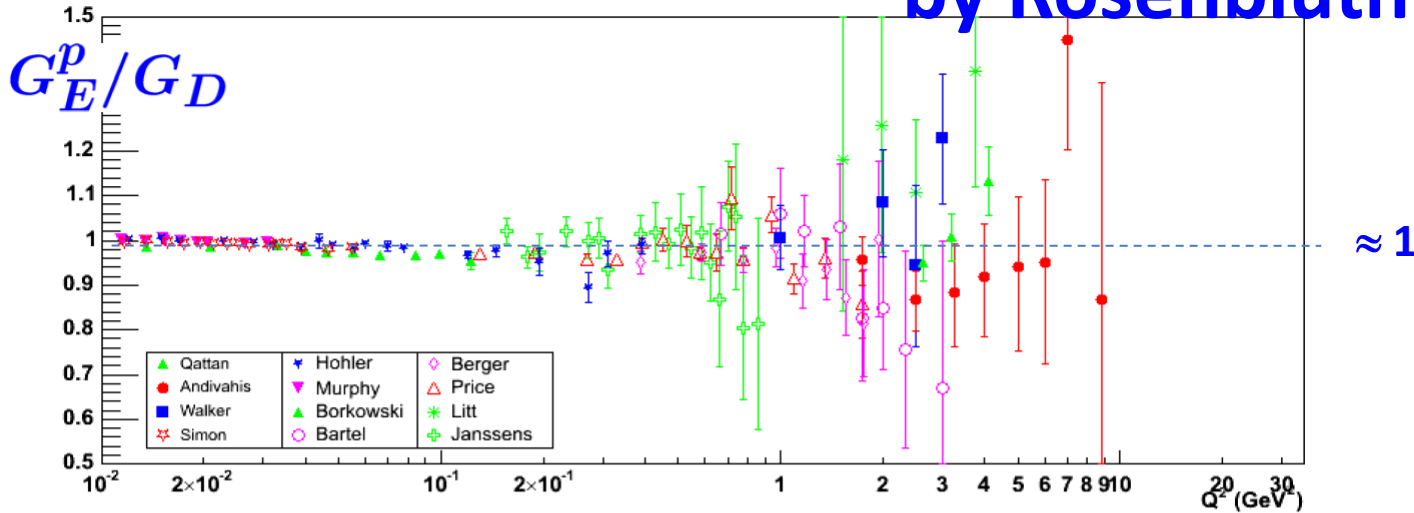
- Different scattering angles  
 $\rightarrow$  large systematic effects on the slope =  $G_E$
- At large  $Q^2$   $G_E^2 \ll \tau G_M^2$   
 $\rightarrow$  Method to determine  $G_M$





# Measurement of proton Form Factors ( $Q^2$ )

by Rosenbluth separation

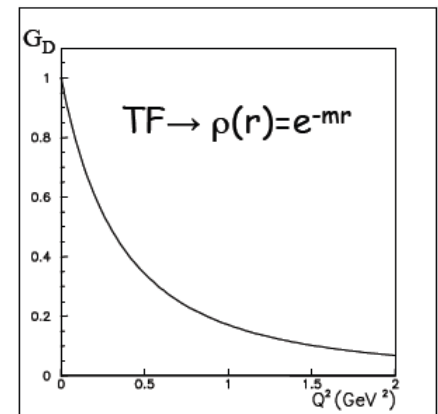


deviation from the dipolar approximation

Dipolar approximation with  $G_D$ :

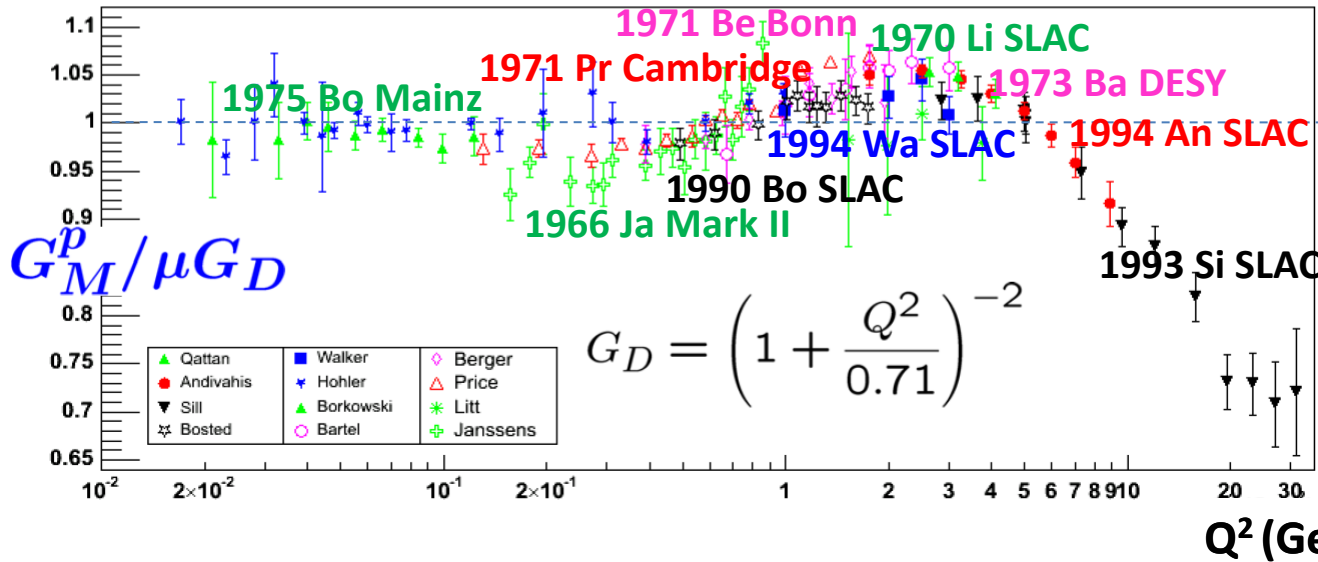
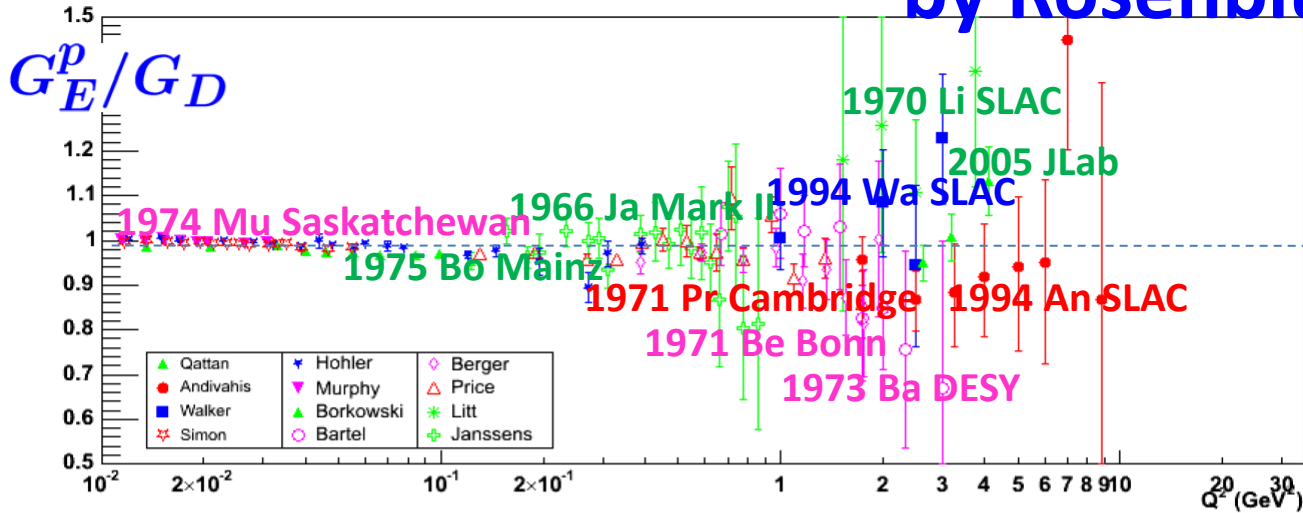
Contribution of an effective double pole in the time-like region

FT  $\rightarrow$  exponential distribution  $\rho(r) = \exp(-0.84 r)$  non physical



# Measurement of proton Form Factors ( $Q^2$ )

## by Rosenbluth separation



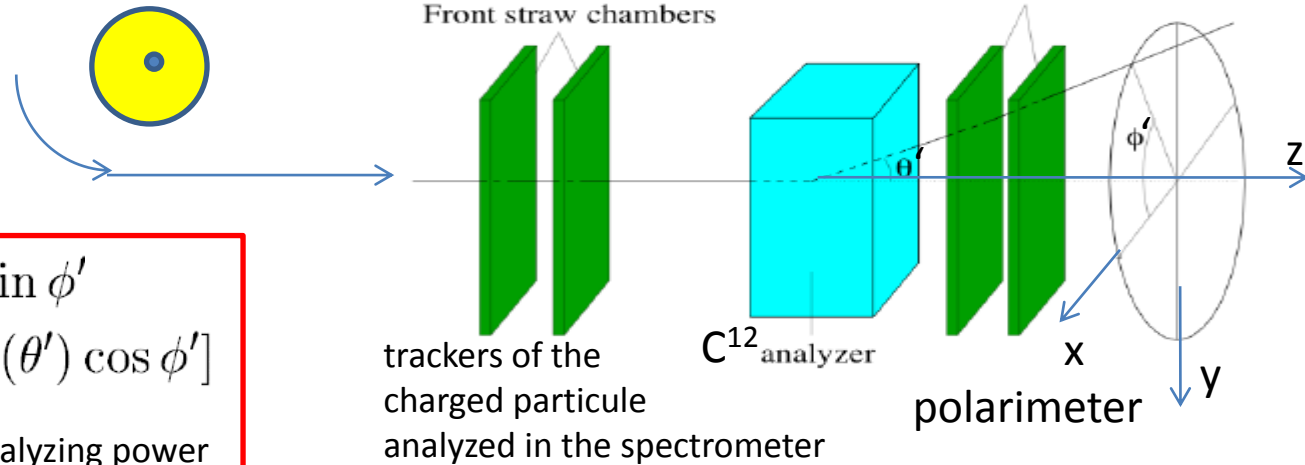
deviation from the dipolar approximation

# Measurement of Form Factors ( $Q^2$ )

## New Double Polarization technique:

polarized e- beam + unpolarized target

→ recoil proton analyzed in a magnetic spectrometer and a polarimeter



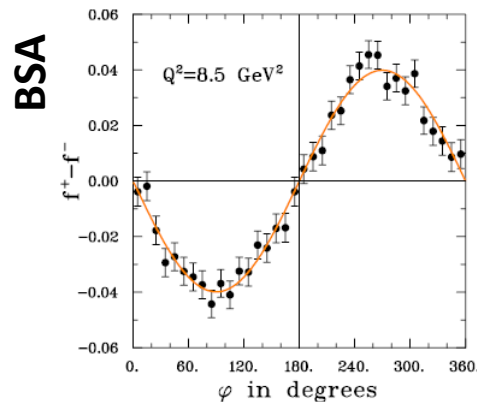
$$\sigma = \sigma_0 [1 + P_e P_x A_y(\theta') \sin \phi' + P_e P_z \sin \chi A_y(\theta') \cos \phi']$$

$\downarrow$  Beam polarization       $\downarrow$  Analyzing power  
 $\downarrow$  Spin precession

$$P_x = \frac{-\sqrt{\tau} \sqrt{2\epsilon(1-\epsilon)} G_E G_M}{\epsilon G_E^2 + \tau G_M^2}$$

$$P_z = \frac{\tau \sqrt{1-\epsilon} G_M^2}{\epsilon G_E^2 + \tau G_M^2}$$

Proposed in 1967 Akhiezer Rekalov but only experimentally possible since high duty cycle facilities advent



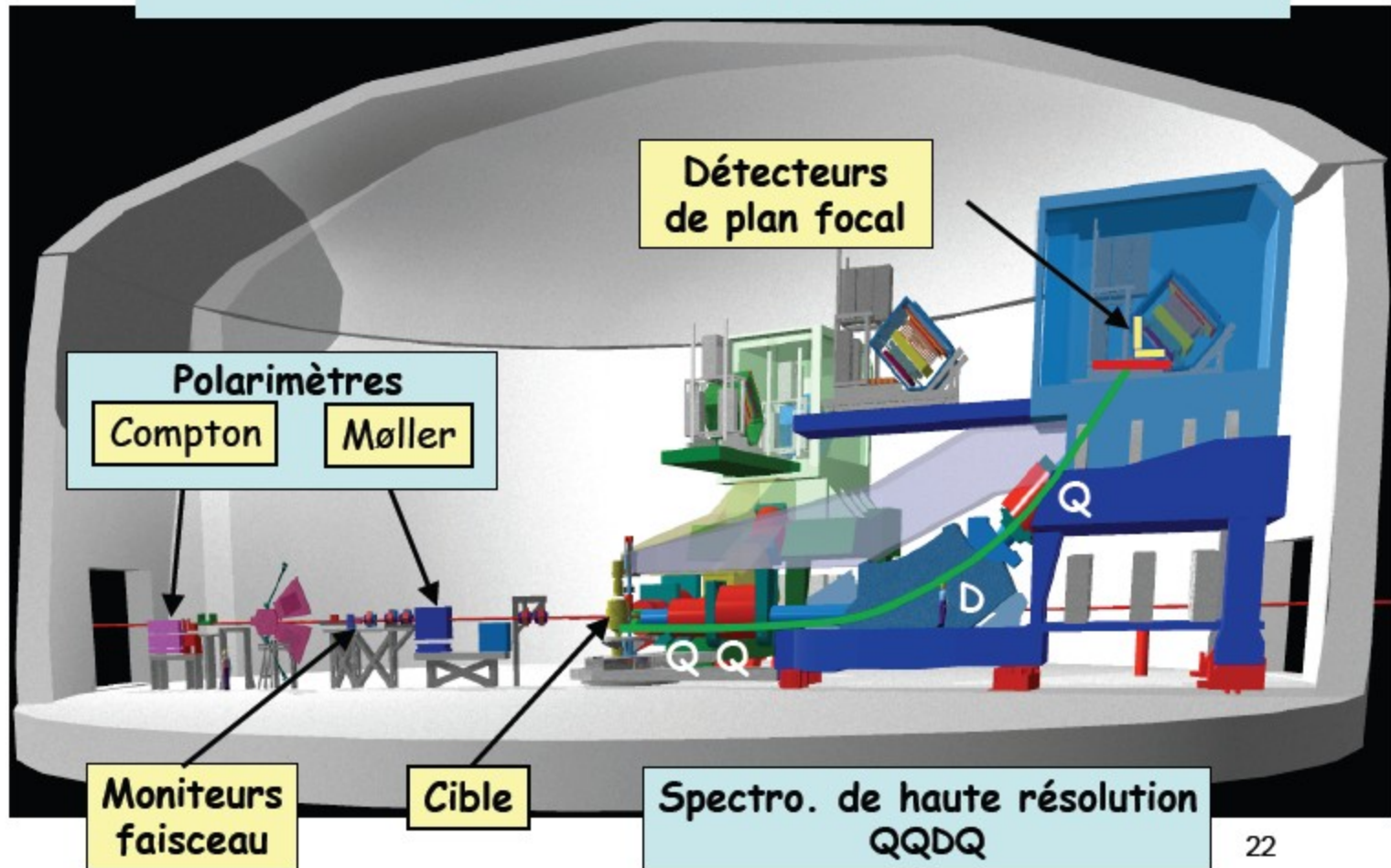
Beam spin asymmetry

$$BSA = \frac{N^+ - N^-}{N^+ + N^-} = a \sin \phi' + b \cos \phi'$$

$$\frac{a}{b} = \frac{P_x}{P_z \sin \chi} = \frac{G_E}{G_M}$$

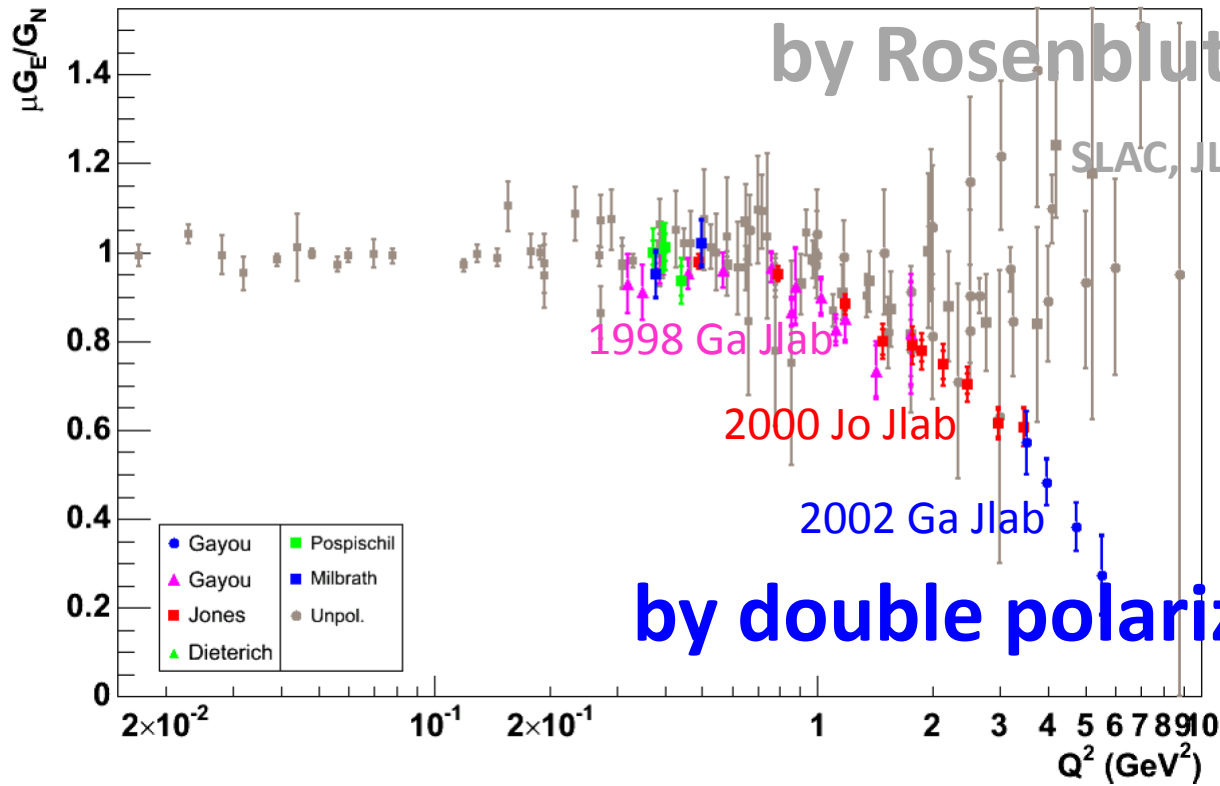
No systematic effects due to Beam polar and analyzing power as they cancel out in the ratio

# JLab - Hall A



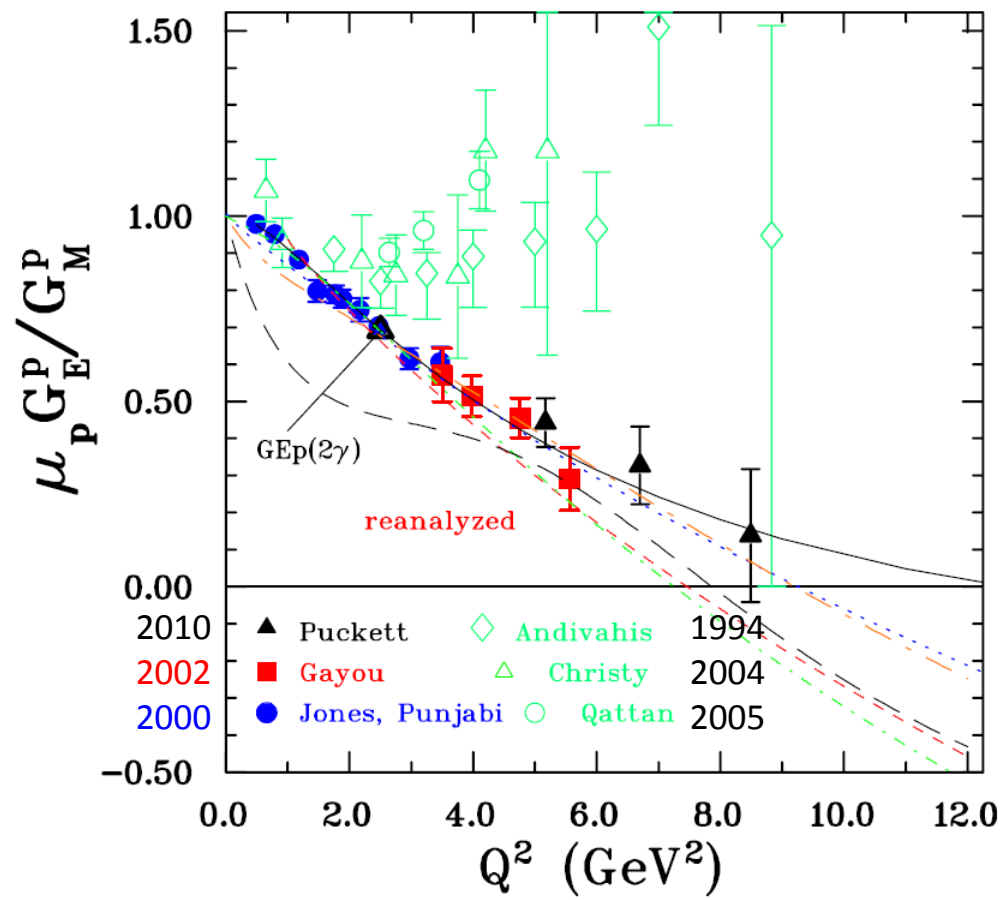
# Measurement of proton Form Factors ( $Q^2$ )

$$\mu_p G_{Ep} / G_{Mp}$$

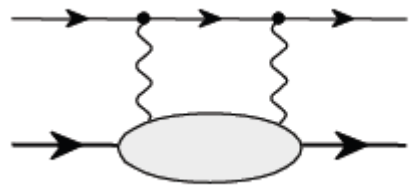


**SURPRISE !!!**

# Difference persists between two techniques

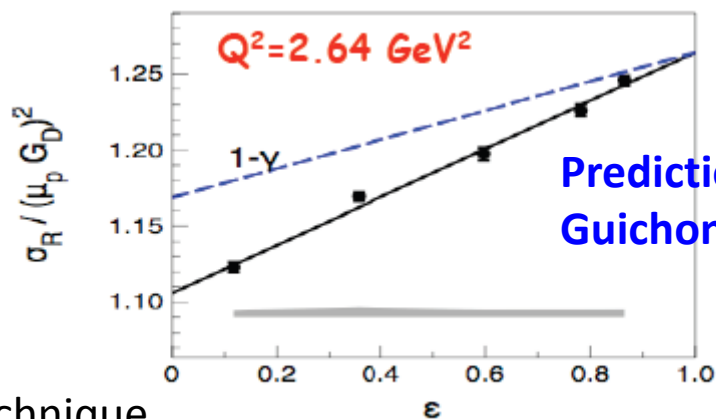
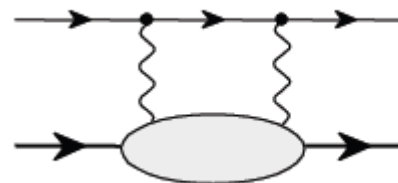


- Jlab Rosenbluth separation data confirm earlier data and global analysis
- No evidence for experimental errors for either of the experimental techniques
- Two photon exchange (TPE) amplitudes can explain a significant part of the discrepancy
- Intensive theoretical and experimental effort on addressing the TPD effect



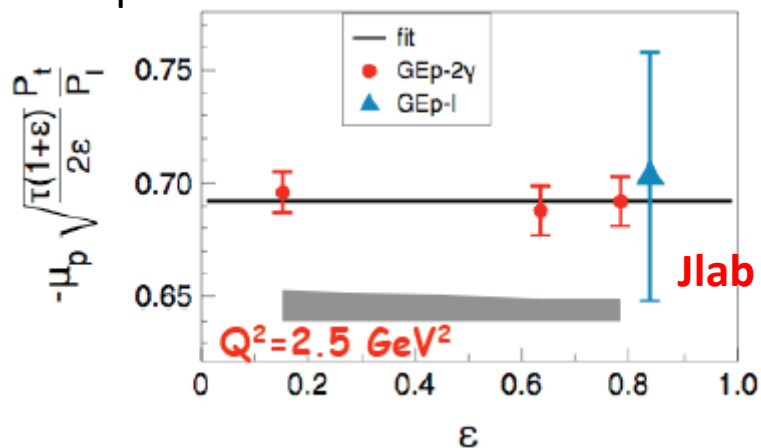
# 2 photon exchange proposed as an explanation

Rosenbluth separation



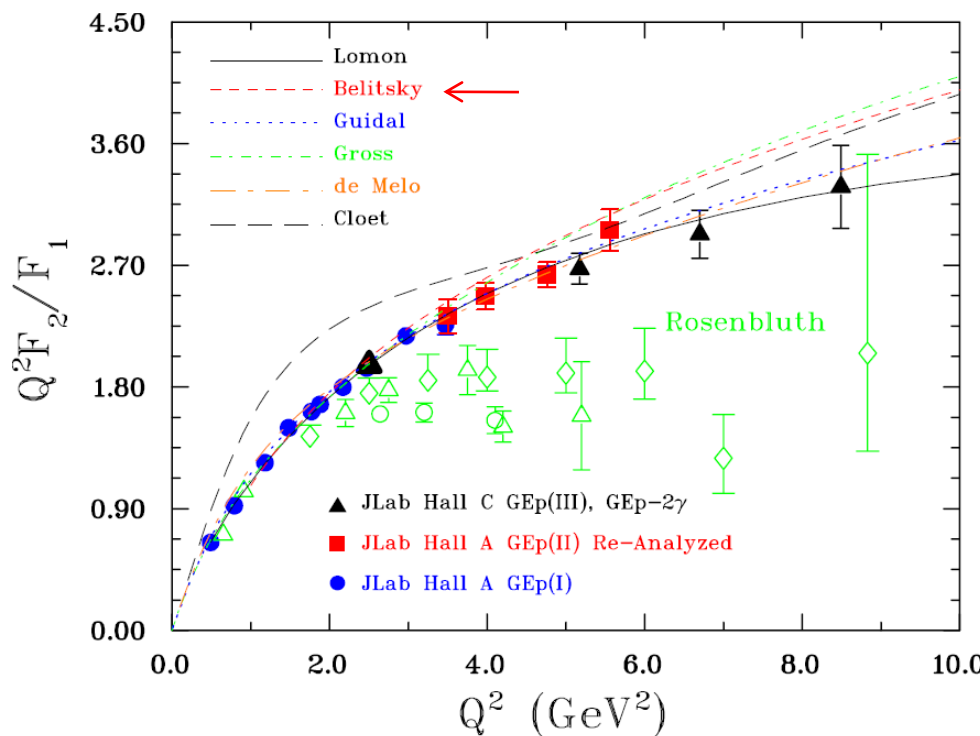
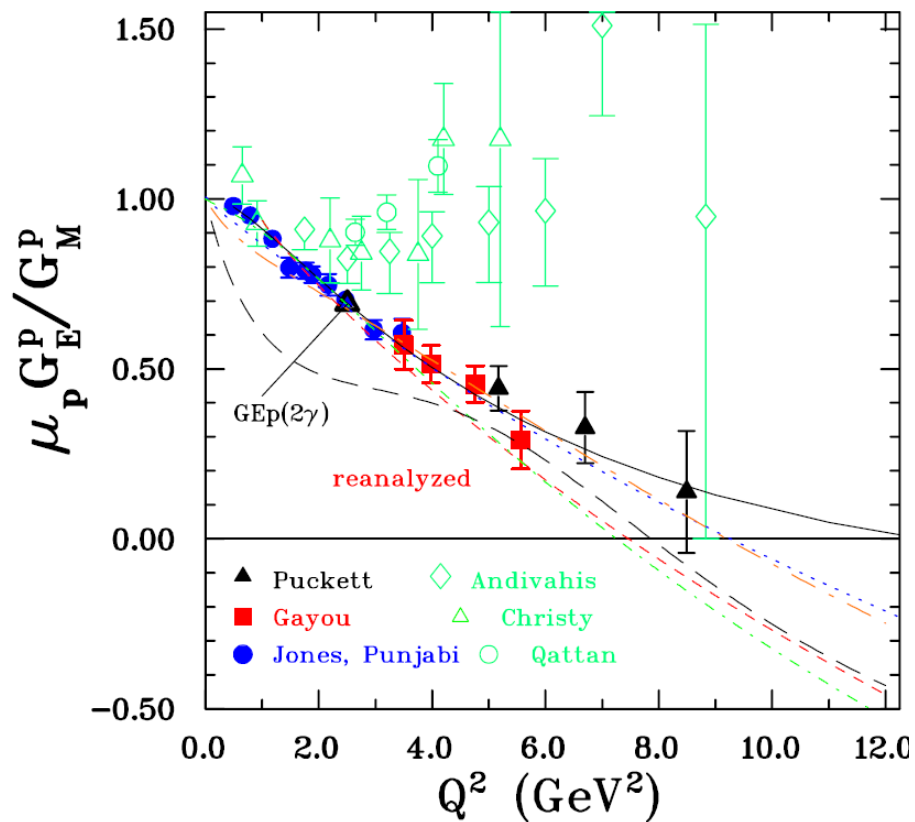
Prediction with two photon exchange  
Guichon, Vanderhaeghen 2003

Polarization technique



Jlab data Mezziane 2011

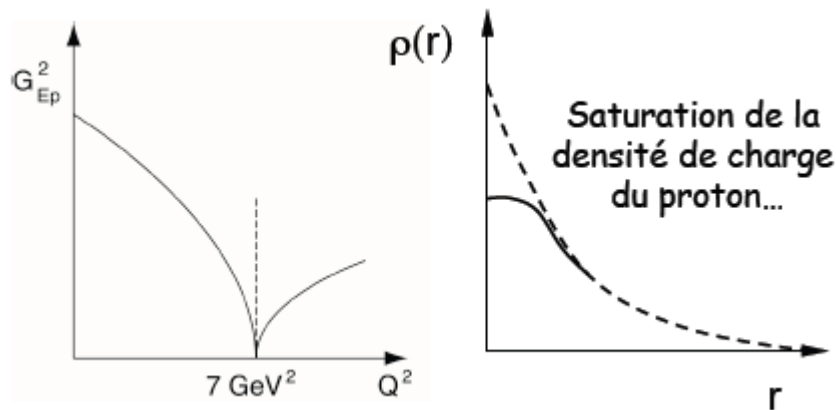
# What we learnt from the $Q^2$ Evolution?



Many models: rQCM, VDM, pQCD

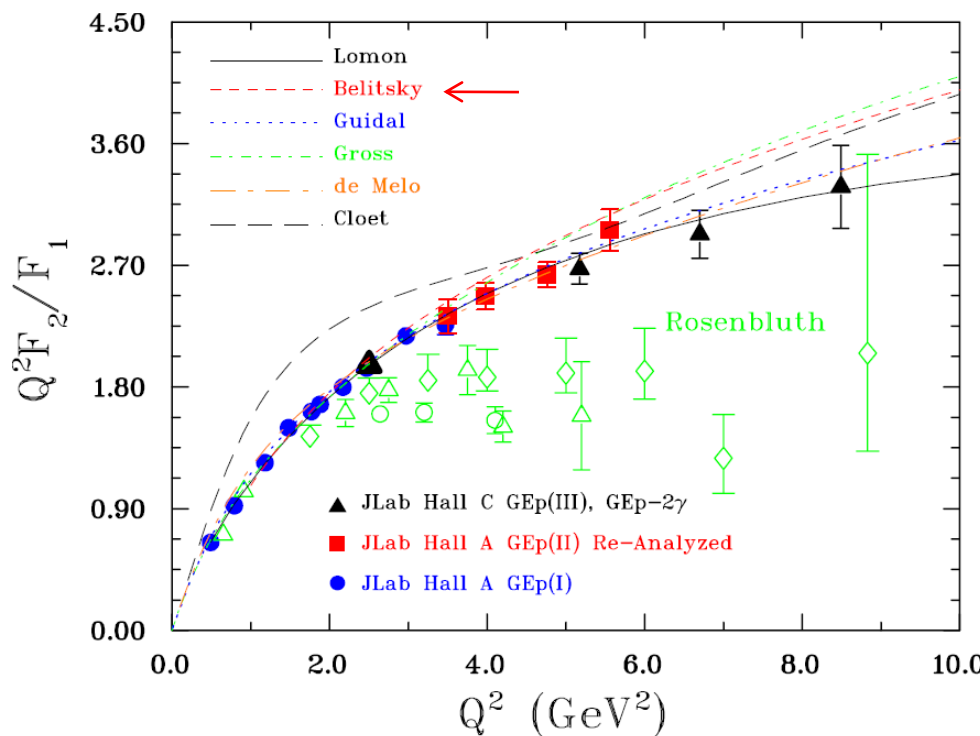
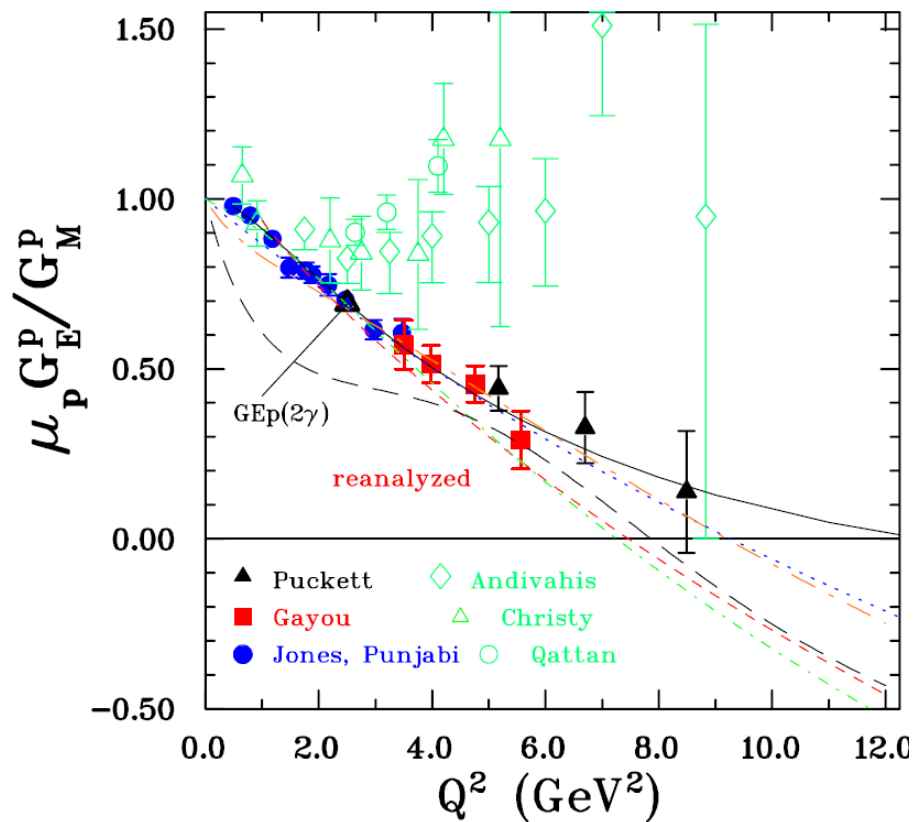
$$\mu_p G_{Ep} / G_{Mp}$$

crossing 0 at  $Q^2=?$   
 saturation of charge density





# What we learnt from the $Q^2$ Evolution?



- Far from the scaling regime even at  $Q^2 = 10 \text{ GeV}^2$
- No early scaling as in DIS

pQCD scaling :  $Q^2 F_2 / F_1$  should be constant

$$F_1(Q^2) \simeq \frac{1}{Q^4}$$

$$F_2(Q^2) \simeq \frac{1}{Q^6}$$

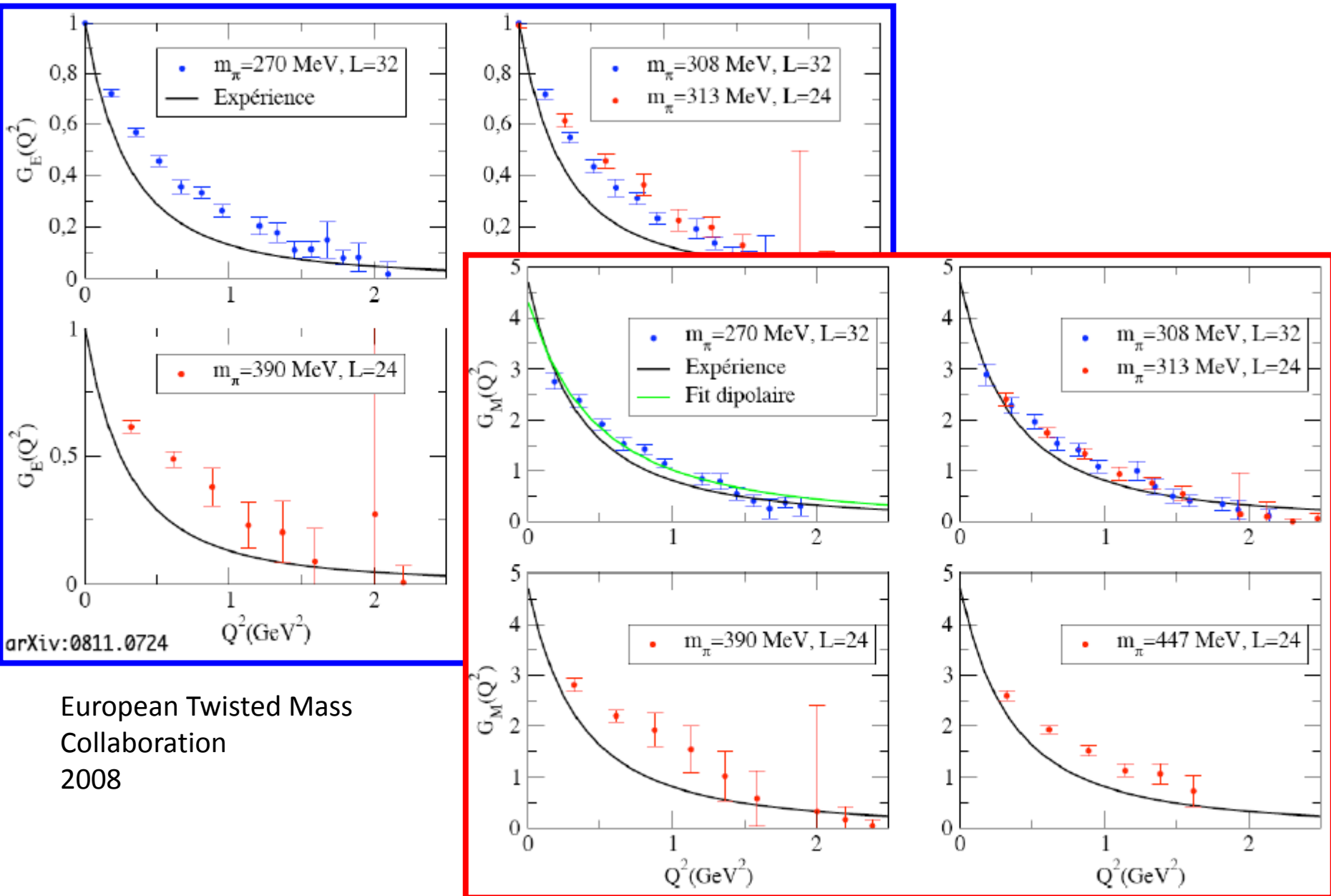
$$G_E \sim G_M \sim \frac{1}{Q^4}$$

Belitsky, Ji, Yuan (2003)

$$Q^2 F_2 / F_1 \rightarrow \ln^2(Q^2 / \Lambda^2)$$

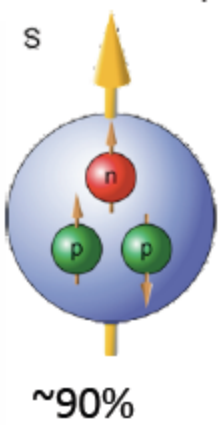
related to quark OAM

# Lattice QCD calculations



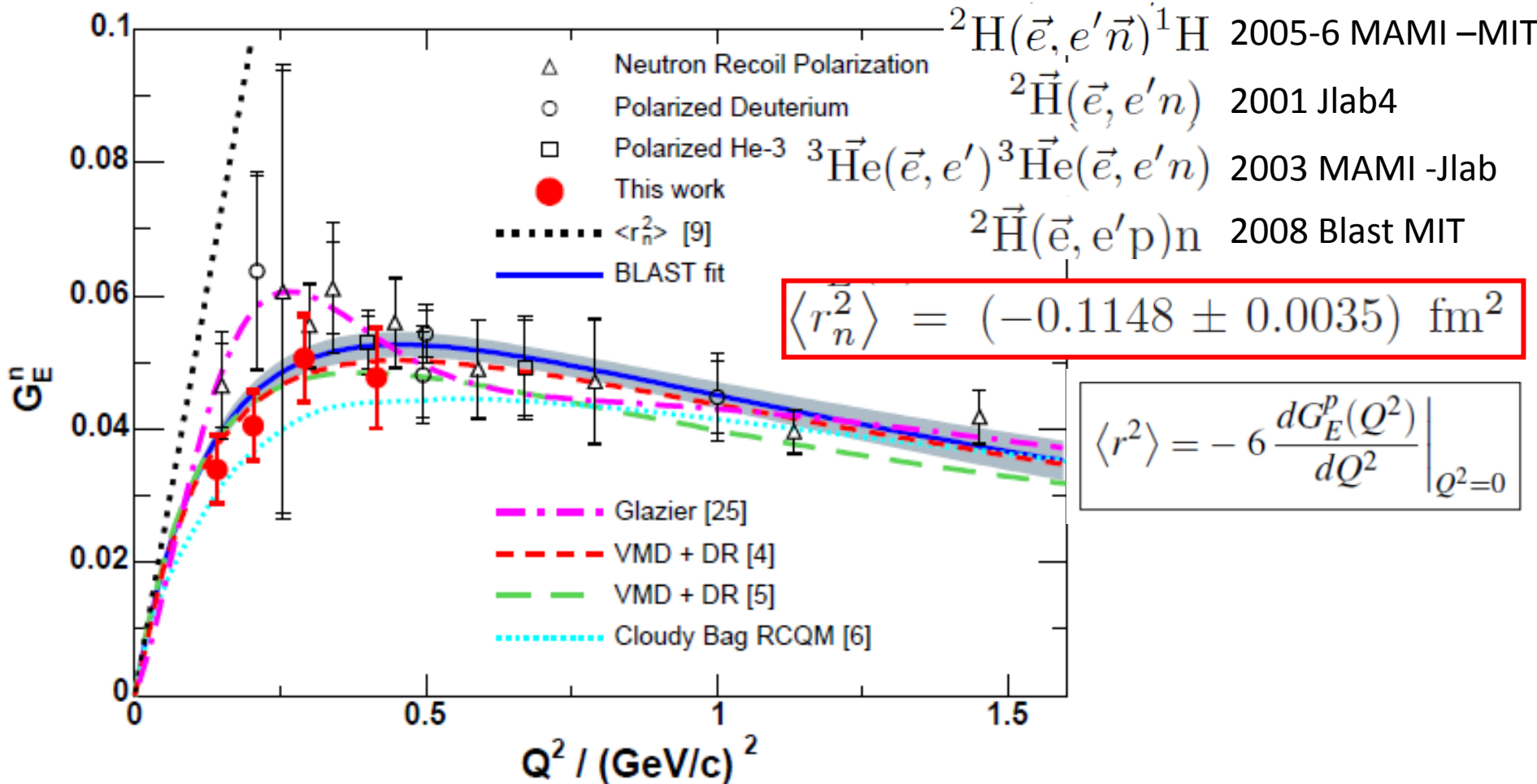
European Twisted Mass  
Collaboration  
2008

# Neutron Form Factor as a function of $Q^2$



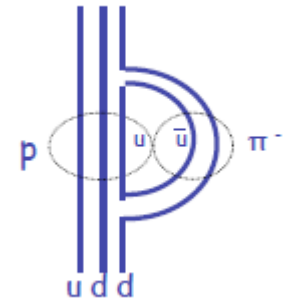
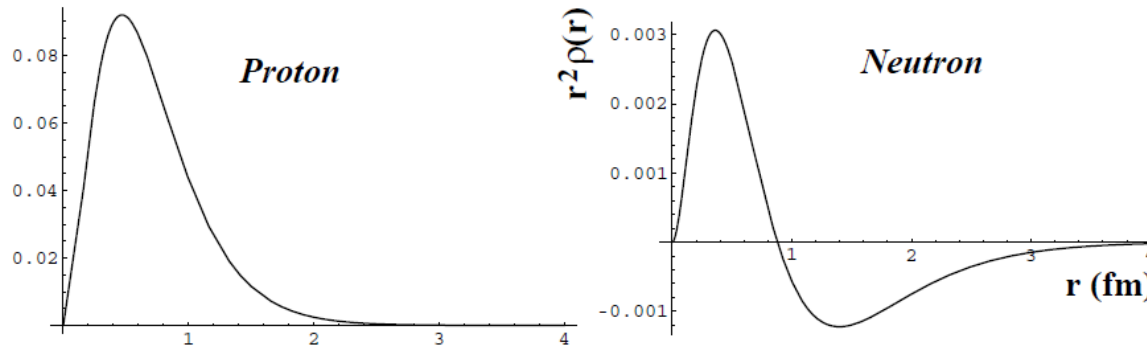
No stable free neutron target

Neutron form factors measured using quasi-elastic electron scattering from deuteron target (p+n) or polarized  $^3\text{H}_e$  target



# Meaning of $G_e^p$ and $G_E^n$

The proton and neutron charge distributions obtained from Fourier Transform of  $G_e^p$  and  $G_E^n$



neutron=proton+  $\pi$  cloud

$$Q = \int \rho(r) 4\pi r^2 dr = 0$$

$$\langle r_n^2 \rangle = \int r^2 \rho(r) 4\pi r^2 dr < 0$$

$$\langle \bar{r}_n^2 \rangle = (-0.1148 \pm 0.0035) \text{ fm}^2$$

# What is the real size of the proton ?



The Muonic hydrogen Lamb shift gives the most precise measurement of the proton charge radius with an unprecedented precision of 0.1%

This value is much smaller (5 standard deviations) than the other measurements using e-p scattering and standard Lamb shift.

## UNEXPECTED !

Publication: 8 July 2010  
and quickly spread on all the media

Then:

- ✓ 16 theoretical papers in 2 months
- ✓ several planned experiments

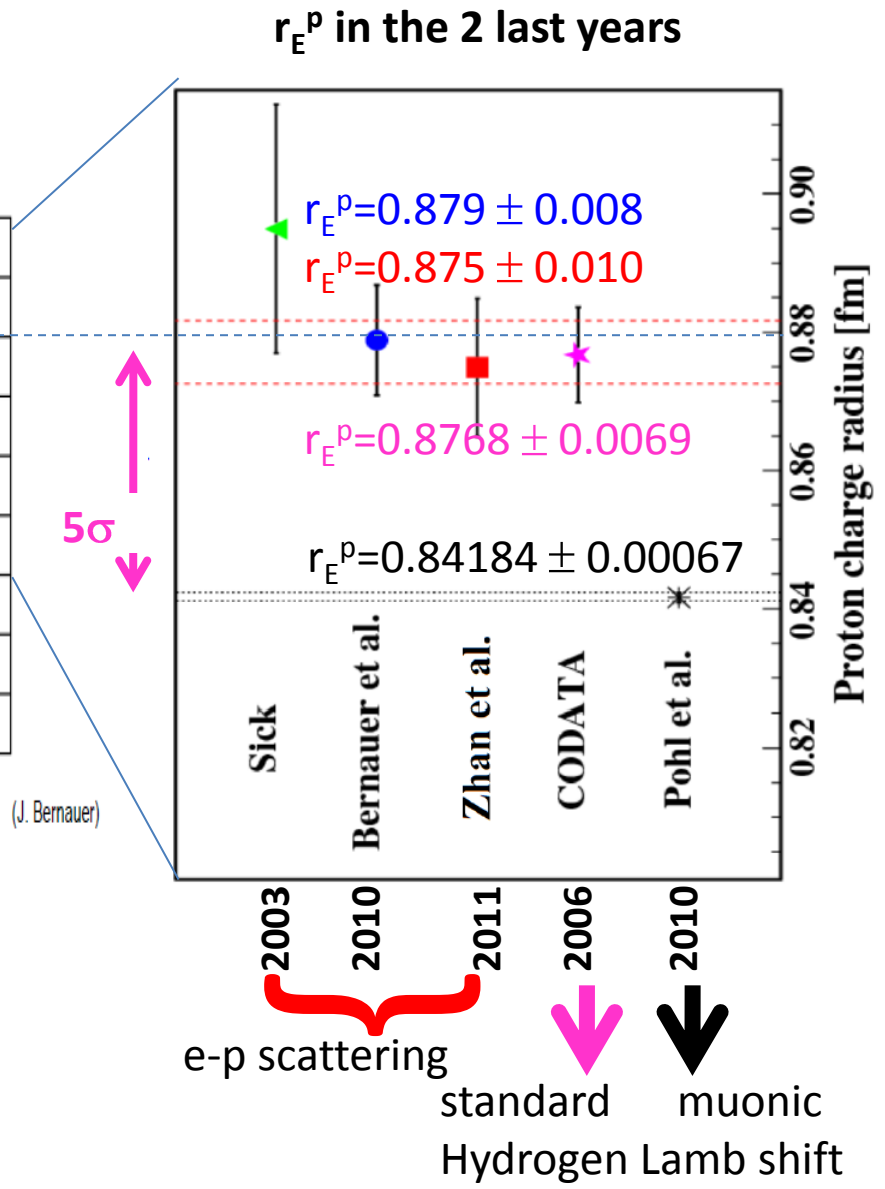
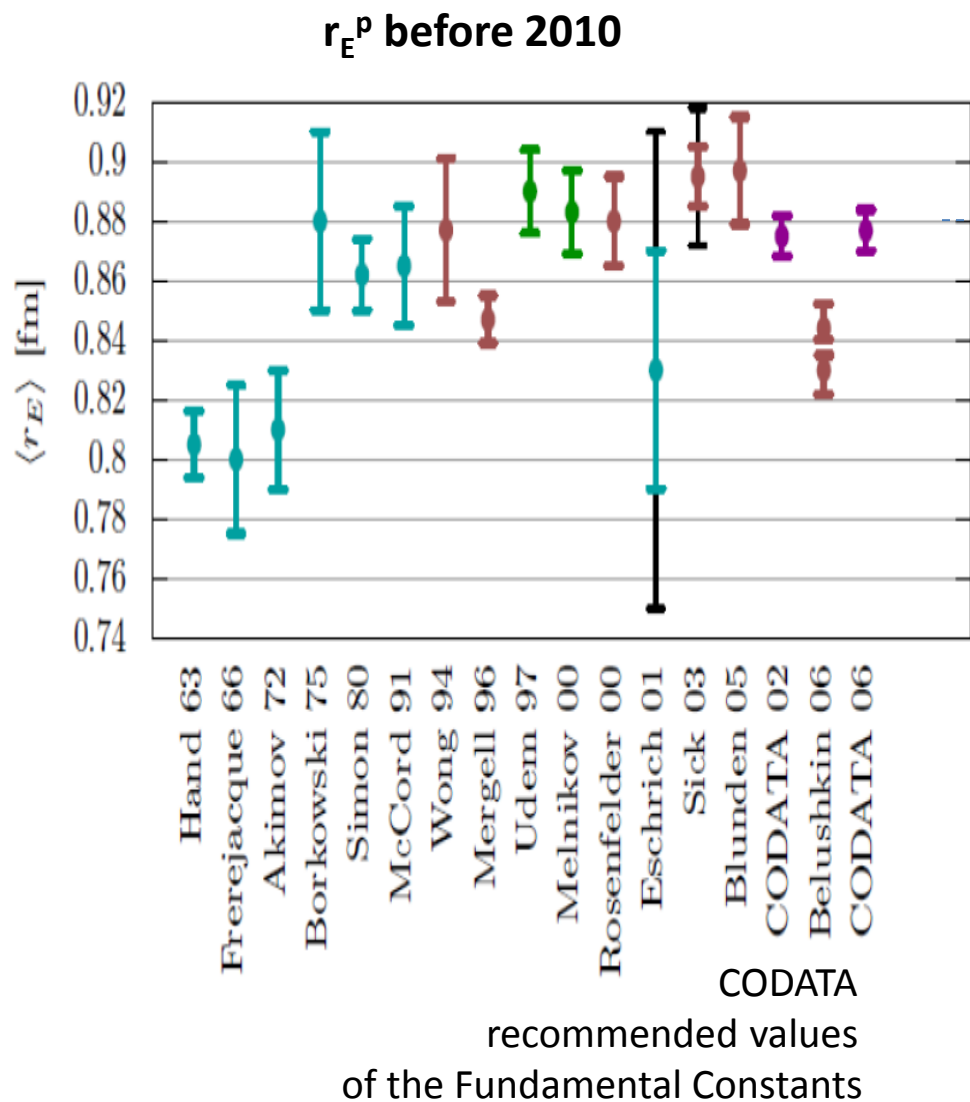
# Proton Form Factor

Focus on measurements at large  $Q^2$

but what is the situation at low and very low  $Q^2$  ?

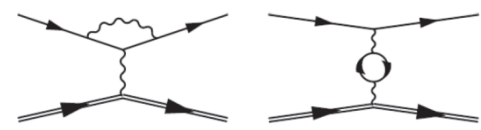
- 2010 (*PRL105 MAMI Bernauer et al.*) :  
The lowest  $Q^2$  is  $4 \times 10^{-3} \text{ GeV}^2$  , super Rosenbluth separation
- 2011 (*Jlab Zhan et al.*) :  
 $Q^2 \in [0.3 : 0.7] \text{ GeV}^2$  , Polarization technique
- Planned experiment  $Q^2 [2 \times 10^{-4} : 2 \times 10^{-2}] \text{ GeV}^2$  in the future at JLab

# Results for the proton charge radius

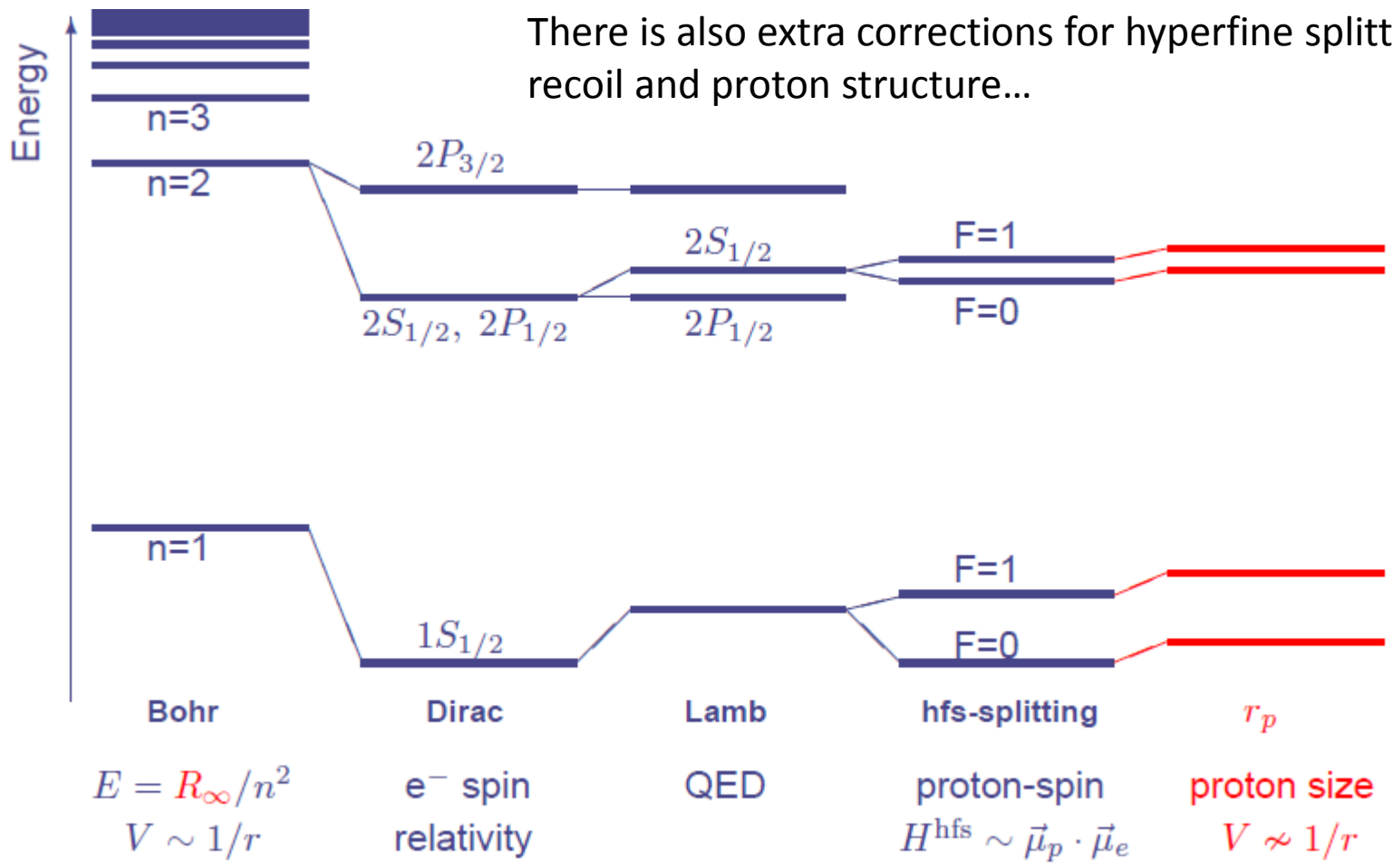


# Lamb shift

(1947) subtle difference between the binding energies of the  $2S_{1/2}$  and  $2P_{1/2}$  (pure radiative QED effects such as 'self energy' and 'vacuum polarization')



There is also extra corrections for hyperfine splitting, recoil and proton structure...





# Muonic hydrogen Lamb shift and proton radius

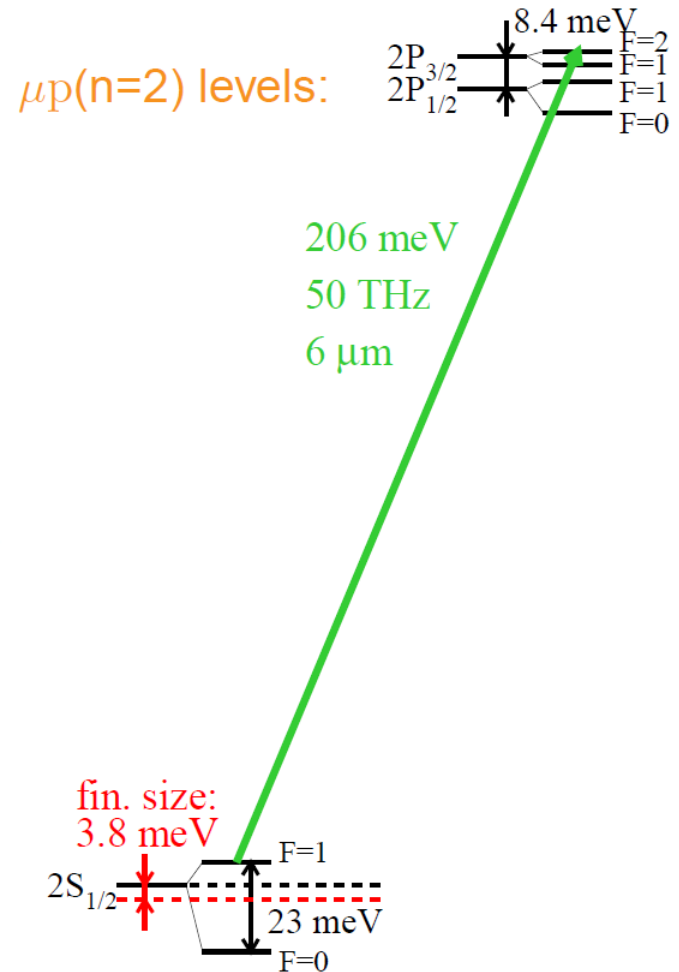
muonic hydrogen =  $\mu^- p$  mass  $m_\mu = 207 m_e$

$$\Delta E_{\text{finite size}}(nl) = \frac{2(Z\alpha)^4 c^4}{3\hbar^2 n^3} m_r^3 r_p^2 \delta_{l0}$$

Lamb shift in  $\mu p$ :  $\Delta E(2P_{3/2}^{F=2} - 2S_{1/2}^{F=1}) =$

$$209.9779(49) - 5.2262 r_p^2 + 0.0347 r_p^3 \text{ [meV]}$$

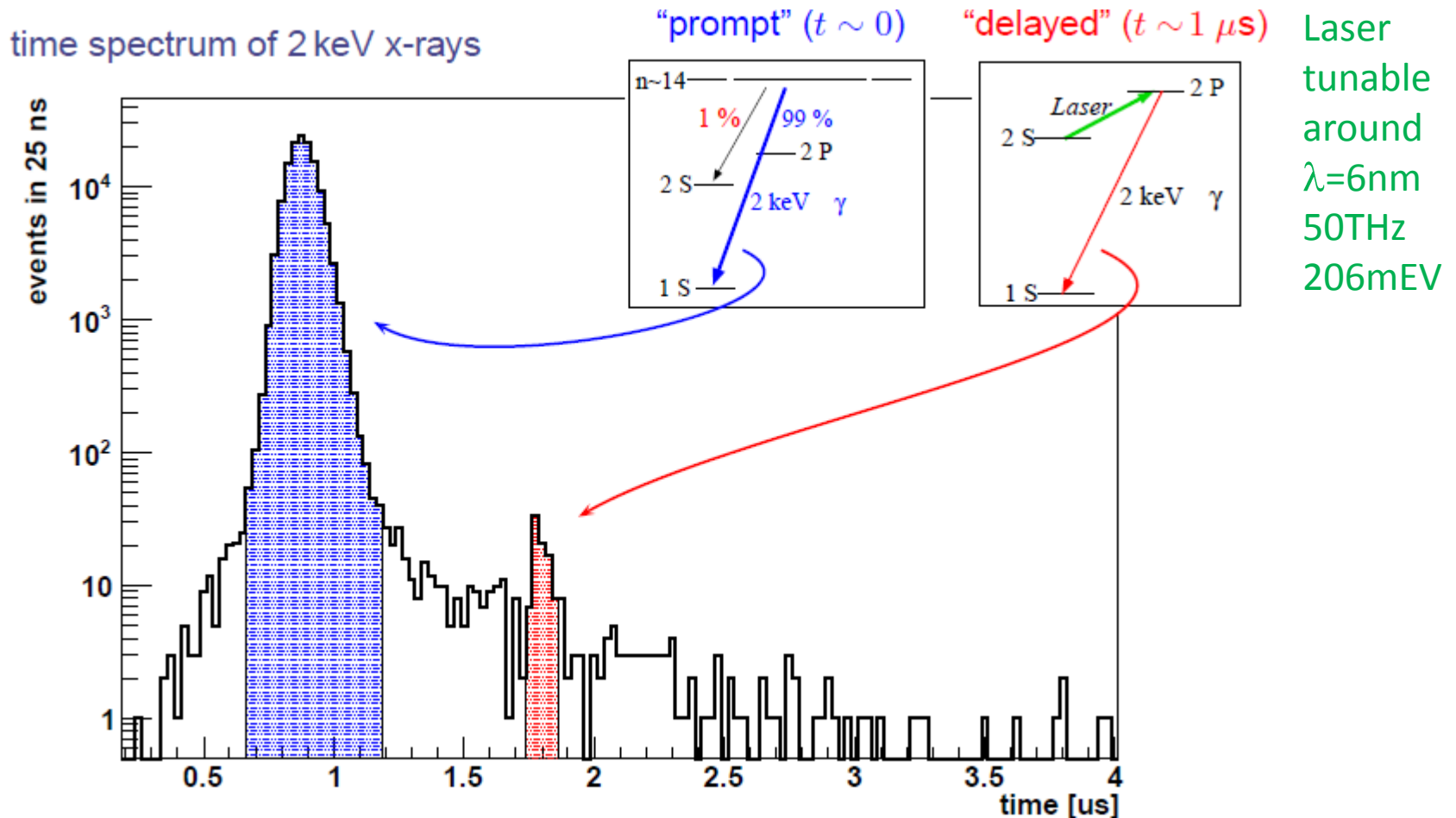
finite size contribution is 2% of the  $\mu p$  Lamb shift



# $\mu\text{p}$ Lamb shift experiment: principle

New 5keV muon beam line at PSI

Muons stopped in  $\text{H}_2$  gas at low pressure  $\rightarrow$  excited  $\mu\text{p}$  atoms ( $n=14$ ) are formed

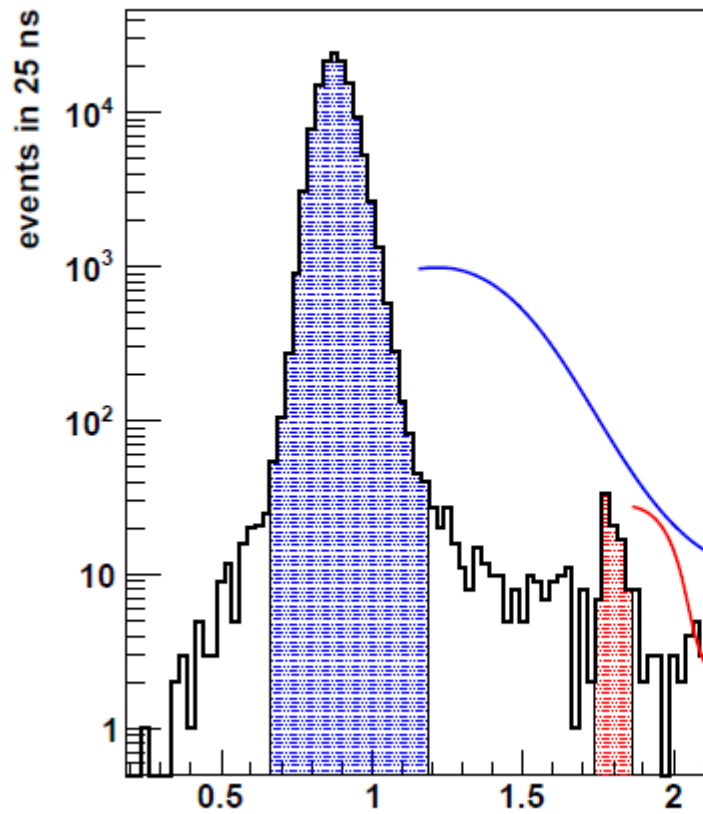


# $\mu\text{p}$ Lamb shift experiment: principle

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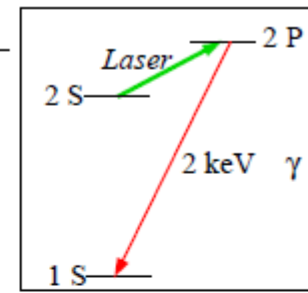
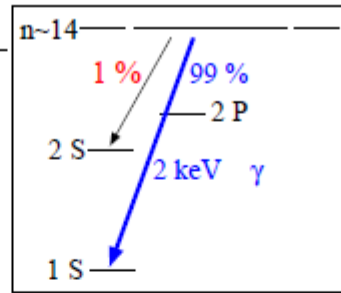
Muons stopped in H<sub>2</sub> gas at low pressure → excited  $\mu\text{p}$  atoms ( $n=14$ ) are formed

time spectrum of 2 keV x-rays



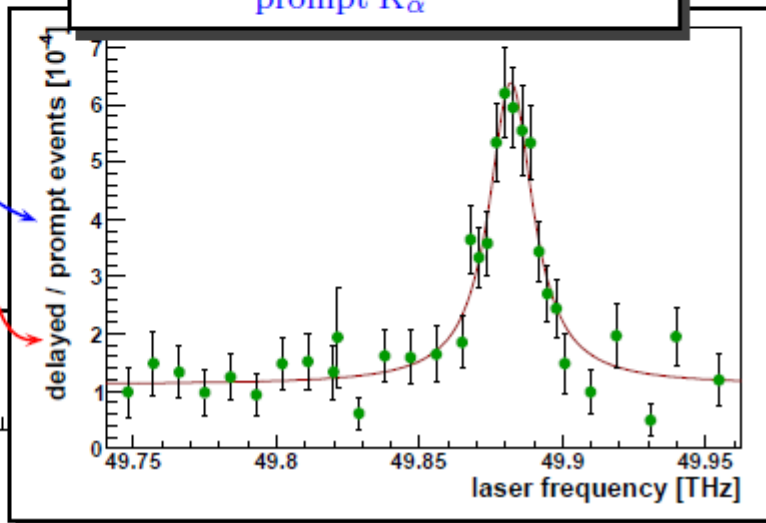
“prompt” ( $t \sim 0$ )

“delayed” ( $t \sim 1 \mu\text{s}$ )

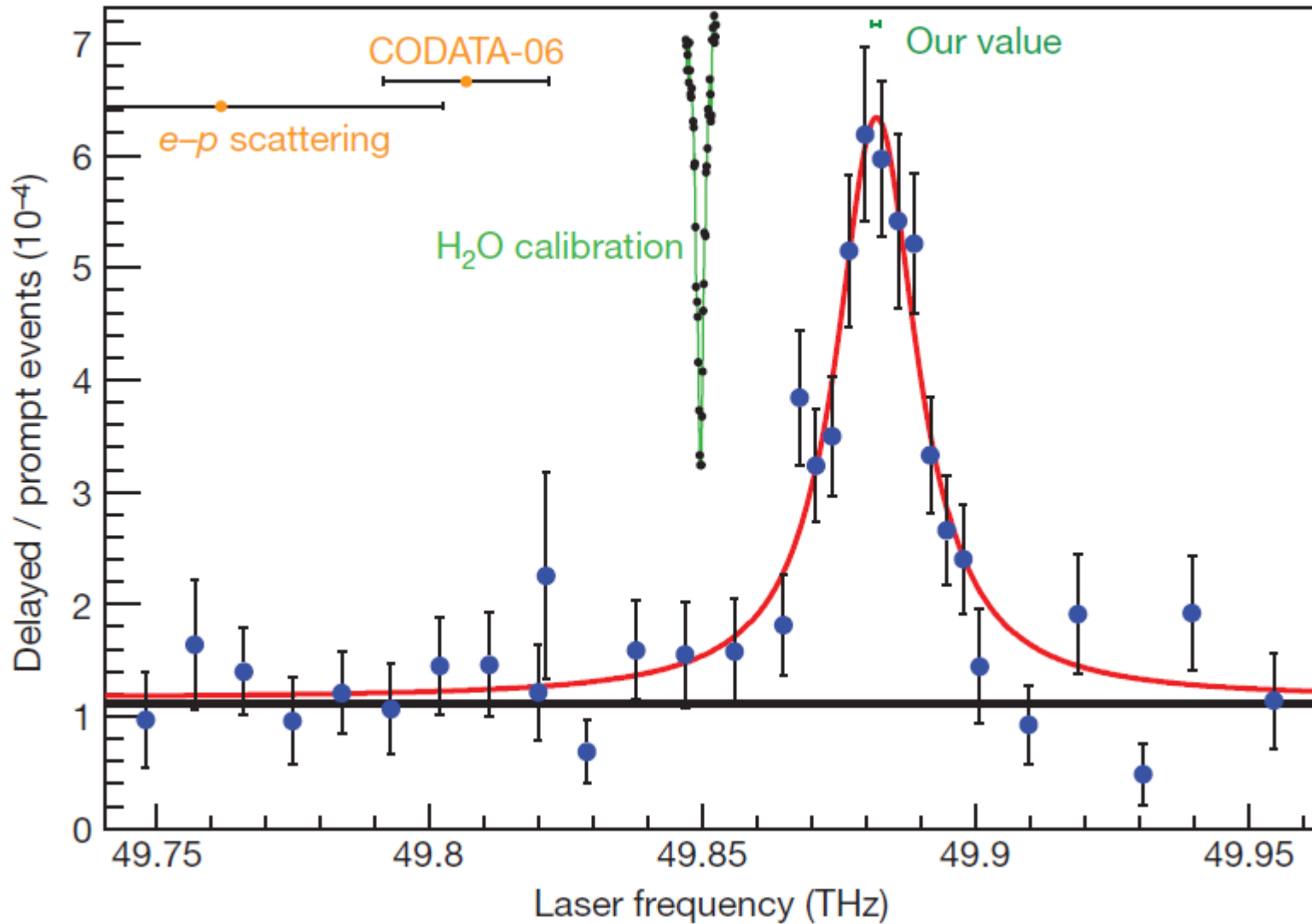


Laser  
tunable  
around  
 $\lambda=6\text{nm}$   
50THz  
206mEV

normalize  $\frac{\text{delayed } K_{\alpha}}{\text{prompt } K_{\alpha}} \Rightarrow \text{Resonance}$



# $\mu p$ Lamb shift result



# UNEXPECTED ! Skrinking the proton

Conclusion: what is wrong ?

- Spectroscopy: Missing element in the QED corrections  
in the bound  $\mu p$  system...
- Lepton Scattering: Need of very precise and very low  $Q^2$  data...
- ...

The proton, already an old-fashioned objet,  
but still embedded in exciting challenges

# Deep Inelastic Scattering

## Parton Distribution Functions (PDF (x))

unpolarized => High precision

polarized => Nucleon Spin Crisis Consolidation

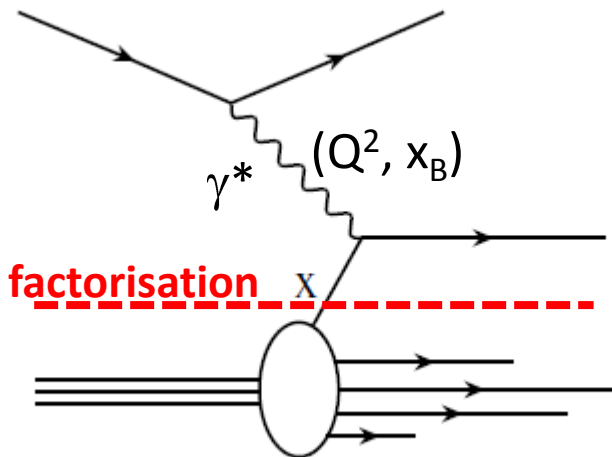
## More on transverse information

momentum: Transverse Momentum Dependent PDF => Exploration in 3D

position: Generalized Parton Distributions (GPD)

# Probing: Lepton-nucleon scattering

## Deep Inelastic Scattering:



longitudinal size contracted  
time dilatation

$\text{time}_{\text{hadronization}} \gg \text{time}_{\gamma^*q \text{ interaction}}$

$$\sigma_{\text{DIS}}(ep \rightarrow e X) = \sum_q \sigma_{\text{elastic}}(eq \rightarrow eq)_{\text{incoherent}}$$

## Quark parton model (QPM)

- Point-like, non-interacting partons
- Collinear to the nucleon movement in photon-nucleon collision (longitudinal direction)
- Each parton carries a fraction  $x$  of the nucleon momentum and for the struck parton:  $x = x_B$
- Scaling: observables function of  $x$  (at first order)

The inclusive cross section is described by 4 structure functions:

**unpolarized**

$F_1(x)$ ,  $F_2(x)$ ,

$$F_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 q(x) = \frac{F_2(x)}{2x}$$

$$q(x) = q^+(x) + q^-(x)$$

probability of finding a quark with a fraction  $x$  of the nucleon longitudinal momentum

**polarized**

$g_1(x)$ ,  $g_2(x)$

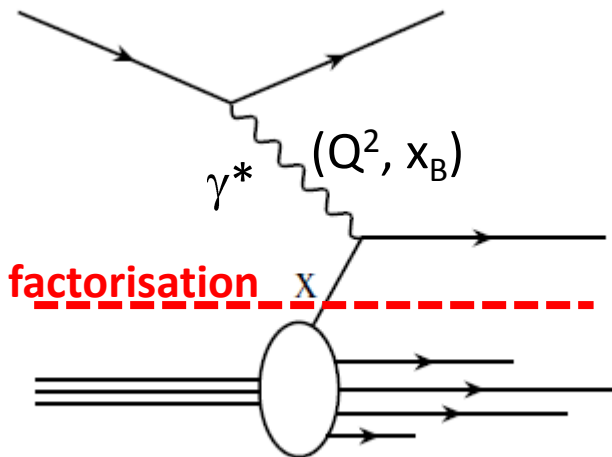
$$g_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x) \quad g_2(x) = 0$$

$$\Delta q(x) = q^+(x) - q^-(x)$$

probability of finding a quark with a momentum fraction  $x$  and spin // to that of the nucleon

# Probing: Lepton-nucleon scattering

## Deep Inelastic Scattering:



longitudinal size contracted  
time dilatation

$\text{time}_{\text{hadronization}} \gg \text{time}_{\gamma^*q \text{ interaction}}$

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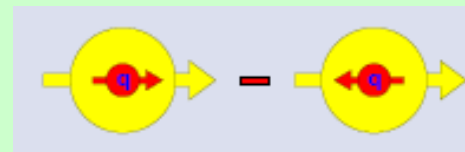
**polarized**

$g_1(x)$ ,  $g_2(x)$

$$g_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x)$$

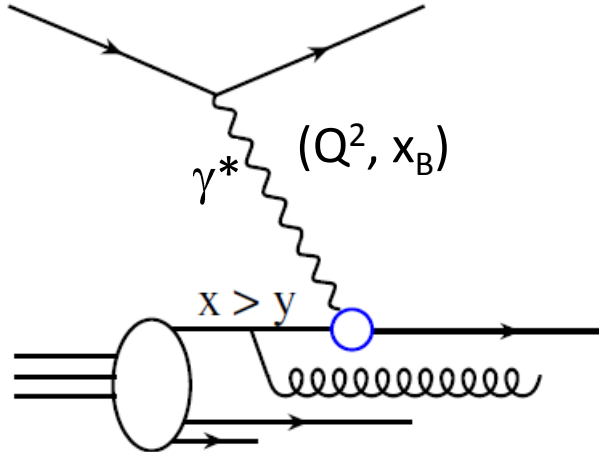
$$g_2(x) = 0$$

$$\Delta q(x) = q^+(x) - q^-(x)$$





# Probing: Lepton-nucleon scattering



## Deep Inelastic Scattering:

$$\sigma_{\text{DIS}}(ep \rightarrow e X) = \sum_q \sigma_{\text{elastic}}(eq \rightarrow eq)$$

incoherent

## QCD improved parton model

- at finite  $Q^2$ , partons interact
- ➔ PDFs and structure functions depend on  $Q^2$
- ➔ gluons are visible (in the  $Q^2$  evolution)

The inclusive cross section is described by 4 structure functions:

**unpolarized**

$$F_1(x, Q^2), F_2(x, Q^2),$$

$$F_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 q(x) = \frac{F_2(x)}{2x}$$

$$q(x) = q^+(x) + q^-(x)$$

➔ quark PDF  $q(x, Q^2)$   
 and gluon PDF  $G(x, Q^2)$  from  $F_1$  evolution

**polarized**

$$g_1(x, Q^2), g_2(x, Q^2)$$

$$g_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x)$$

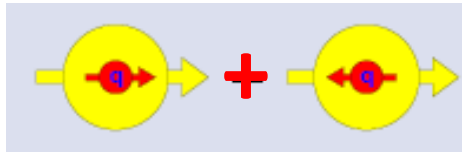
$$g_2(x) = 0$$

$$\Delta q(x) = q^+(x) - q^-(x)$$

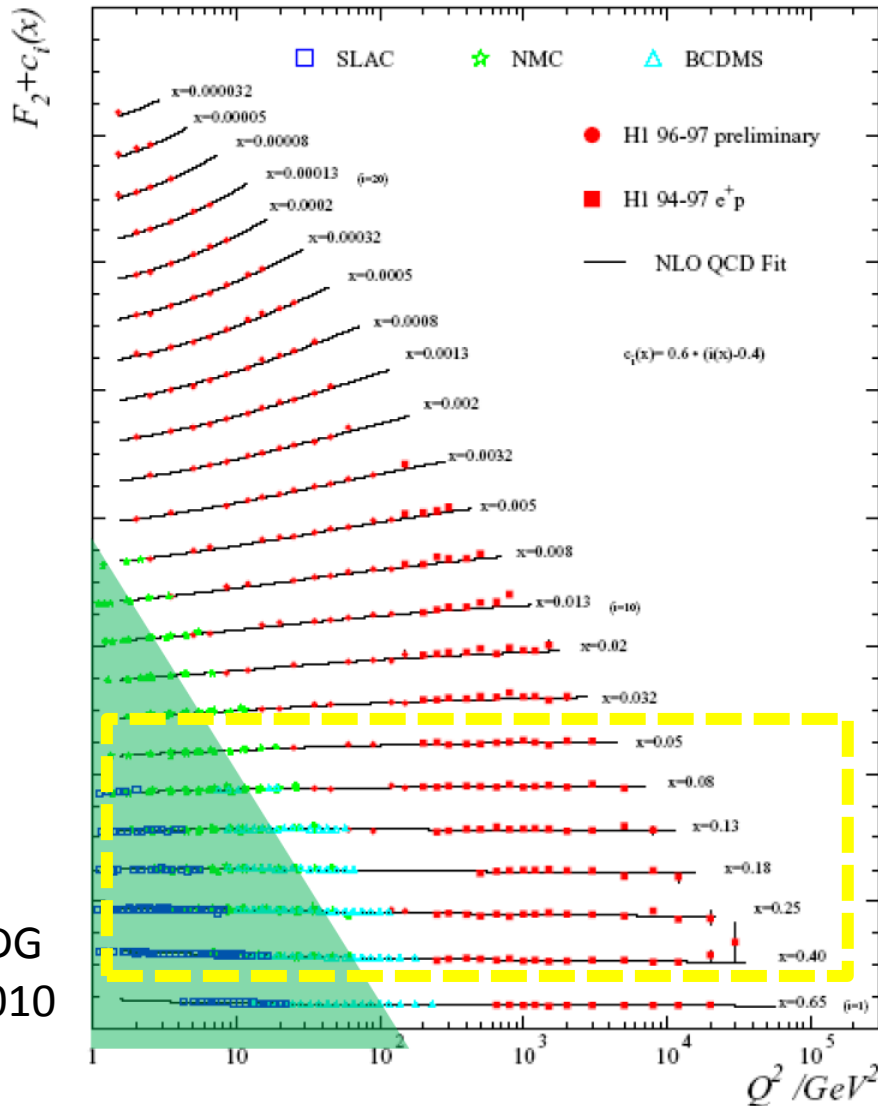
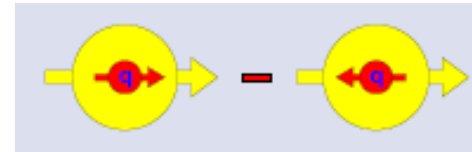
$$\Delta q(x, Q^2)$$

$$\Delta G(x, Q^2) \text{ from } g_1 \text{ evolution}$$

# $F_2(x, Q^2)$

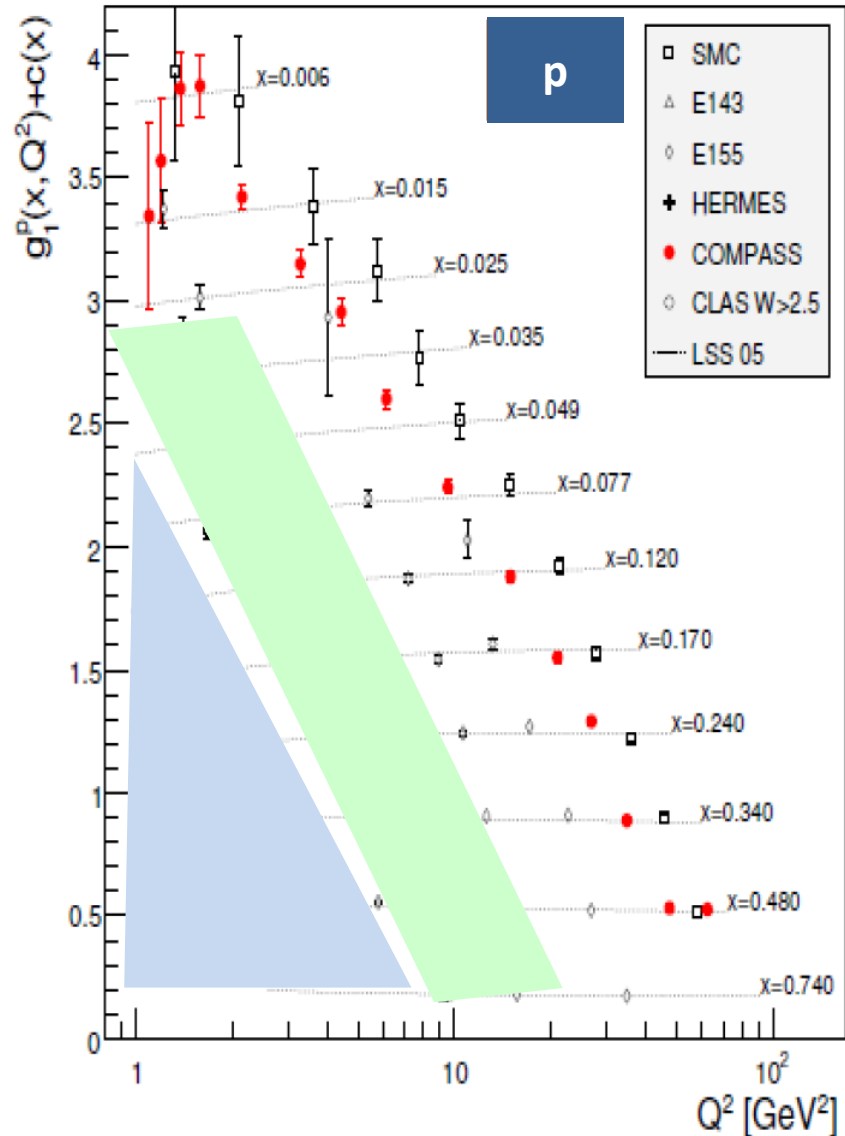


# $g_1(x, Q^2)$



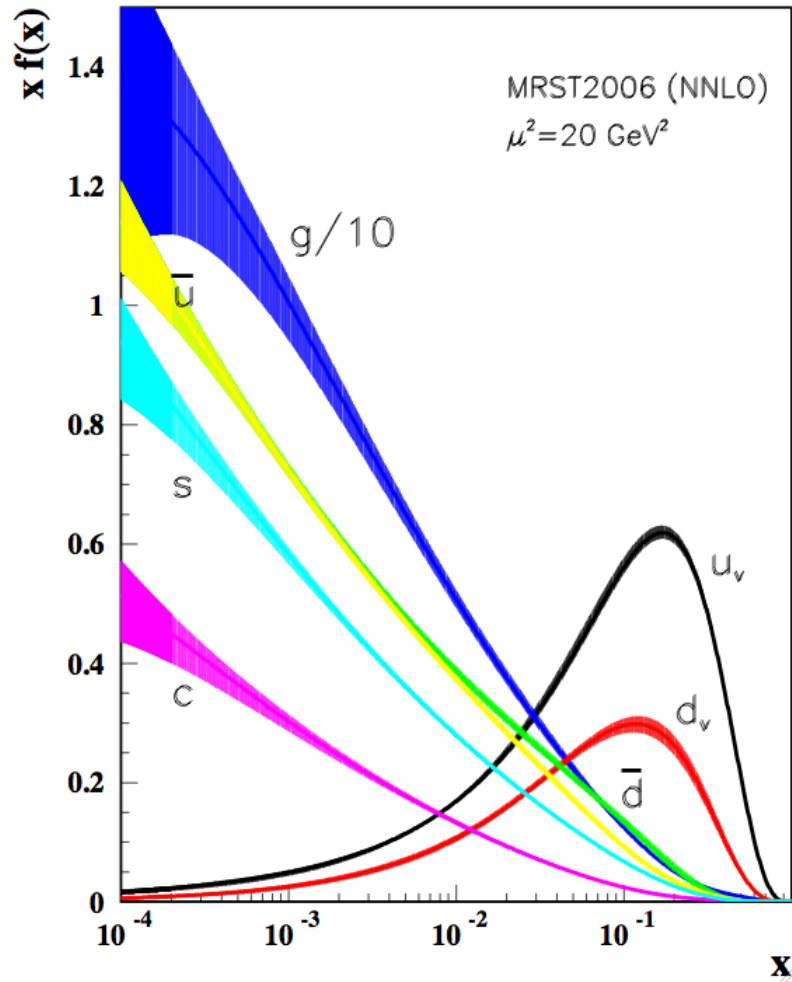
PDG 2010

Very gentle QCD evolution  
Factorisation scale  $Q^2$  few  $\text{GeV}^2$



Still not so well known

# Unpolarized quark and gluon PDF

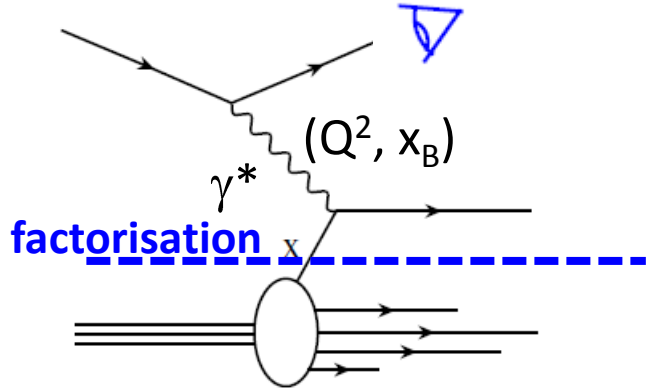


Global fit  
on the world data

# Universality of PDF

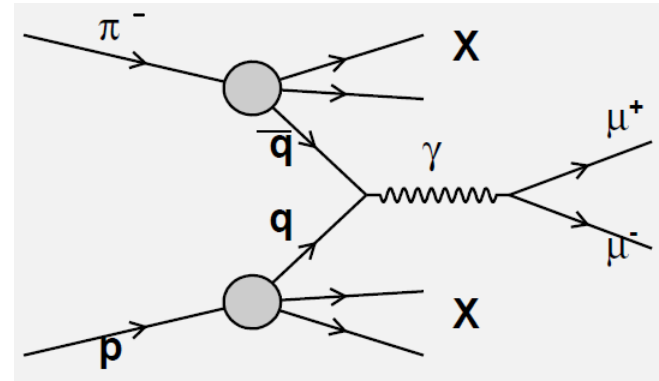
Physical cross section = cross section for partonic process ( $\gamma^* q \rightarrow q$  or  $q q \bar{q} \rightarrow \gamma^*$ )  $\times$  PDF

Deep Inelastic Scattering (DIS)  $\ell p \rightarrow \ell' X$



*Factorisation: scheme dependent!*

Drell Yan (DY)  $\pi p \rightarrow \ell^+ \ell^- X$



# nucleon spin crisis

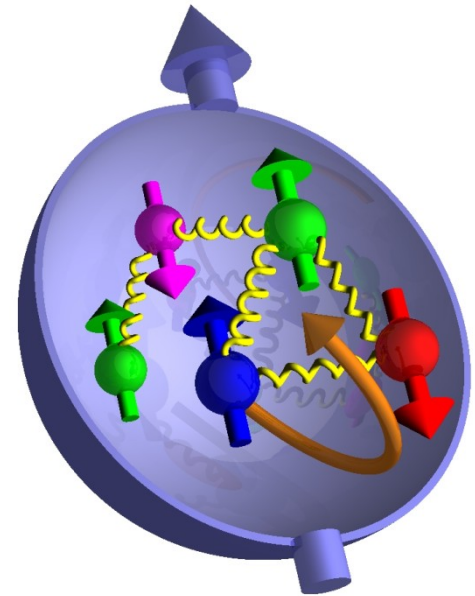
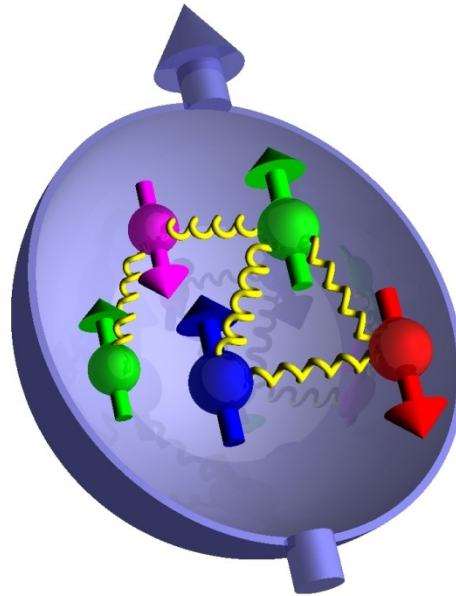
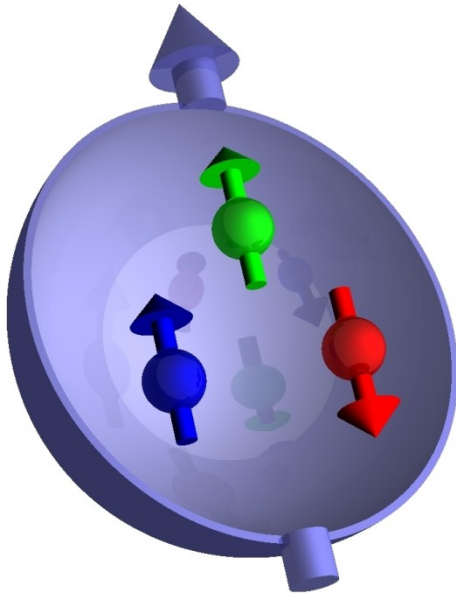
$$\Delta q(x) \quad \text{and} \quad \Delta q = \int_0^1 \Delta q(x) dx$$

$$\Delta G(x) \quad \text{and} \quad \Delta G = \int_0^1 \Delta G(x) dx$$

Need of longitudinally polarized beam

and longitudinally polarized target

# The spin of the nucleon



Naive quark parton model

$$\Delta\Sigma = \Delta u_v + \Delta d_v = 1$$

Note  $\Delta$  means  $\int_0^1 dx$

QCD: sea quarks and gluons

$$\Delta q_s, \Delta G$$

Orbital angular momentum

$$L_z^q, L_z^g$$

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + L_z^q + L_z^g$$

$$\Delta\Sigma = \Delta u + \Delta d + \Delta s$$

$\sim 0.6$  QCD with  $\Delta s = 0$  (Ellis-Jaffe)

$\sim 0.12 \pm 0.17$  EMC (1987) surprise!

# The structure function $g_1$ and its first moment

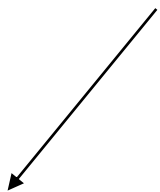
$$g_1(\mathbf{x}) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x) \quad \Gamma_1 = \int_0^1 g_1(\mathbf{x}) dx$$

In naive QPM + SU(3)

$$\begin{aligned} \Gamma_1^p &= \frac{1}{2} \left\{ \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right\} \quad \text{with} \quad \Delta q = \int \Delta q(x) dx \\ &= \frac{1}{12} \underbrace{(\Delta u - \Delta d)}_{a_3} + \frac{1}{36} \underbrace{(\Delta u + \Delta d - 2\Delta s)}_{\sqrt{3}a_8} + \frac{1}{9} \underbrace{(\Delta u + \Delta d + \Delta s)}_{a_0} \end{aligned}$$

General sum rule (OPE)

$$\Gamma_1^{p,n} = \frac{1}{12} \left( \pm a_3 + \frac{1}{\sqrt{3}} a_8 \right) + \frac{1}{9} a_0$$



Neutron decay $g_A/g_V \rightarrow a_3$	hyperon decay $3F-D \rightarrow a_8$
--	---

?  $Q^2 \neq \infty$

$$a_0 = \Delta u + \Delta d + \Delta s \equiv \Delta \Sigma$$

measured first  
by EMC in 1987

$a_0 = 0.12 \pm 0.17$  surprise!

# The structure function $g_1$ and its first moment

$$\mathbf{g}_1(\mathbf{x}) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x) \quad \Gamma_1 = \int_0^1 \mathbf{g}_1(\mathbf{x}) dx$$

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measured first  
by EMC in 1987

A crisis in the parton model:  
where, oh where is the proton's spin?

E. Leader<sup>1</sup> and M. Anselmino<sup>2</sup>

Birkbeck College, University of London, London, UK  
Dipartimento di Fisica Teorica, Università di Torino, I-10125 Torino, Italy

Received 18 March 1988

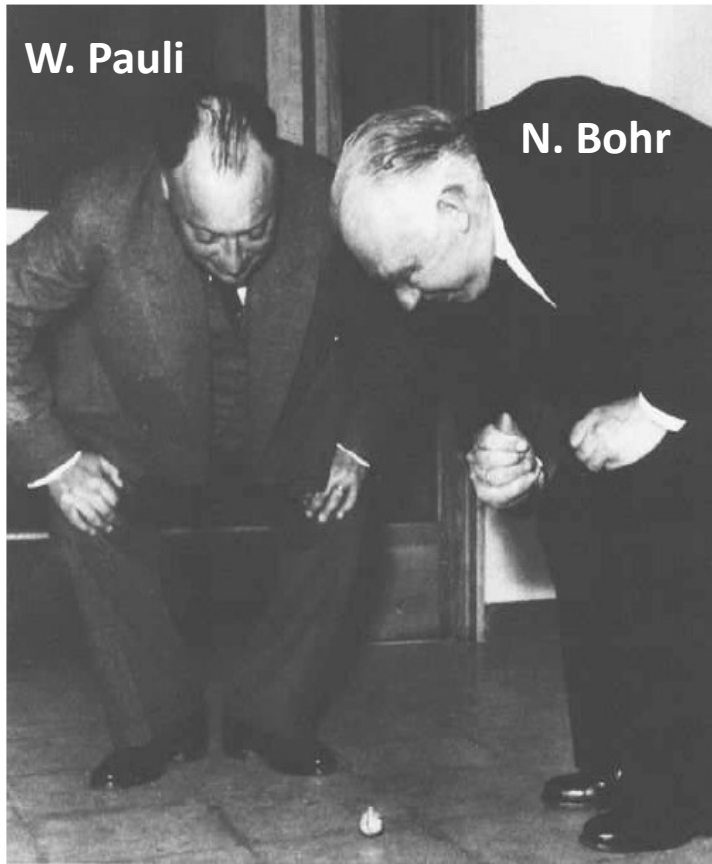
?  $Q^2 \neq \infty$

$$a_0 = \Delta u + \Delta d + \Delta s \equiv \Delta \Sigma$$

$$a_0 = 0.12 \pm 0.17 \quad \text{surprise!}$$



# How to solve the spin crisis?



Pauli and Bohr wondering about a tip top toy (1955)

In polarised DIS, one measures  $a_0$  flavor singlet axial matrix element at  $Q^2 \neq \infty$

- ✓ In the  $\overline{MS}$  renormalisation scheme
- ✓ While in the Adler-Bardeen and JET schemes (axial anomaly)

$$a_0 = \Delta\Sigma - (\alpha_s/2\pi)n_f \Delta G$$

So a large positive value of  $\Delta G$  ( $\sim 3$  at  $Q^2=3 \text{ GeV}^2$ ) might resolve the spin crisis ?



Huge effort during  
the past 20 years  
Today: COMPASS, HERMES, RHIC

- new measurements of  $g_1$ ,  $\Delta q$
- first measurements of  $\Delta G$

# Polarized Beam + Polarized Target ${}^6\text{LiD(d)}$ and $\text{NH}_3(\text{p})$

- to be measured:

$$A_{\parallel} = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}}$$

- flux normalization:

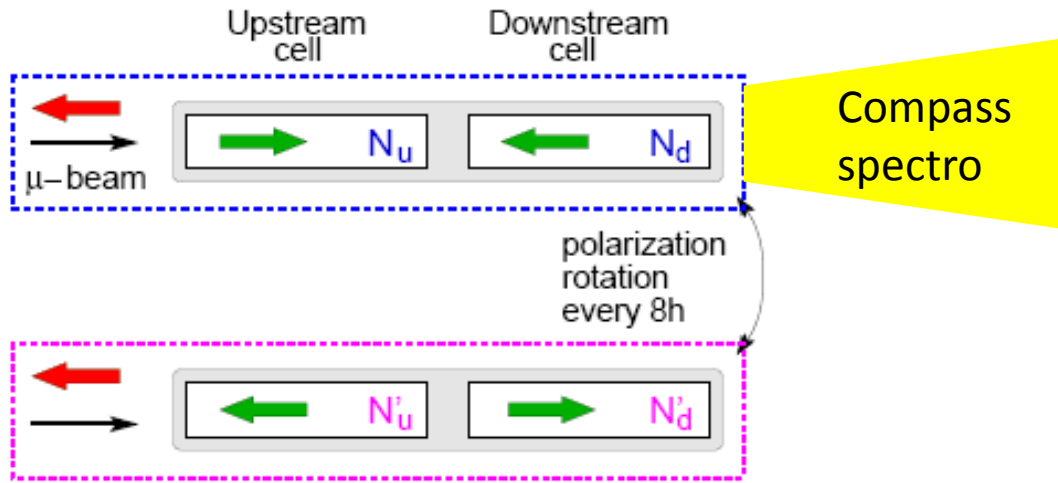
$$A_{\text{exp}} = \frac{N_u - N_d}{N_u + N_d}$$

- acceptance difference:  
Polarisation rotation

- take average asymmetry:

$$\Rightarrow A_{\text{exp}} = \frac{A + A'}{2} = \frac{1}{2} \left( \frac{N_u - N_d}{N_u + N_d} + \frac{N'_d - N'_u}{N'_u + N'_d} \right)$$

$\Rightarrow$  minimization of bias



**elegant and efficient strategy**

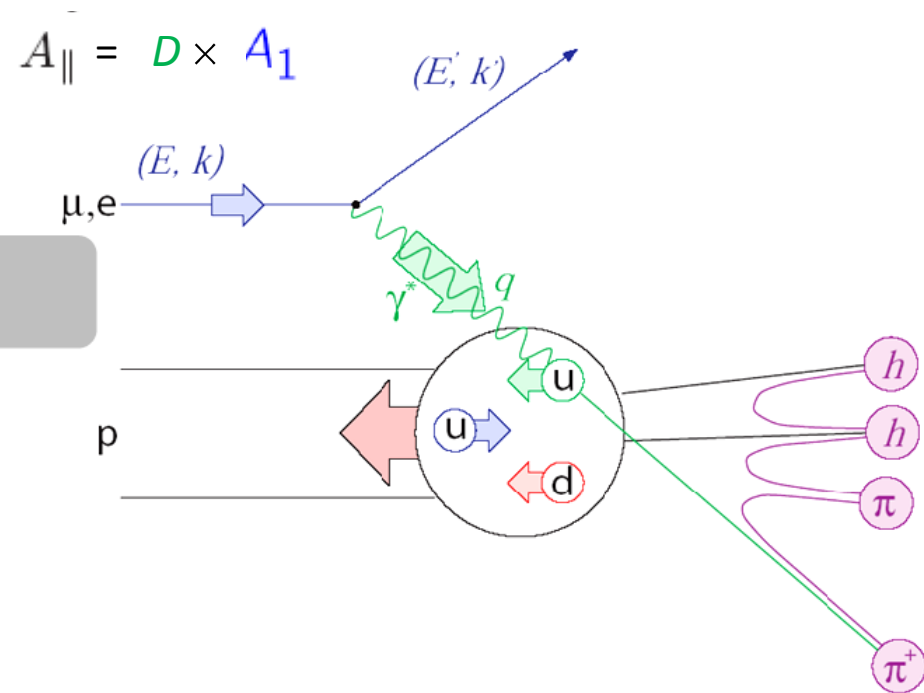
# Asymmetry measurement

$$\frac{A_{\text{exp}}}{f P_{\mu} P_T D} \simeq A_1$$

- Inclusive scattering

$$A_1 = \frac{\sum_q e_q^2 g_1^q(x, Q^2)}{\sum_q e_q^2 f_1^q(x, Q^2)}$$

$D$  depolarization factor  
 $P_{\mu}$  target polarization  
 $P_T$  target polarization  
 $f$  dilution factor



# Spin Structure Function $g_1(x, Q^2)$

Using inclusive data

$$g_1(\mathbf{x}) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x)$$

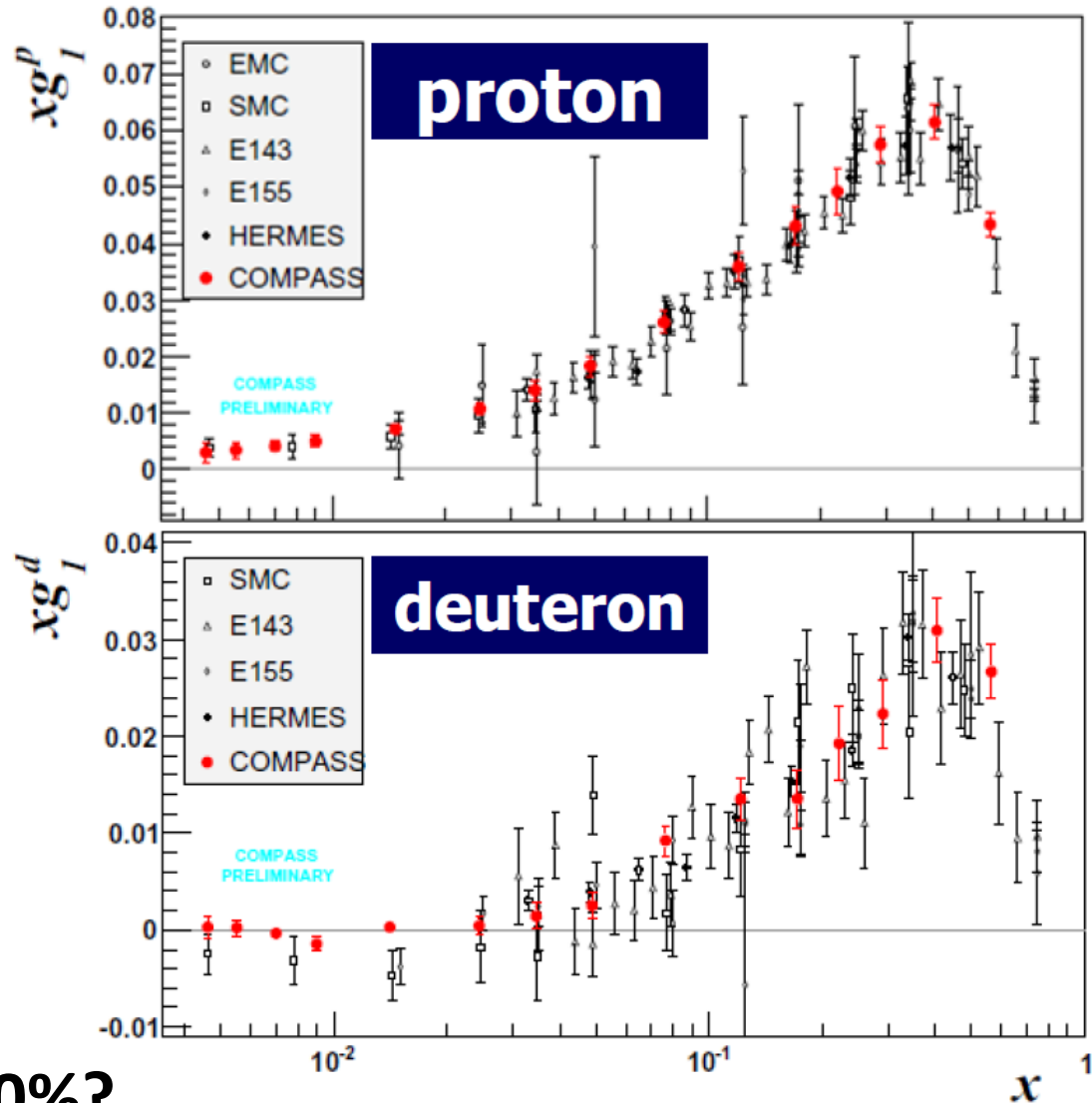
- Very precise data
- Only COMPASS for  $x < 0.01$  ( $Q^2 > 1$ )
- COMPASS Deuteron data from  $\Gamma_1$  @  $Q^2 \rightarrow \infty$

$$a_0 = \Delta \Sigma = 0.33 \pm 0.03 \pm 0.05$$

$$\Delta s + \Delta \bar{s} = 1/3 (a_0 - a_8)$$

$$= -0.08 \pm 0.01 \pm 0.02$$

What makes up  
the missing 70%?



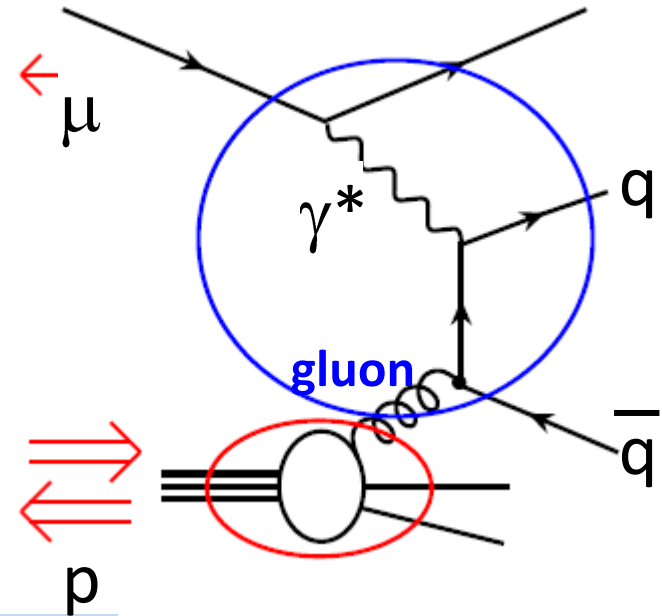
# Direct Access to the Gluon polarisation (SIDIS)

$\ell p \rightarrow \ell' h X$

Measured in **photon-gluon fusion (PGF)**

$$A_{||} = R_{pgf} \hat{a}_{pgf} \frac{\Delta G}{G} + A_{bdf}$$

Fraction of process  $\downarrow$  Analyzing power calculable at LO and NLO  
 Background

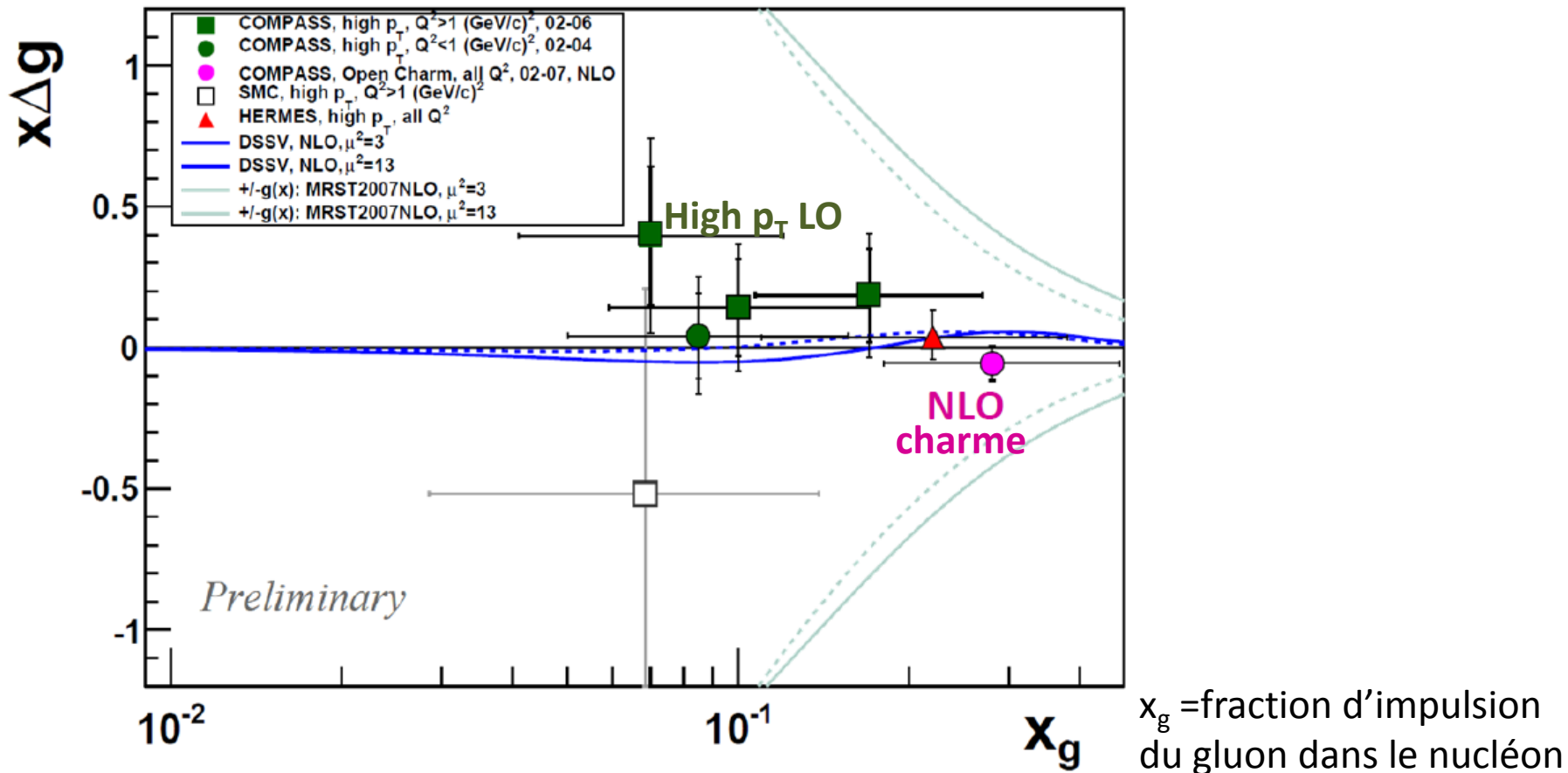


2 signatures to suppress the background:

➤  $q=c$  and charm hadronization in meson  $D^0 (c\bar{u}) \rightarrow K \pi$   
*Detection of  $D^0$  mesons*  
*Very clean signal but limited statistics*

➤ soit  $q=u,d,s$  and  $q \bar{q} \rightarrow \text{hadron1} + \text{hadron2}$   
*Detection of a high  $p_T$  hadron pair*  
*Physical background, better described at high  $Q^2$*

# Gluon Polarization: from LO to NLO



- ❖ All results compatible with  $a_0 = 0$  ! ( $a_0 = \Delta \Sigma$ )
- ❖ Confirmed by RHIC results in pp

Let's go to measure  $L_z$ !

So far the partons were considered collinear to the nucleon movement in photon-nucleon collision (longitudinal direction), but what else about the transverse information?

## **Transverse spin and transverse momentum**

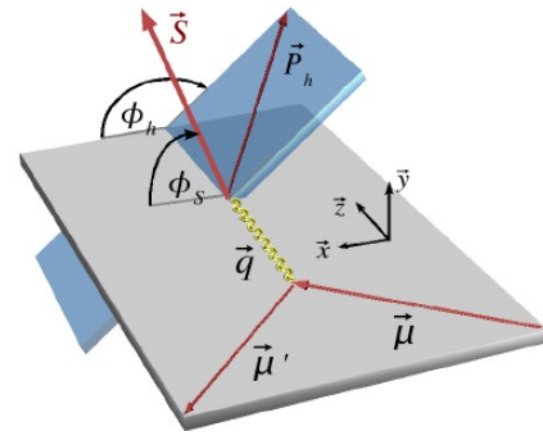
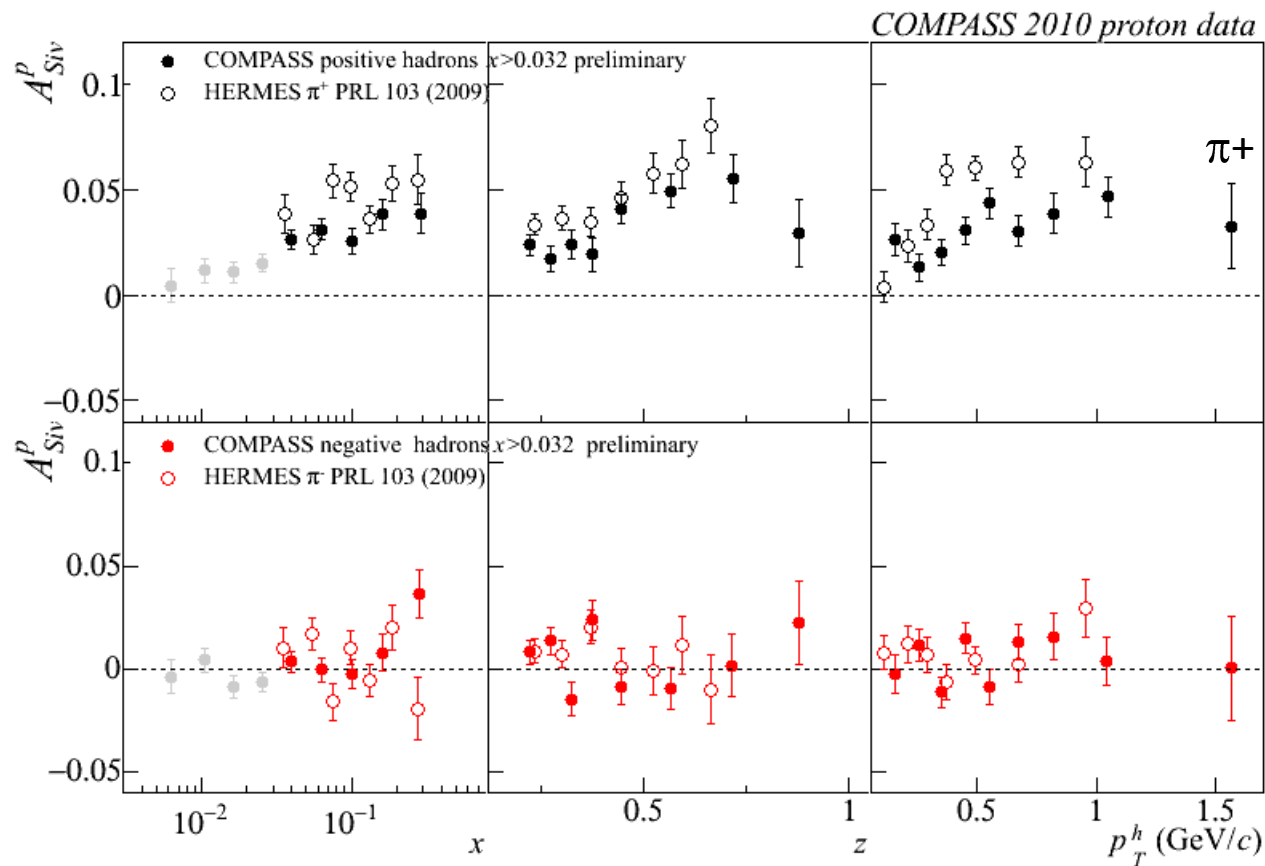


# Proton Sivers effect in tranv. pol. SIDIS: $\ell p \uparrow \rightarrow \ell' h X$

Azimuthal cross-section asymmetry:

$$\frac{\Delta\sigma}{\sigma} \propto A_{Siv} \sin \Phi_S$$

$$\Phi_S = \phi_h - \phi_S$$

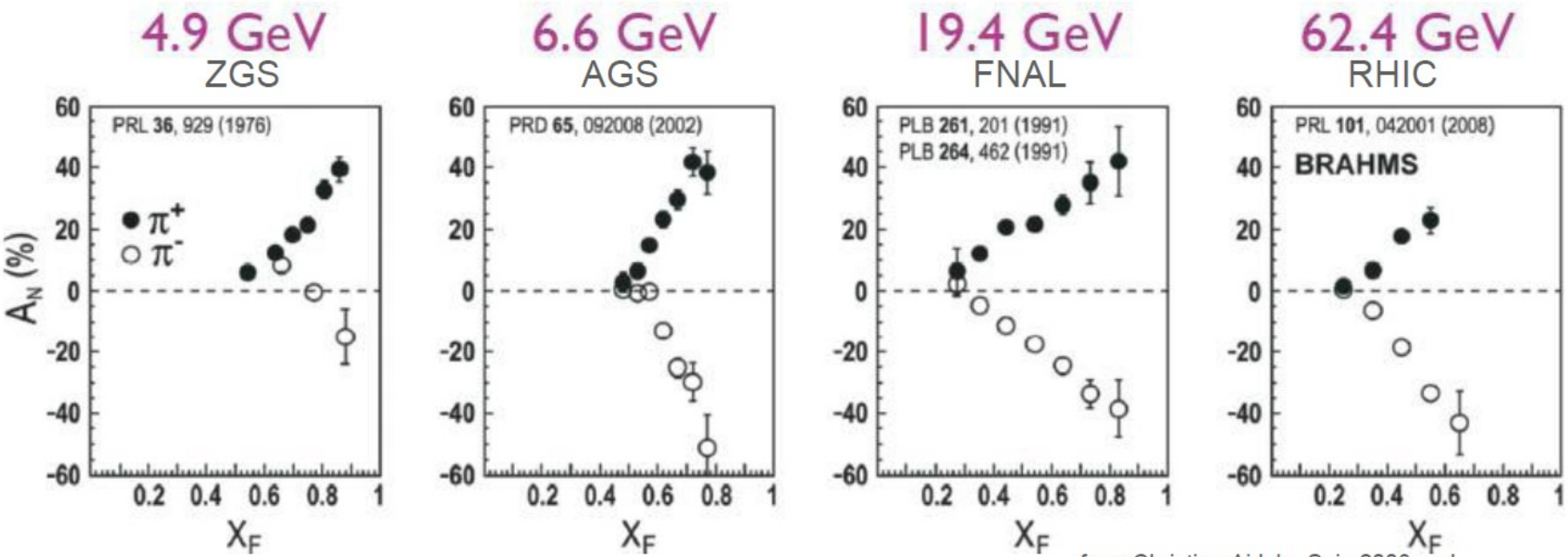


After a long debate  
this effect is actually  
observed

# Observation of large transverse single spin asymmetries

$$p \uparrow \rightarrow \pi^{+/-/0} + X$$

persisting at large energy



from Christine Aidala, Spin 2008 and Don Crabb & Alan Krisch in then Spin 2008 Summary, CERN Courier, 6-2009

CERN Courier, June 2009

# and what's next

## Focus on transverse structure of the nucleon

### ➤ Transverse Momentum Dependent TMD PDF

→ Study of SIDIS and DY

### ➤ Transverse size and orbital angular momentum : Generalized Parton Distributions (GPDs)

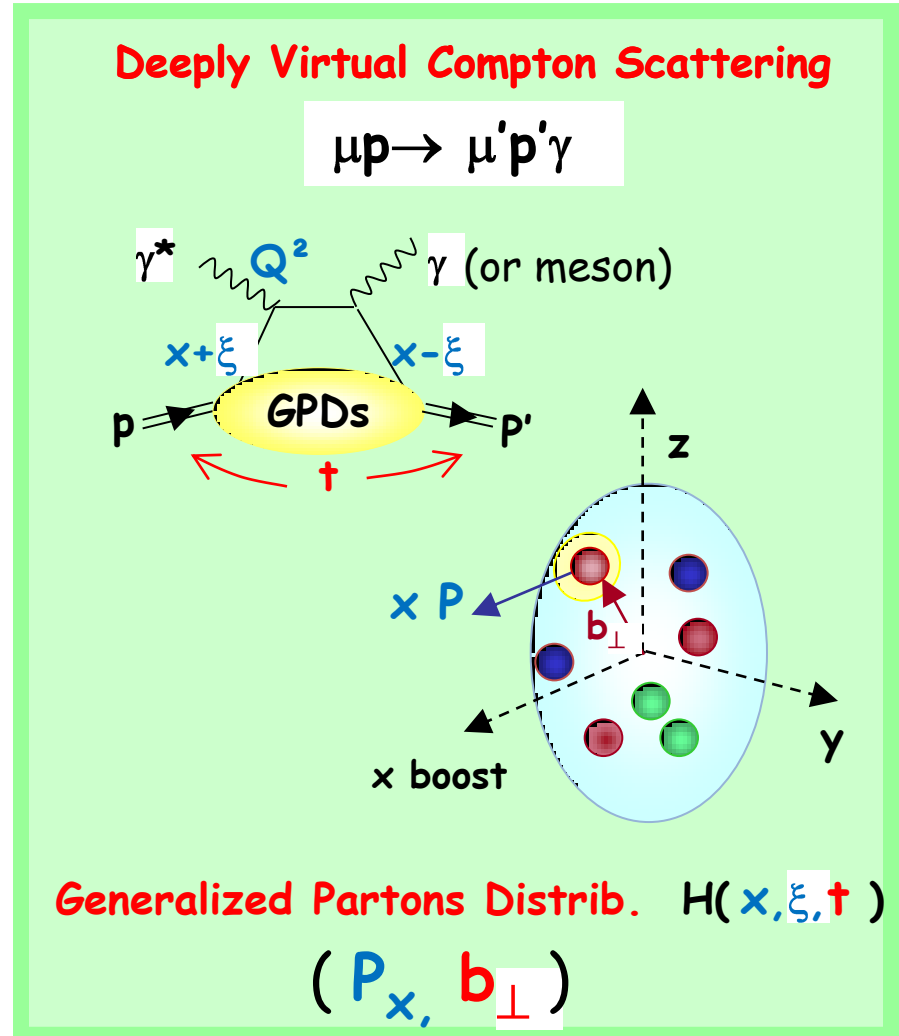
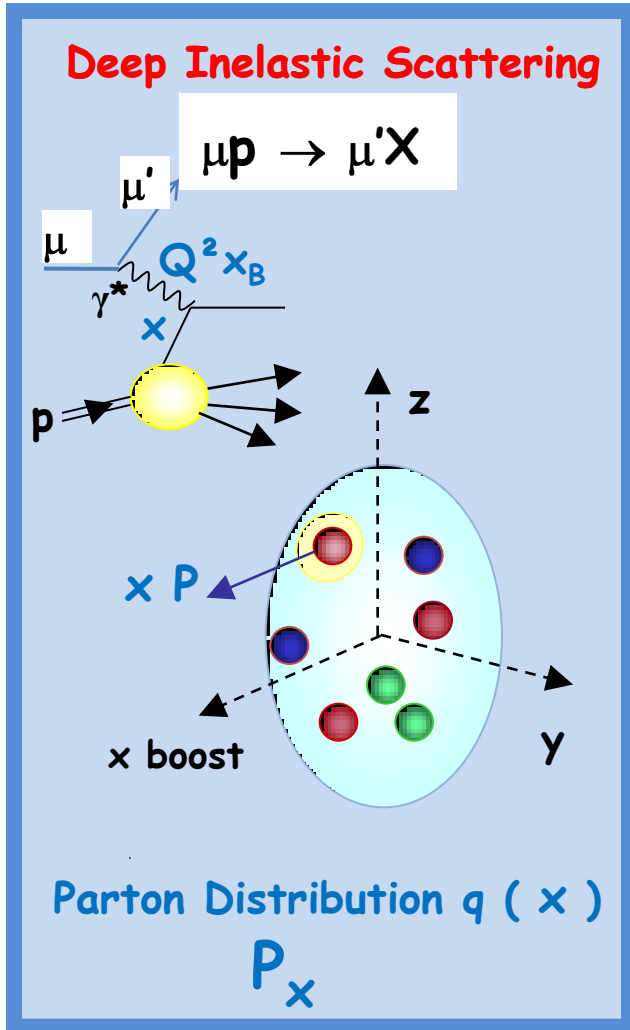
→ Study of Exclusive reactions

- COMPASS-II programme (2012-2016)
- Jlab 11 GeV (start in 2014)
- RHIC
- JPARC
  
- and more future ENC, eRHIC/ELIC, NICA

Coming from  
the general Wigner  
phase-space-distributions

from inclusive reactions

to exclusive reactions



Observation of the Nucleon Structure  
in 1 dimension

in 1+2 dimensions

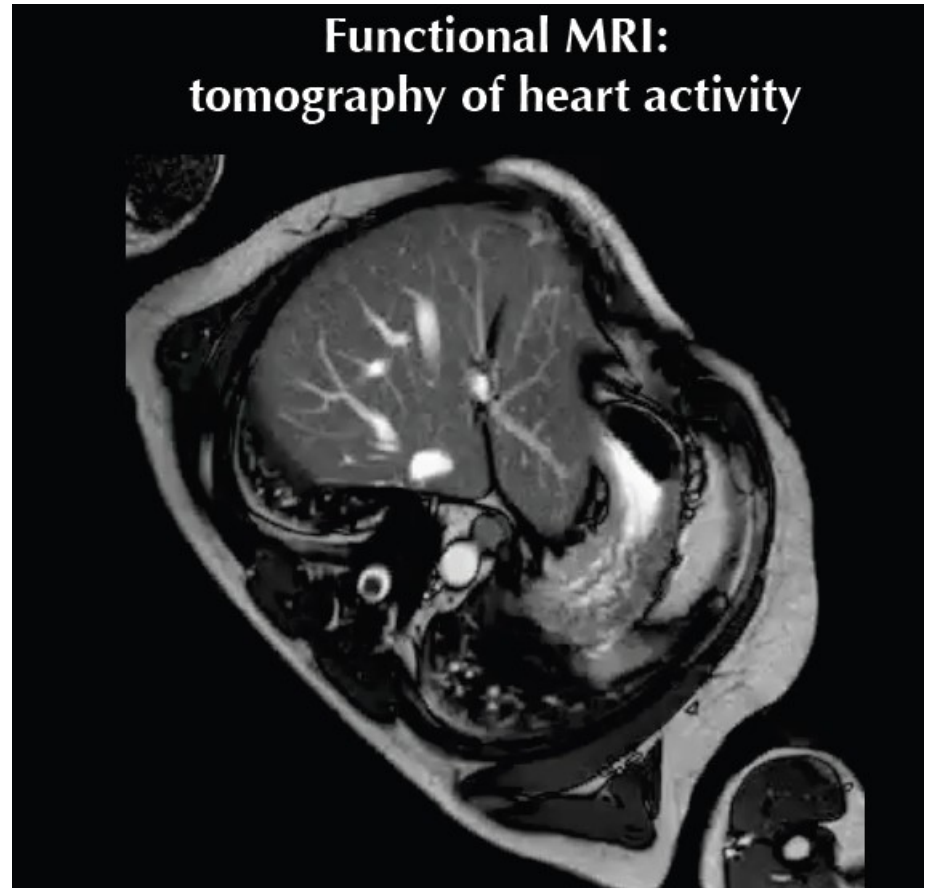
# From 1D

ECG: monodimensional information on heart activity



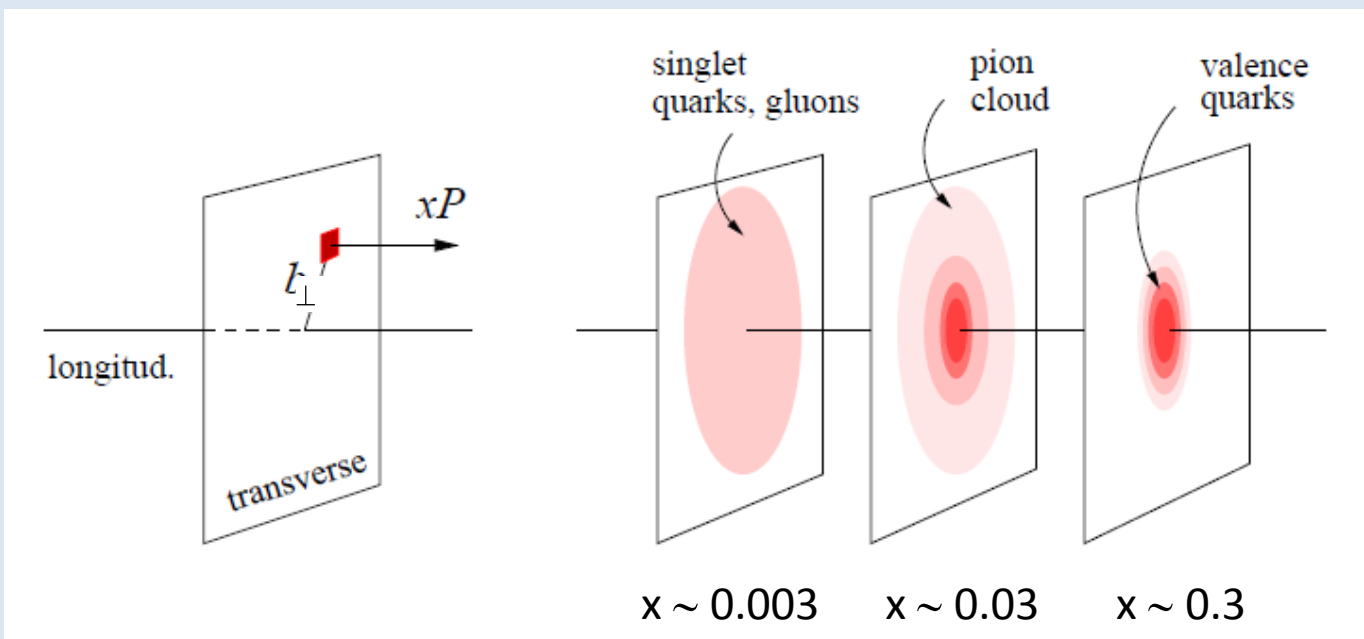
# to 3D

Functional MRI:  
tomography of heart activity



# Generalized Partons Distributions (H, E, ...)

- Allow for a unified description of form factors and parton distributions
- Allow for **transverse imaging (nucleon tomography)** and give access to **the quark angular momentum** (through E)

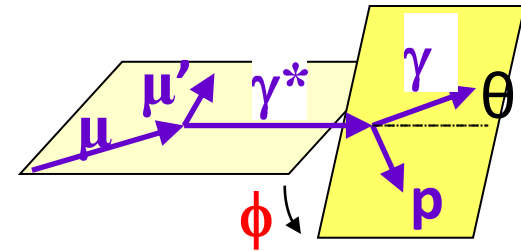
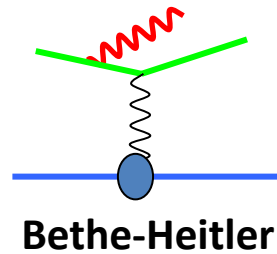
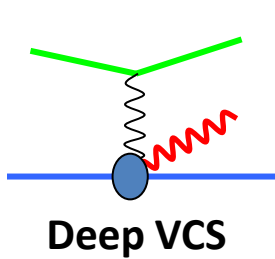


Impact parameter  $b_{\perp}$

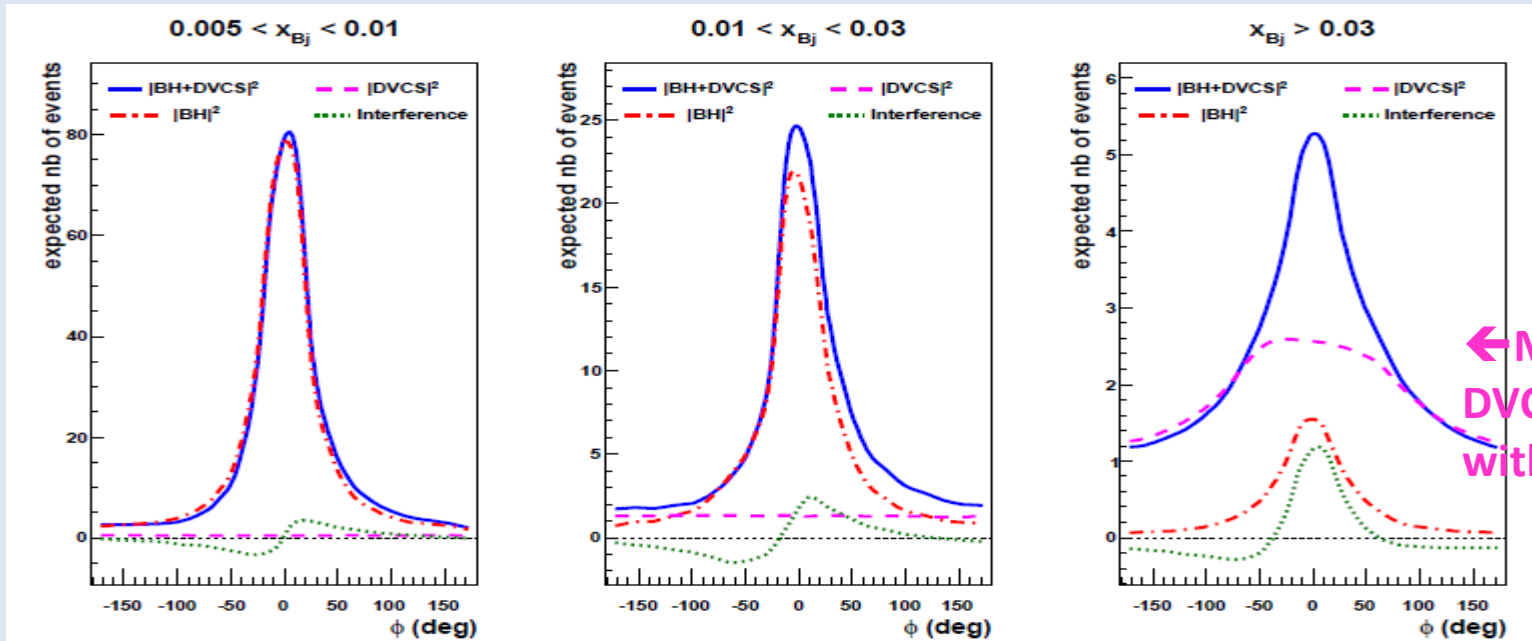
Longitudinal momentum fraction  $x$

Tomographic parton images of the nucleon

# Contributions of DVCS and BH at $E_{\mu} = 160$ GeV



$$d\sigma \propto |T^{\text{DVCS}}|^2 + |T^{\text{BH}}|^2 + \text{Interference Term}$$



Monte-Carlo Simulation for COMPASS set-up with only ECAL1+2

← Missing DVCS acceptance without ECAL0

**BH dominates**

excellent  
reference yield

**study of Interference**

→  $\text{Re } T^{\text{DVCS}}$   
or  $\text{Im } T^{\text{DVCS}}$

**DVCS dominates**

study of  $d\sigma^{\text{DVCS}}/dt$   
→ Transverse Imaging

# Transverse imaging at COMPASS

$$d\sigma^{\text{VCS}}/dt \sim \exp(-B|t|)$$

$$B(x_B) = \frac{1}{2} \langle r_{\perp}^2(x_B) \rangle$$

distance between the active quark and the center of momentum of spectators

## Transverse size of the nucleon

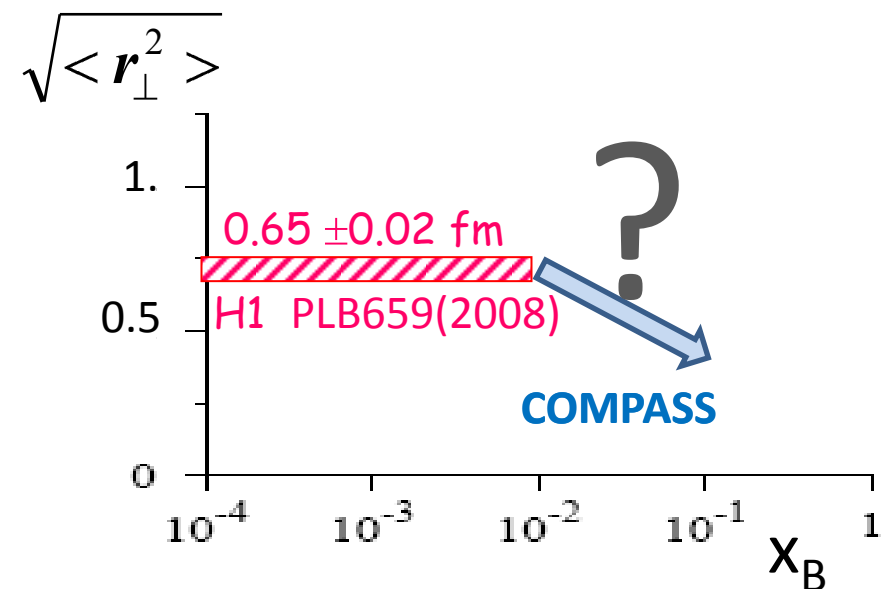
mainly dominated by  $H(x, \xi=x, t)$

$$\text{related to } \frac{1}{2} \langle b_{\perp}^2(x_B) \rangle$$

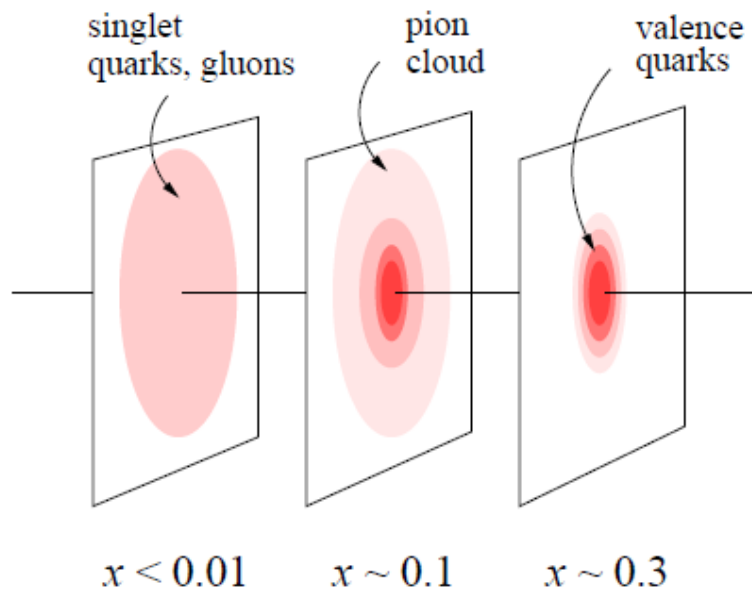
distance between the active quark and the center of momentum of the nucleon

## Impact Parameter Representation

$$q(x, b_{\perp}) \leftrightarrow H(x, \xi=0, t)$$



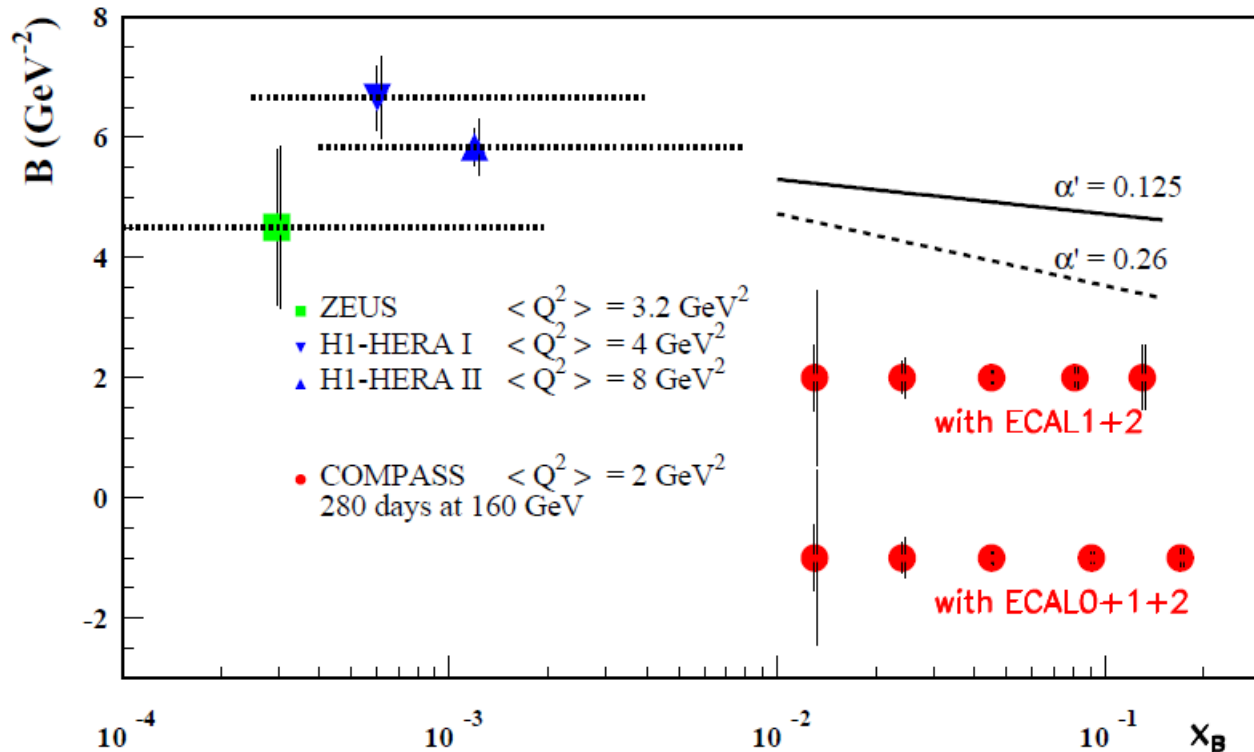
$$0.65 = \sqrt{2/3} \cdot 0.8$$





# Transverse imaging at COMPASS

$$d\sigma^{\text{VCS}}/dt \sim \exp(-B|t|)$$



**2 years of data**

160 GeV muon beam

2.5m LH<sub>2</sub> target

$\epsilon_{\text{global}} = 10\%$

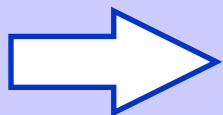
ansatz at small  $x_B$

inspired by

Regge Phenomenology:

$$B(x_B) = b_0 + 2 \alpha' \ln(x_0/x_B)$$

$\alpha'$  slope of Regge trajet



without any model we can extract  $B(x_B)$

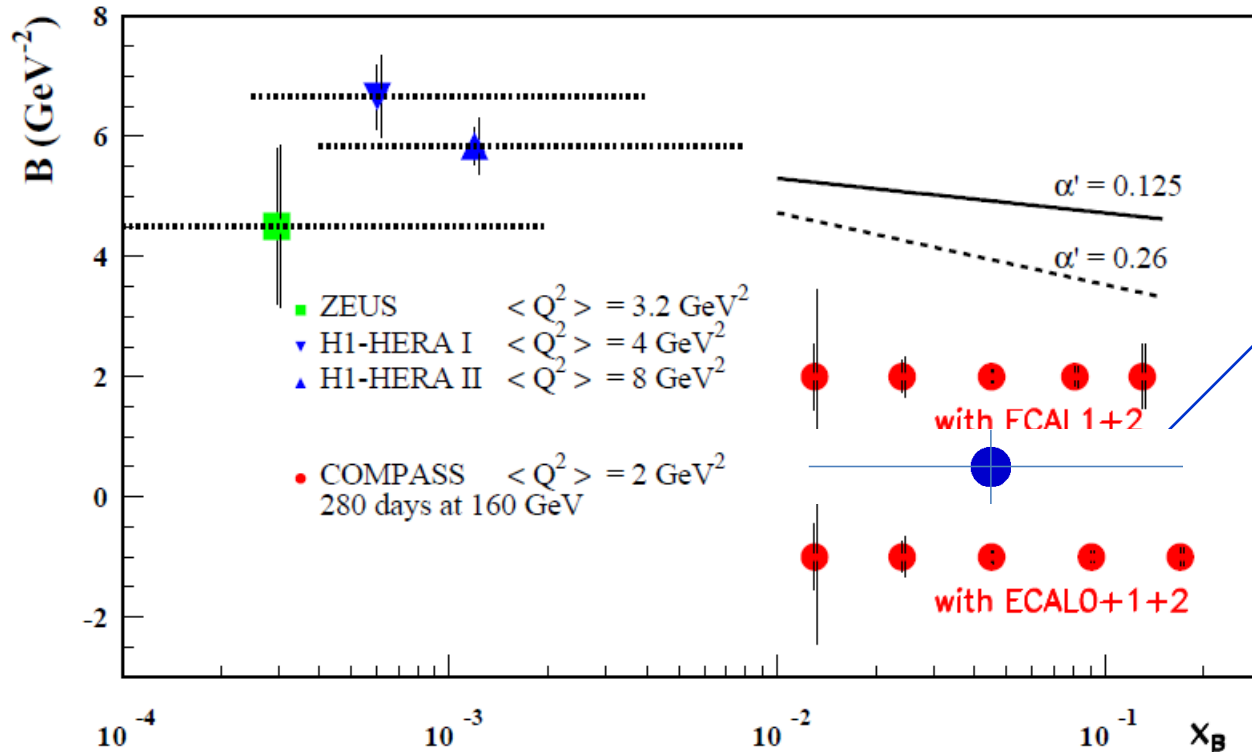
$$B(x_B) = \frac{1}{2} \langle r_{\perp}^2(x_B) \rangle$$

$r_{\perp}$  is the transverse size of the nucleon

Accuracy > 2.5% if  $\alpha' = 0.125$  and full ECALS

# Transverse imaging at COMPASS

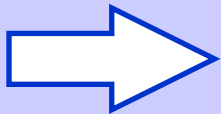
$$d\sigma^{\text{VCS}}/dt \sim \exp(-B|t|)$$



DVCS test in 2012

With 1 week  
Using the 4m long RPD  
+ the 2.5m long LH2 target

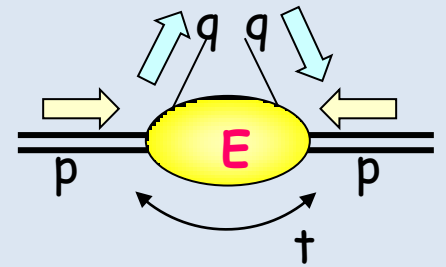
1/40 of the complete  
statistics



**2012: we can determine one mean value of B in the COMPASS kinematic range**

# The GPD E is the 'Holy-Grail' of the GPD quest

the GPD **E** allows nucleon helicity flip  
so it is related to the angular momentum



Ji sum rule:  $2J_q = \int x (H^q(x, \xi, 0) + E^q(x, \xi, 0)) dx$

## Constraints on the GPD E

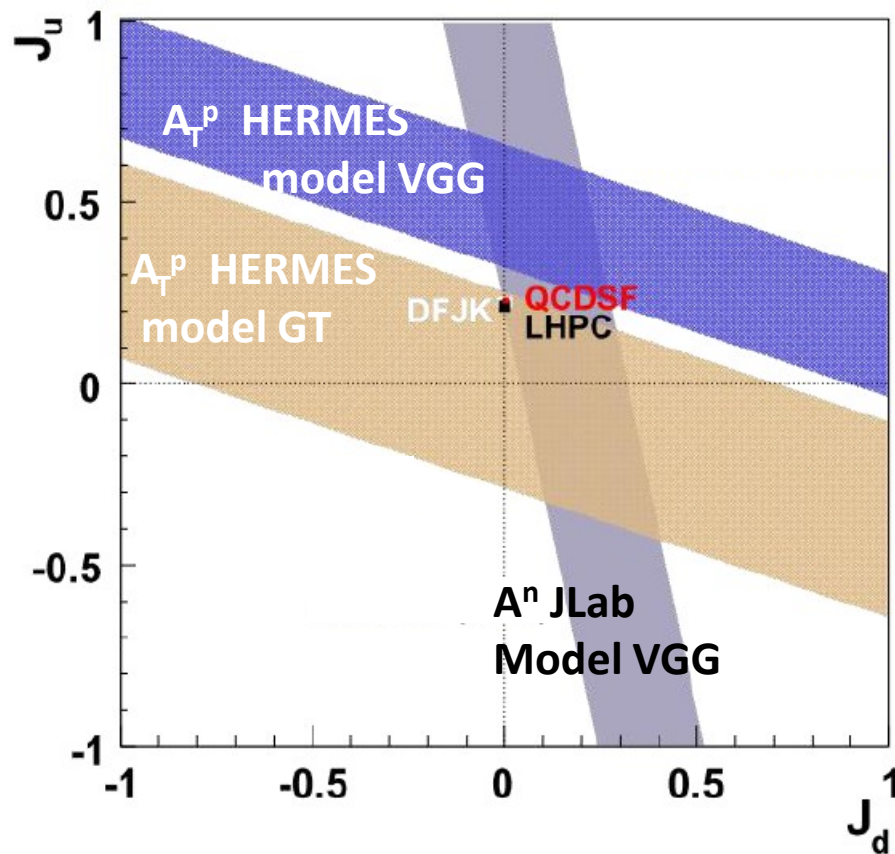
-Using a neutron target  $A^n \propto F_1(t) \cdot H - \frac{t}{4M^2} F_2(t) \cdot E$

-Using a transversely polarized proton target

$$A_T^p \propto F_2(t) \cdot H - F_1(t) \cdot E$$

# Potential of these experiments

A model-dependent case-study



JHEP06 (2008) 066

In the near future

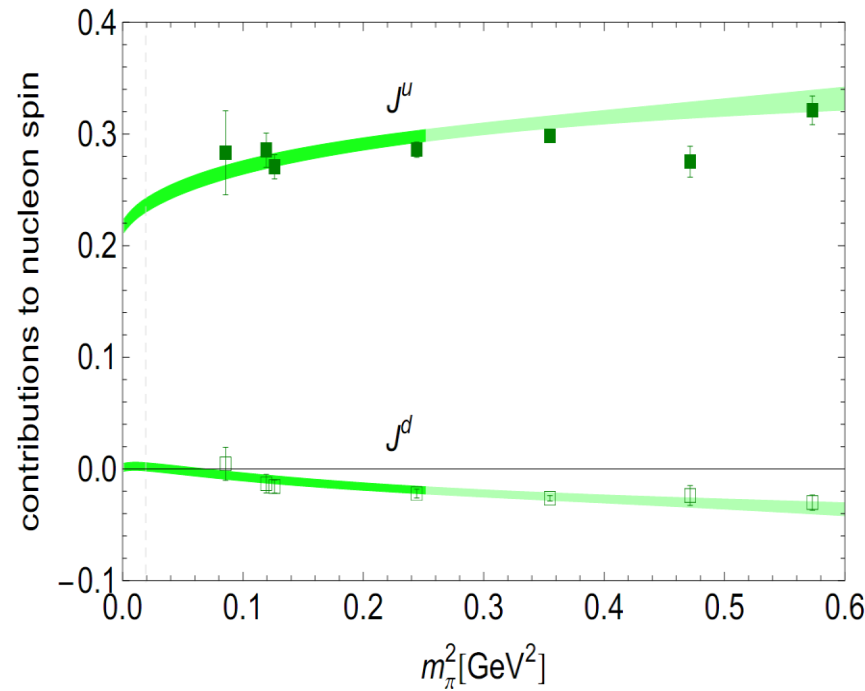
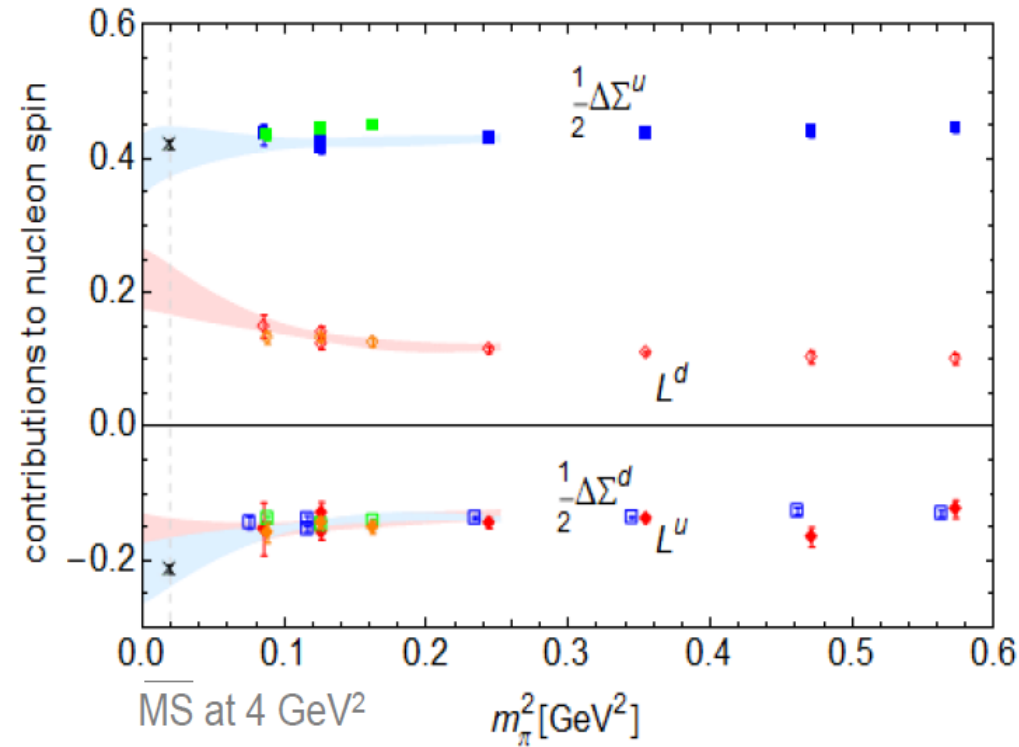
Two main actors

- COMPASS (sea quarks and gluons)
- Jlab (valence domain, with a huge luminosity)

An important activity

For global fits on the world data  
(Mueller, Guidal, Moutarde, ...)

# Lattice QCD calculations



Ph. Haegler, 2010

# Nucleon Spin is fun!



Still a long  
and exciting trip