Probing the nucleon structure

What have we learnt during the last 30 years?

Nicole d’Hose, Irfu – CEA Saclay
La Colle sur Loup, Sept 16, 2011
Probing the nucleon structure

A brief story of the nucleon

Proton and neutron are the basic building blocks of *the visible matter of the universe*

Nucleon Identity card (PDG):

<table>
<thead>
<tr>
<th>Proton</th>
<th>Neutron</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q=+e$</td>
<td>$Q=0$</td>
</tr>
<tr>
<td>$M_p=938.27$ MeV</td>
<td>$M_n=939.57$ MeV</td>
</tr>
<tr>
<td>$S=\frac{1}{2}$</td>
<td>$S=\frac{1}{2}$</td>
</tr>
<tr>
<td>$\mu_p=2.79 \mu_N$ ($\kappa_p=1.79$)</td>
<td>$\mu_n=-1.91 \mu_N$ ($\kappa_n=-1.91$)</td>
</tr>
</tbody>
</table>
Probing the nucleon structure

*A brief story of the nucleon*

Identified in 1919 (proton) and 1932 (neutron)

Until 1933: they were thought as a point-like particle, like electron
Probing the nucleon structure

A brief story of the nucleon

1933-1960: extented object

1933: Stern (NP 1943) measured the anomalous magnetic moment 1rst evidence the proton is not point -like

1955: Stanford : e⁻ beam of 1 GeV → elastic scattering Hofstadter (NP 1961) measured the charge radius of the proton ≈ 0.8fm
Probing the nucleon structure

A brief story of the nucleon

1960-1980: nucleon composed of **quarks and gluons**

1964: Gell-Mann and Zweig (NP 1969) postulated that there are 3 quarks in the proton: $u u d$

$\rightarrow$ hadron spectroscopy and classification

1969: Stanford: $e^-$ beam of 20 GeV $\rightarrow$ **deep inelastic scattering**

*Friedman, Kendall, Taylor (NP 1990)* found quarks in the proton

*Feynman, Bjorken*  
*Gross, Politzer, Wilczek (NP 2004)*

**Quantum ChromoDynamics (QCD)**

✓ Asymptotic Freedom
✓ Confinement
Probing the nucleon structure

In 1980:

We know a lot and we know little

Proton is made of 2 up quarks (e=2/3) + 1 down quark (e=-1/3)
  + any number of quark-antiquarks
  + any number of gluons

Still fundamental questions?

Origin of mass?
  \[ M_p \approx 2000 \times M_e. \]
  \~10\% from Higgs interaction
  \~90\% comes from the motion of quarks and gluons

Proton spin crisis 1987: the valence quarks contribute very little to the proton spin
QCD: still unsolved in non-perturbative region

Gross, Politzer, Wilczek (NP 2004)
Asymptotic freedom

Non perturbative regime of QCD
One of the top 10 challenges for physics
Nucleon structure provides much insight about how QCD works in the confinement regime
Understanding the nucleon structure

Solving QCD

✓ Numerically simulation, lattice calculations continue to make advances in techniques and computing power

✓ Effective field theories (chiral physics, large Nc, ...)

✓ (Phenomenological models, fits of structure functions...)

Experimental probes

✓ Require clean reaction mechanisms with photons, electrons

✓ through low and high-energy scattering off the nucleon
Probing the nucleon structure

What has been done during the last 30 years?

Old Chinese compass

Exploration
Probing the nucleon structure

What has been done during the last 30 years?

Hand-held compass

Consolidation
Probing the nucleon structure

What has been done during the last 30 years?

GPS compass

Precision
Probing: Lepton-nucleon scattering

Hypotheses:
- One photon exchanged
- $M_e << $ (electric charge of electron)

$$Q^2 = -q^2 = 4 E E' \sin^2 \theta / 2 > 0$$

$$x_B = \frac{Q^2}{(2p.q)} = \frac{Q^2}{(2M_p(E-E'))}$$

Elastic Scattering: $X=\text{proton}$

$$s = (p+q)^2 = M_p^2 - Q^2 + 2p.q = M_p^2 \Rightarrow x_B = 1$$

at fixed beam energy $E$, only one variable $Q^2$ ($E'$ and $\theta$ are not independent)

- Form Factors ($Q^2$) $\Rightarrow$ Consolidation and Exploration at higher $Q^2$

- Nucleon radius (from $Q^2 \to 0$) $\Rightarrow$ High Precision, but also need of Consolidation
Probing: Lepton-nucleon scattering

Hypotheses:

- One photon exchanged
- $M_e << q^2 = 4E E' \sin^2 \theta/2 > 0$

$$Q^2 = -q^2 = 4E E' \sin^2 \theta/2 > 0$$

$$x_B = \frac{Q^2}{2p.q} = \frac{Q^2}{2M_p(E-E')}$$

Deep Inelastic Scattering (DIS): the proton is broken in many debris $X$

$$s = (p+q)^2 = M_p^2 - Q^2 + 2p.q > M_p^2 \Rightarrow 0 < x_B < 1$$

at fixed beam energy $E$, 2 variables ($E'$, $\theta$) or ($Q^2$, $x_B$)

We learnt that: $$\sigma_{\text{DIS}}(ep \rightarrow e X) = \sum_{\text{incoherent}} \sigma_{\text{elastic}}(eq \rightarrow eq)$$
Probing: Lepton-nucleon scattering

Deep Inelastic Scattering (DIS):

\[ \sigma_{\text{DIS}}(ep \rightarrow e X) = \sum_q \sigma_{\text{elastic}}(eq \rightarrow eq) \]

Quark parton model (QPM)
- Point-like, non-interacting partons
- Collinear to the nucleon movement in photon-nucleon collision (longitudinal direction)
- Each parton carries a fraction \( x \) of the nucleon momentum and for the struck parton: \( x = x_B \)
- Scaling: observables function of \( x_B \) (at first order)

Parton Distribution Functions (PDF (\( x \))):
- unpolarized \( \Rightarrow \) High precision
- polarized \( \Rightarrow \) Nucleon Spin Crisis Consolidation

More on transverse information
- momentum: Transverse Momentum Dependent PDF
- position: Generalized Parton Distributions (GPD) \( \Rightarrow \) Exploration in 3D
Lepton nucleon or nucleus scattering

Elastic: \[ \Delta x = \frac{\hbar c}{|q|} \] exploration of distance of 1 fm with 200 MeV

Deep Inelastic: \( q \) and \( \omega \) varie independently
\[ \Delta t = \frac{\hbar c}{\omega} \] instantaneous picture to observe free proton
The main facilities in the world (1980-2015)

- SLAC
- CERN
- DESY
- RHIC
- JLab

- 12 GeV
- 49 GeV e^-
- 160/280 GeV
- 27 GeV e^±
- 250+250 GeV pp
- 6 GeV e^-
- +KEK 6 GeV e^± (Belle)
- + MAMI (Mainz Microtron) 1.5 GeV e^-
The main facilities in the world (1980-2015)

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- 160/280 GeV
- 6 GeV e
- 12 GeV
- \(\rightarrow\) 12 GeV
Kinematic domain \((Q^2, x_B)\) for DIS

\[
Q^2 (\text{GeV}^2) \\
\begin{array}{c}
\text{COMPASS 160 GeV} \\
\text{HERMES 27 GeV} \\
\text{JLab 11 GeV} \\
\text{ZEUS +H1}
\end{array}
\]

\[
E = 160 \text{ GeV} \\
E = 27 \text{ GeV} \\
E = 11 \text{ GeV}
\]

\[
x_B > (2 \text{GeV})^2
\]
Elastic Scattering

- Form Factors ($Q^2$)  =>  Consolidation and Exploration at higher $Q^2$

- nucleon radius (from $Q^2\rightarrow 0$)  =>  Very High Precision and Consolidation
Elastic Scattering

Spin-less point-like electron in the static Coulomb field given by a charge $Z$ of infinite mass

$$\frac{d\sigma}{d\Omega} = |f(q)|^2 = \left(\frac{Z\alpha}{2k}\right)^2 \frac{1}{\sin^4 \theta/2} |F(q)|^2 = \sigma_{\text{Rutherford}} |F(q)|^2$$

Relativistic calculation for a spin $\frac{1}{2}$ electron in the EM field given by a charge $+e$ and finite mass

$$\frac{d\sigma}{d\Omega} = \sigma_{\text{Mott}} \left(\frac{E'}{E}\right)^2 \times |F(q)|^2 \quad \text{with} \quad \sigma_{\text{Mott}} = \frac{\alpha^2 \cos^2(\theta/2)}{4E^2 \sin^4(\theta/2)}.$$ 

The Form Factor measures the deviations from a structure less particle

- Point particle: $\rho(\vec{r}) = \delta(\vec{r})$ \quad $F(q) = 1$

- Spherical distribution: $\rho(\vec{r}) = \rho(r)$ \quad $F(q) = \frac{4\pi}{q} \int_0^\infty \rho(r) \sin(qr) r \, dr$

$$\langle r^2 \rangle = -6 \frac{\partial F}{\partial q^2} |q^2=0$$

Quadratic mean charge radius of the nucleon
Form Factors for sphere or nucleus

Minima due to the discontinuity of $\rho(r)$

Real nucleus without discontinuity => Diffractive minimum partly filled
Scattering on a spin $\frac{1}{2}$ particle: $e p \rightarrow e p$

\[ -i M^{(e p)} = -j_{\mu} \frac{-i g_{\mu\nu}}{q^2} J^\nu \]

\[ |M^{(e,p)}|^2 = \frac{1}{4} \sum_{s_e, s_{e'}, s_p, s_{p'}} |M^{(e_p)}|^2 \]

General expression for the hadronic current according symmetries of the interaction

\[ J_{\mu} = (-ie) \bar{u'}(p') \left[ F_1(q^2) \gamma_{\mu} + F_2(q^2) \frac{i}{2M} \sigma_{\mu\nu} q^\nu \right] u(p) \]

Dirac and Pauli Form Factors

With Gordon identity

\[ J_{\mu} = \frac{-ie}{2Mp} \bar{u'}(p') \left[ \underbrace{F_1(q^2) P_{\mu}}_{\text{charge } Q} + i\sigma_{\mu\nu} q^\nu \left( F_1(q^2) + F_2(q^2) \right) \right] u(p) \]

\[ Q^2 = -q^2 > 0 \]

\[ F_1(0) = Q \]

\[ F_2(0) = \kappa \]

\[ \frac{\mu}{\mu_N} = Q + \kappa \]
Scattering on a spin $\frac{1}{2}$ particle: $ep \rightarrow ep$

\[
\left( \frac{d\sigma}{d\Omega} \right)_{lab} = \sigma_{\text{Mott}} \frac{E'}{E} \left[ F_1^2 + \tau F_2^2 + \tau(F_1 + F_2)^2 \tan^2 \frac{\theta}{2} \right]
\]

\[
\left( \frac{d\sigma}{d\Omega} \right)_{lab} = \sigma_{\text{Mott}} \frac{E'}{E} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} + 2\tau G_M^2 \tan^2 \frac{\theta}{2} \right]
\]

\[
\left( \frac{d\sigma}{d\Omega} \right)_{lab} = \sigma_{\text{Mott}} \frac{E'}{E} \left[ \frac{\epsilon G_E^2 + \tau G_M^2}{\epsilon(1 + \tau)} \right]
\]

\[
\epsilon = \left[ 1 + 2\left( \frac{\bar{q}^2}{Q^2} \right) \tan^2 \left( \frac{\theta}{2} \right) \right]^{-1}
\]

\[
\tau = \frac{Q^2}{4M_p^2}
\]

**Electric and Magnetic Sachs Form Factors** (Fourier Transform (TF) of the charge and magnetization distribution in the breit frame or brick-wall frame)

\[
G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)
\]

\[
G_E(0) = Q
\]

\[
G_M(Q^2) = F_1(Q^2) + F_2(Q^2)
\]

\[
G_M(0) = \mu_N
\]
Measurement of Form Factors ($Q^2$)

Rosenbluth separation:

We plot $\sigma_R = \epsilon(1 + \tau)\frac{d\sigma}{d\Omega} \frac{E}{E'} \frac{1}{\sigma_{Mott}} = \epsilon G_E^2 + \tau G_M^2$

Fixed $Q^2 \Rightarrow$ use of different beam energies to get different $\epsilon$

- Different scattering angles
  $\Rightarrow$ large systematic effects on the slope = $G_E$

- At large $Q^2$$G_E^2 \ll \tau G_M^2$
  $\Rightarrow$ Method to determine $G_M$
Measurement of proton Form Factors $(Q^2)$

by Rosenbluth separation

\[ \frac{G_E^p}{G_D} \]

\[ \approx 1 \]

Dipolar approximation with $G_D$: contribution of an effective double pole in the time-like region

\[ G_D = \left(1 + \frac{Q^2}{0.71} \right)^{-2} \]

FT $\rightarrow$ exponential distribution $\rho(r) = e^{-0.84r}$ non physical

Deviation from the dipolar approximation

\[ G_D \]

TF $\rightarrow$ $\rho(r) = e^{-mr}$
Measurement of proton Form Factors ($Q^2$) by Rosenbluth separation

\[ G_E^p / G_D \]

\[ G_M^p / \mu G_D \]

\[ G_D = \left( 1 + \frac{Q^2}{0.71} \right)^{-2} \]

\[ Q^2 (GeV^2) \]
Measurement of Form Factors ($Q^2$)

**New Double Polarization technique:**

- polarized e- beam + unpolarized target
- recoil proton analyzed in a magnetic spectrometer and a polarimeter

\[
\sigma = \sigma_0 \left[ 1 + P_e P_x A_y(\theta') \sin \phi' + P_e P_z \sin \chi A_y(\theta') \cos \phi' \right]
\]

Beam polarization \[\downarrow\]
Analyzing power \[\downarrow\]
Spin precession

\[
P_x = \frac{-\sqrt{\tau} \sqrt{2\epsilon(1-\epsilon)G_E G_M}}{\epsilon G_E^2 + \tau G_M^2}
\]

\[
P_z = \frac{\tau \sqrt{1-\epsilon}G_M^2}{\epsilon G_E^2 + \tau G_M^2}
\]

Proposed in 1967 Akhiezer Rekalo but only experimentally possible since high duty cycle facilities advent

Beam spin asymmetry

\[
BSA = \frac{N^+-N^-}{N^++N^-} = a \sin \phi' + b \cos \phi'
\]

\[
a = \frac{P_x}{P_z \sin \chi} = \frac{G_E}{G_M}
\]

No systematic effects due to Beam polar and analyzing power as they cancel out in the ratio
Measurement of proton Form Factors ($Q^2$)

by Rosenbluth separation

by double polarization technique

SURPRISE !!!
Difference persists between two techniques

- Jlab Rosenbluth separation data confirm earlier data and global analysis
- No evidence for experimental errors for either of the experimental techniques
- Two photon exchange (TPE) amplitudes can explain a significant part of the discrepancy
- Intensive theoretical and experimental effort on addressing the TPD effect
2 photon exchange proposed as an explanation

Rosenbluth separation

Prediction with two photon exchange
Guichon, Vanderhaeghen 2003

Polarization technique

Jlab data Meziane 2011
What we learnt from the $Q^2$ Evolution?

Many models: rQCM, VDM, pQCD

$$\mu_p G_{Ep} / G_{Mp}$$ crossing 0 at $Q^2=?$ saturation of charge density
What we learnt from the $Q^2$ Evolution?

pQCD scaling: $Q^2 F_2/F_1$ should be constant

$F_1(Q^2) \simeq \frac{1}{Q^4}$

$F_2(Q^2) \simeq \frac{1}{Q^6}$

$G_E \sim G_M \sim \frac{1}{Q^4}$

Belitsky, Ji, Yuan (2003)

- Far from the scaling regime even at $Q^2 = 10$ GeV^2
- No early scaling as in DIS

related to quark OAM
Lattice QCD calculations

European Twisted Mass Collaboration
2008
Neutron Form Factor as a function of $Q^2$

No stable free neutron target

Neutron form factors measured using quasi-elastic electron scattering from deuteron target $(p+n)$ or polarized $^3\text{He}$ target

\[
\langle r_n^2 \rangle = (-0.1148 \pm 0.0035) \text{ fm}^2
\]

\[
\langle r^2 \rangle = -6 \left( \frac{dG_E^p(Q^2)}{dQ^2} \right)_{Q^2=0}
\]
Meaning of $G_e^p$ and $G_E^n$

The proton and neutron charge distributions obtained from Fourier Transform of $G_e^p$ and $G_E^n$

$Q = \int r \rho(r) 4\pi r^2 dr = 0$

$<r^2>_n = \int r^2 \rho(r) 4\pi r^2 dr < 0$

$\langle r_{n}^{2} \rangle = (-0.1148 \pm 0.0035) \text{ fm}^2$

neutron = proton + $\pi$ cloud

2008 Blast MIT
What is the real size of the proton?

The Muonic hydrogen Lamb shift gives the most precise measurement of the proton charge radius with an unprecedented precision of 0.1%

This value is much smaller (5 standard deviations) than the other measurements using e-p scattering and standard Lamb shift.

UNEXPECTED!

Publication: 8 July 2010 and quickly spread on all the media

Then:

- 16 theoretical papers in 2 months
- several planned experiments
Proton Form Factor

Focus on measurements at large $Q^2$

but what is the situation at low and very low $Q^2$?

- **2010 (PRL105 MAMI Bernauer et al.):**
  The lowest $Q^2$ is $4 \times 10^{-3} \text{ GeV}^2$, super Rosenbluth separation

- **2011 (Jlab Zhan et al.):**
  $Q^2 \in [0.3 : 0.7] \text{ GeV}^2$, Polarization technique

- Planned experiment $Q^2 [2 \times 10^{-4} : 2 \times 10^{-2}] \text{ GeV}^2$ in the future at JLab
Results for the proton charge radius

CODATA recommended values of the Fundamental Constants

$\langle r_E \rangle$ before 2010

$r_E^p$ in the 2 last years

$\langle r_E \rangle$ [fm]

$0.74$ $0.76$ $0.78$ $0.8$ $0.82$ $0.84$ $0.86$ $0.88$ $0.9$ $0.92$

CODATA

Sick

Bernauer et al.

Zhan et al.

CODATA

Pohl et al.

2003

2010

2011

2006

2010

e-p scattering

standard

muonic Hydrogen Lamb shift

$5\sigma$

$0.84184 \pm 0.00067$

$0.8768 \pm 0.0069$

$0.875 \pm 0.010$

$0.879 \pm 0.008$

$r_E^p$
Lamb shift

(1947) subtle difference between the binding energies of the $2S\frac{1}{2}$ and $2P\frac{1}{2}$ (pure radiative QED effects such as ‘self energy’ and ‘vacuum polarization’)

There is also extra corrections for hyperfine splitting, recoil and proton structure...
Muonic hydrogen = $\mu^- + p$, mass $m_\mu = 207 \, m_e$

\[
\Delta E_{\text{finite size}}(nl) = \frac{2(Z\alpha)^4 e^4}{3\hbar^2 n^3} m^3 \rho^2 \delta l_0
\]

Lamb shift in $\mu p$: $\Delta E(2P_{3/2}^{F=2} - 2S_{1/2}^{F=1}) = 209.9779(49) - 5.2262 \rho^2_p + 0.0347 \rho^3_p \quad \text{[meV]}
$

finite size contribution is 2% of the $\mu p$ Lamb shift

$\mu p(n=2)$ levels:

- $2P_{3/2}$
- $2P_{1/2}$
- $2S_{1/2}$

8.4 meV
206 meV
50 THz
6 \, \mu m

fin. size:
3.8 meV
23 meV
F=0
F=1
F=1
μp Lamb shift experiment: principle

New 5keV muon beam line at PSI
Muons stopped in H₂ gas at low pressure ⇒ excited μp atoms (n=14) are formed

Laser tunable around λ=6nm 50THz 206mEV
μp Lamb shift experiment: principle

New 5keV muon beam line at PSI
Muons stopped in H2 gas at low pressure \(\rightarrow\) excited μp atoms (n=14) are formed

Laser tunable around \(\lambda=6\text{nm}\) 50THz 206mEV
μp Lamb shift result

CODATA-06

e–p scattering

H₂O calibration

Our value

Delayed / prompt events (10⁻⁴)

Laser frequency (THz)

49.75  49.8  49.85  49.9  49.95
Conclusion: what is wrong?

- Spectroscopy: Missing element in the QED corrections in the bound \( \mu p \) system...
- Lepton Scattering: Need of very precise and very low Q2 data...
- ...

The proton, already an old-fashioned objet, but still embedded in exciting challenges
Deep Inelastic Scattering

Parton Distribution Functions (PDF (x))
  unpolarized  =>  High precision
  polarized    =>  Nucleon Spin Crisis Consolidation

More on transverse information
  momentum:  Transverse Momentum Dependent PDF  =>  Exploration in 3D
  position:  Generalized Parton Distributions (GPD)
Probing: Lepton-nucleon scattering

**Deep Inelastic Scattering:**

\[
\sigma_{\text{DIS}}(ep \rightarrow e X) = \sum_q \sigma_{\text{elastic}}(eq \rightarrow eq)
\]

incoherent

**Quark parton model (QPM)**

- Point-like, non-interacting partons
- Collinear to the nucleon movement in photon-nucleon collision (longitudinal direction)
- Each parton carries a fraction \(x\) of the nucleon momentum and for the struck parton: \(x = x_B\)
- Scaling: observables function of \(x\) (at first order)

The inclusive cross section is described by 4 structure functions:

**unpolarized**

\[F_1(x), F_2(x),\]

\[
F_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 q(x) = \frac{F_2(x)}{2x}
\]

\[q(x) = q^+(x) + q^-(x)\]

**polarized**

\[g_1(x), g_2(x)\]

\[
g_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x)
\]

\[g_2(x) = 0\]

\[\Delta q(x) = q^+(x) - q^-(x)\]

*probability of finding a quark with a fraction \(x\) of the nucleon longitudinal momentum*
Probing: Lepton-nucleon scattering

Deep Inelastic Scattering:

\[ \sigma_{\text{DIS}}(ep \rightarrow e X) = \sum_q \sigma_{\text{elastic}}(eq \rightarrow eq) \]

incoherent

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\[ F_1(x), \quad F_2(x), \]
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polarized
\[ g_1(x), \quad g_2(x) \]
\[ g_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x) \]
\[ g_2(x) = 0 \]
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Probing: Lepton-nucleon scattering

Deep Inelastic Scattering:

\[
\sigma_{\text{DIS}}(e p \rightarrow e X) = \sum_q \sigma_{\text{elastic}}(e q \rightarrow e q)
\]

QCD improved parton model

- at finite \(Q^2\), partons interact
- PDFs and structure functions depend on \(Q^2\)
- gluons are visible (in the \(Q^2\) evolution)

The inclusive cross section is described by 4 structure functions:

- unpolarized
  \(F_1(x, Q^2), \ F_2(x, Q^2)\),

  \[
  F_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 q(x) = \frac{F_2(x)}{2x}
  \]

  \[
  q(x) = q^+(x) + q^-(x)
  \]

- polarized
  \(g_1(x, Q^2), \ g_2(x, Q^2)\)

  \[
  g_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x)
  \]

  \[
  g_2(x) = 0
  \]

  \[
  \Delta q(x) = q^+(x) - q^-(x)
  \]

  \[
  \Delta q(x, Q^2)
  \]

  \[
  \Delta G(x, Q^2) \text{ from } g_1 \text{ evolution}
  \]

and

- quark PDF \(q(x, Q^2)\)
- gluon PDF \(G(x, Q^2)\) from \(F_1\) evolution
$F_2(x,Q^2) + g_1(x,Q^2)$

Very gentle QCD evolution
Factorisation scale $Q^2$ few GeV$^2$

Still not so well known
Unpolarized quark and gluon PDF

Global fit on the world data
Universality of PDF

Physical cross section \( = \) cross section for partonic process \( (\gamma^* q \rightarrow q \text{ or } q\bar{q} \rightarrow \gamma^*) \times \text{PDF} \)

Deep Inelastic Scattering (DIS) \( \ell p \rightarrow \ell' X \)

Drell Yan (DY) \( \pi p \rightarrow \ell^+\ell^- X \)

Factorisation: scheme dependent!
nucleon spin crisis

\[ \Delta q(x) \quad \text{and} \quad \Delta q = \int_0^1 \Delta q(x) \, dx \]
\[ \Delta G(x) \quad \text{and} \quad \Delta G = \int_0^1 \Delta G(x) \, dx \]

Need of longitudinally polarized beam
and longitudinally polarized target
The spin of the nucleon

Naive quark parton model
\[ \Delta \Sigma = \Delta u_v + \Delta d_v = 1 \]
Note \( \Delta \) means \( \int_0^1 dx \)

QCD: sea quarks and gluons
\[ \Delta q_s, \Delta G \]

Orbital angular momentum
\[ L_{zq}, L_{zg} \]

\[ \frac{1}{2} = \frac{1}{2} \Delta \Sigma + \Delta G + L_{zq} + L_{zg} \]

\[ \Delta \Sigma = \Delta u + \Delta d + \Delta s \]
\(~ 0.6 \) QCD with \( \Delta s = 0 \) (Ellis-Jaffe)
\(~ 0.12 \pm 0.17 \) EMC (1987) surprise!
The structure function $g_1$ and its first moment

$$g_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x)$$

$$\Gamma_1 = \int_0^1 g_1(x) \, dx$$

In naive QPM + SU(3)

$$\Gamma_1^p = \frac{1}{2} \left\{ \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right\}$$

$$\Gamma_1^n = \frac{1}{12} (\Delta u - \Delta d) + \frac{1}{36} (\Delta u + \Delta d - 2 \Delta s) + \frac{1}{9} (\Delta u + \Delta d + \Delta s)$$

General sum rule (OPE)

$$\Gamma_{1,p,n}^p = \frac{1}{12} \left( \pm a_3 + \frac{1}{\sqrt{3}} a_8 \right) + \frac{1}{9} a_0$$

- Neutron decay $g_A/g_V \rightarrow a_3$
- Hyperon decay $3F-D \rightarrow a_8$

$Q^2 \neq \infty$

$\Delta u + \Delta d + \Delta s = \Delta \Sigma$

$a_0 = 0.12 \pm 0.17$ surprise!

measured first by EMC in 1987
The structure function $g_1$ and its first moment

$$g_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x)$$

$$\Gamma_1 = \int_0^1 g_1(x) \, dx$$

In naive QPM + SU(3)

$$\Gamma_1^p = \frac{1}{2} \left\{ \frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s \right\}$$

$$with \quad \Delta q = \int \Delta q(x) \, dx$$

$$= \frac{1}{12} \left( \Delta u - \Delta d \right) + \frac{1}{36} \left( \frac{\Delta u + \Delta d - 2\Delta s}{\sqrt{3} a_8} \right) + \frac{1}{9} \left( \frac{\Delta u + \Delta d + \Delta s}{a_0} \right)$$

General sum rule (OPE)

$$\Gamma_1^{p,n} = \frac{1}{12} \left( \pm a_3 + \frac{1}{\sqrt{3}} a_8 \right) + \frac{1}{9} a_0$$

A crisis in the parton model: where, oh where is the proton’s spin?

E. Leader and M. Anselmino
Birkbeck College, University of London, London, UK
Dipartimento di Fisica Teorica, Università di Torino, I-10125 Torino, Italy
Received 18 March 1988
How to solve the spin crisis?

In polarised DIS, one measures $a_0$ flavor singlet axial matrix element at $Q^2 \neq \infty$

- In the MS renormalisation scheme
  \[ a_0 = \Delta \Sigma \]

- While in the Adler-Bardeen and JET schemes (axial anomaly)
  \[ a_0 = \Delta \Sigma - \left( \frac{\alpha_s}{2\pi} \right) n_f \Delta G \]

So a large positive value of $\Delta G$ (~3 at $Q^2 = 3 \text{ GeV}^2$) might resolve the spin crisis?
Huge effort during the past 20 years
Today: COMPASS, HERMES, RHIC

- new measurements of $g_1$, $\Delta q$
- first measurements of $\Delta G$
Polarized Beam + Polarized Target $^6\text{LiD(d)}$ and $\text{NH}_3(p)$

- to be measured:
  \[ A_\parallel = \frac{\sigma^{\uparrow\downarrow} - \sigma^{\uparrow\uparrow}}{\sigma^{\uparrow\downarrow} + \sigma^{\uparrow\uparrow}} \]

- flux normalization:
  \[ A_{\text{exp}} = \frac{N_u - N_d}{N_u + N_d} \]

- acceptance difference:
  Polarisation rotation

- take average asymmetry:
  \[ \Rightarrow A_{\text{exp}} = \frac{A + A'}{2} = \frac{1}{2} \left( \frac{N_u - N_d}{N_u + N_d} + \frac{N'_d - N'_u}{N'_u + N'_d} \right) \]
  \[ \Rightarrow \text{minimization of bias} \]

- elegant and efficient strategy
Asymmetry measurement

\[
\frac{A_{\text{exp}}}{f P_\mu P_T D} \approx A_1
\]

- Inclusive scattering

\[
A_1 = \frac{\sum_q e_q^2 g_1^q(x, Q^2)}{\sum_q e_q^2 f_1^q(x, Q^2)}
\]

\( D \) depolarization factor
\( P_\mu \) target polarization
\( P_T \) target polarization
\( f \) dilution factor
Spin Structure Function $g_1(x,Q^2)$

Using inclusive data

$$g_1(x) = \frac{1}{2} \sum_{q=u,d,s} e_q^2 \Delta q(x)$$

- Very precise data
- Only COMPASS for $x<0.01$ ($Q^2>1$)
- COMPASS Deuteron data from $1 @ Q^2 \to \infty$

$$a_0 = \Delta \Sigma = 0.33 \pm 0.03 \pm 0.05$$
$$\Delta s^+ \Delta s = 1/3 (a_0-a_8) = -0.08 \pm 0.01 \pm 0.02$$

What makes up the missing 70%?
Direct Access to the Gluon polarisation (SIDIS) \( \ell p \rightarrow \ell' h X \)

Measured in photon-gluon fusion (PGF)

\[
A_{||} = R_{pgf} \hat{a}_{pgf} \frac{\Delta G}{G} + A_{bdf}
\]

2 signatures to suppress the background:

1. \( q=c \) and charm hadronization in meson \( D^0 (c \bar{u}) \rightarrow K \pi \)
   - Detection of \( D^0 \) mesons
   - Very clean signal but limited statistics

2. soit \( q=u,d,s \) and \( q \bar{q} \rightarrow \) hadron1 + hadron2
   - Detection of a high \( p_T \) hadron pair
   - Physical background, better described at high \( Q^2 \)
Gluon Polarization: from LO to NLO

- All results compatible with 0!  \( a_0 = \Delta \Sigma \)
- Confirmed by RHIC results in pp

Let’s go to measure \( L_z \)!
So far the partons were consider collinear to the nucleon movement in photon-nucleon collision (longitudinal direction), but what else about the transverse information?

Transverse spin and transverse momentum
Proton Sivers effect in tranv. pol. SIDIS: $\ell p \uparrow \rightarrow \ell' h X$

Azimuthal cross-section asymmetry:

$$\frac{\Delta \sigma}{\sigma} \propto A_{Siv} \sin \Phi_S$$

$$\Phi_S = \phi_h - \phi_S$$

After a long debate this effect is actually observed.
Observation of large transverse single spin asymmetries

\[ p \uparrow p \rightarrow \pi^+/\pi^-/0 + X \]

persisting at large energy

CERN Courier, June 2009

from Christine Aidala, Spin 2008 and Don Crabb & Alan Krisch in then Spin 2008 Summary, CERN Courier, 6-2009
and what’s next

Focus on transverse structure of the nucleon

- Transverse Momentum Dependent TMD PDF
  → Study of SIDIS and DY

- Transverse size and orbital angular momentum:
  Generalized Parton Distributions (GPDs)
  → Study of Exclusive reactions
  - COMPASS-II programme (2012-2016)
  - Jlab 11 GeV (start in 2014)
  - RHIC
  - JPARC

- and more future ENC, eRHIC/ELIC, NICA

Coming from the general Wigner phase-space-distributions
from inclusive reactions

Deep Inelastic Scattering

\[ \mu p \rightarrow \mu' X \]

Parton Distribution \( q(x) \)

Deeply Virtual Compton Scattering

\[ \mu p \rightarrow \mu' p' \gamma \]

Generalized Partons Distrib. \( H(x, \xi, t) \)

(\( P_x, b_\perp \))

Observation of the Nucleon Structure

in 1 dimension

to exclusive reactions

in 1+2 dimensions
From 1D

ECG: monodimensional information on heart activity

to 3D

Functional MRI: tomography of heart activity
Generalized Partons Distributions (H,E,...)

- Allow for a unified description of form factors and parton distributions
- Allow for transverse imaging (nucleon tomography) and give access to the quark angular momentum (through E)

Impact parameter $b_{\perp}$

Longitudinal momentum fraction $x$

Tomographic parton images of the nucleon

$x \sim 0.003$ $x \sim 0.03$ $x \sim 0.3$
Contributions of DVCS and BH at $E = 160$ GeV

Deep VCS

Bethe-Heitler

$d\sigma \propto |T^{DVCS}|^2 + |T^{BH}|^2 + \text{Interference Term}$

BH dominates

excellent

reference yield

study of Interference

$\rightarrow \Re T^{DVCS}$

or $\Im T^{DVCS}$

DVCS dominates

study of $d\sigma^{DVCS}/dt$

$\rightarrow$ Transverse Imaging

Monte-Carlo Simulation for COMPASS set-up with only ECAL1+2

Missing DVCS acceptance without ECAL0
Transverse imaging at COMPASS

\[ \frac{dVCS}{dt} \sim \exp(-B|t|) \]

\[ B(x_B) = \frac{1}{2} < r_{\perp}^2(x_B) > \]

distance between the active quark and the center of momentum of spectators

Transverse size of the nucleon

mainly dominated by \( H(x, \xi=x, t) \)

\[ \sqrt{\langle r_{\perp}^2 \rangle} \]

related to \( \frac{1}{2} < b_{\perp}^2(x_B) > \)

distance between the active quark and the center of momentum of the nucleon

Impact Parameter Representation

\( q(x, b_{\perp}) \leftrightarrow H(x, \xi=0, t) \)

\[ 0.65 = \sqrt{2/3} \times 0.8 \]

COMPASS

\( H1 \ PLB659(2008) \)

\[ 0.65 \pm 0.02 \text{ fm} \]
Without any model we can extract $B(x_B)$

$$B(x_B) = \frac{1}{2} < r_T^2(x_B) >$$

$r_T$ is the transverse size of the nucleon

Accuracy $> 2.5 \quad \alpha' = 0.125$ and full ECALS
2012: we can determine one mean value of $B$ in the COMPASS kinematic range

$\frac{dI^{\text{DVCS}}}{dt} \sim \exp(-B|t|)$

DVCS test in 2012

With 1 week
Using the 4m long RPD
+ the 2.5m long LH2 target
1/40 of the complete statistics
The GPD $E$ is the 'Holy-Grail' of the GPD quest

the GPD $E$ allows nucleon helicity flip
so it is related to the angular momentum

Ji sum rule: $2J_q = \int x \left( H_q(x,\xi,0) + E_q(x,\xi,0) \right) \, dx$

Constrains on the GPD $E$

- Using a neutron target
  \[ A^n \propto F_1(t) \cdot H - \frac{t}{4M^2} F_2(t) \cdot E \]

- Using a transversely polarized protons target
  \[ A_T^p \propto F_2(t) \cdot H - F_1(t) \cdot E \]
In the near future

Two main actors
- COMPASS (sea quarks and gluons)
- Jlab (valence domain, with a huge luminosity)

An important activity
For global fits on the world data (Mueller, Guidal, Moutarde, ...)

JHEP06 (2008) 066
Lattice QCD calculations

Ph. Haegler, 2010
Nucleon Spin is fun!

Still a long and exciting trip