Nuclear interactions and nuclei

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and OAK RIDGE NATIONAL LABORATORY

Aim of these lectures:

Give overview of nuclear forces from effective field theory, results of first principles computations, and an effective theory for heavy deformed nuclei

30th Ecole Joliot Curie: Physics at the femtometer scale

September 12-17 2011, La Colle Sur Loup / France

Content

1. Lecture: Overview, interactions from chiral effective field theory, and results from nuclear structure computations

2. Lecture: Rest of lecture 1 + effective theory for deformed nuclei

Reading suggestions

More is different, P. W. Anderson, Science **177**, 393 (1972)

Elementary features of nuclear structure, B.R. Mottelson, in: H. Nifenecker, J.P. Blaizot, G. Bertsch, W. Weise, F. David (Eds.), *Trends in Nuclear Physics, 100 Years Later*, North-Holland, Amsterdam, 1998

Chiral effective field theory and nuclear forces

- Machleidt, *Nuclear Forces from chiral effective field theory,* arXiv:0704.0807
- Epelbaum Hammer & Meißner, *Modern theory of nuclear forces,* Rev. Mod. Phys. **81**, 1773 (2009); arXiv:0811.1338

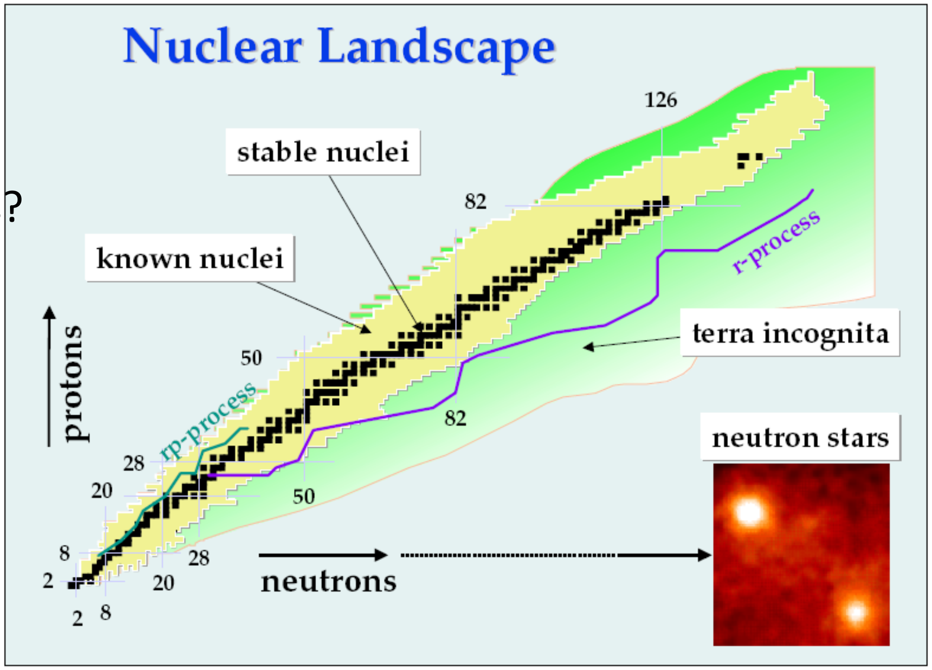
Low-momentum interactions and similarity transforms

- Bogner, Furnstahl & Schwenk, *From low-momentum interactions to nuclear structure,* Prog. Part. Nucl. Phys. **65**, 94 (2010); arXiv:0912.3688

Effective theory for deformed nuclei

- Haruo Ui and Gyo Takeda, *A Class of Simple Hamiltonians with Degenerate Ground State. II A Model of Nuclear Rotation*, Prog. Theor. Phys. **70**, 176 (1983)
- Papenbrock, *Effective theory for deformed nuclei*, Nucl. Phys. A **852**, 36 (2011), arXiv:1011.5026

Science questions



Basic science questions in the physics of nuclei:

1. What binds nucleons into stable nuclei and rare isotopes?
2. What are the limits of nuclear binding?
3. What is the origin of simple modes in complex systems?

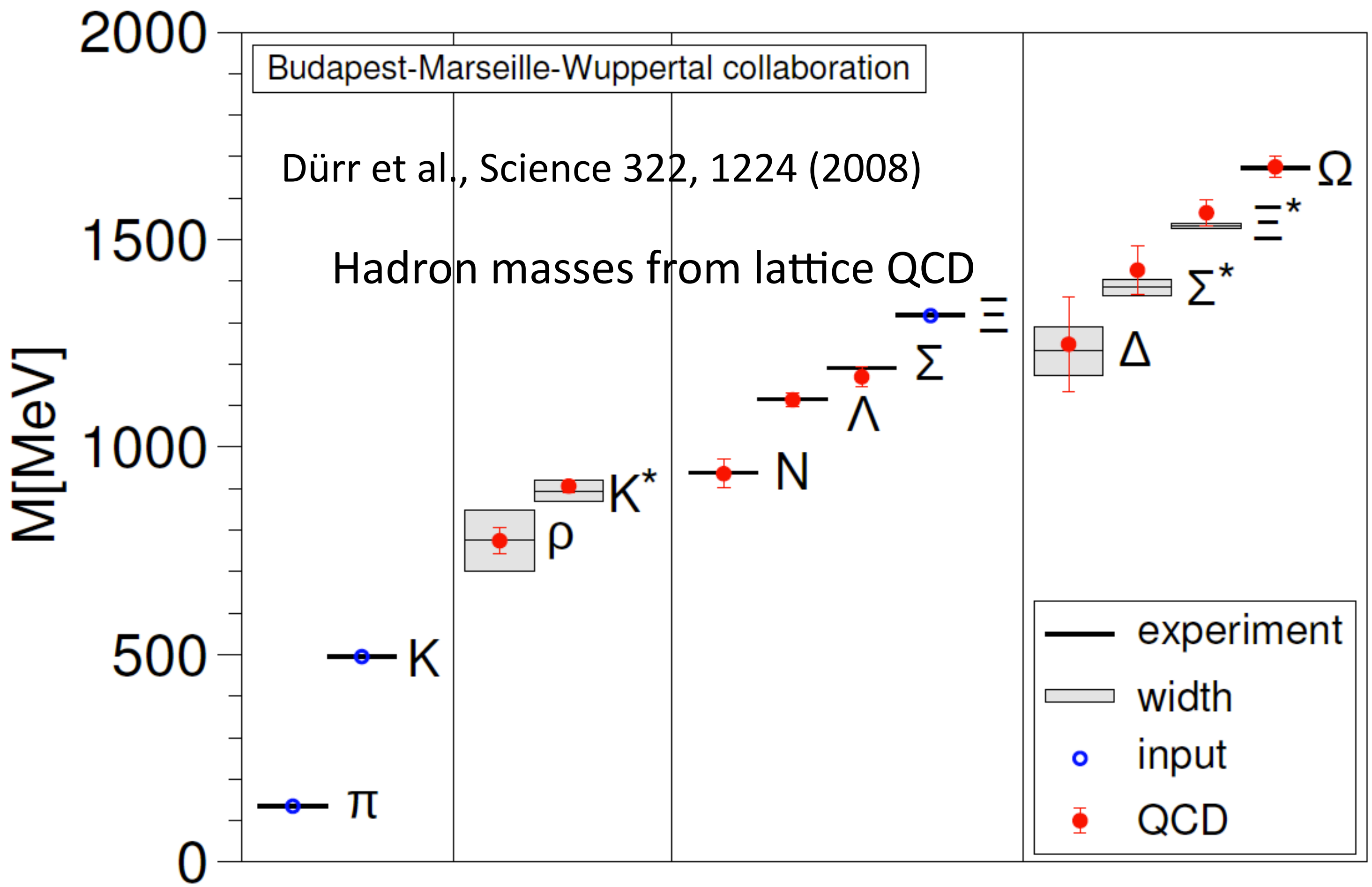
Basic science questions in nuclear astrophysics:

4. How are the elements from iron to uranium made?
5. What is the fate of massive stars?
6. What is the mass of the neutrino, and what is its nature (Dirac / Majorana fermion)?

Understanding of rare isotopes central to addressing four of these questions (1, 2, 4, 5).

Rare isotopes also relevant for medical applications (imaging and therapy), and energy production

Quantum chromo dynamics – theory of the strong interaction



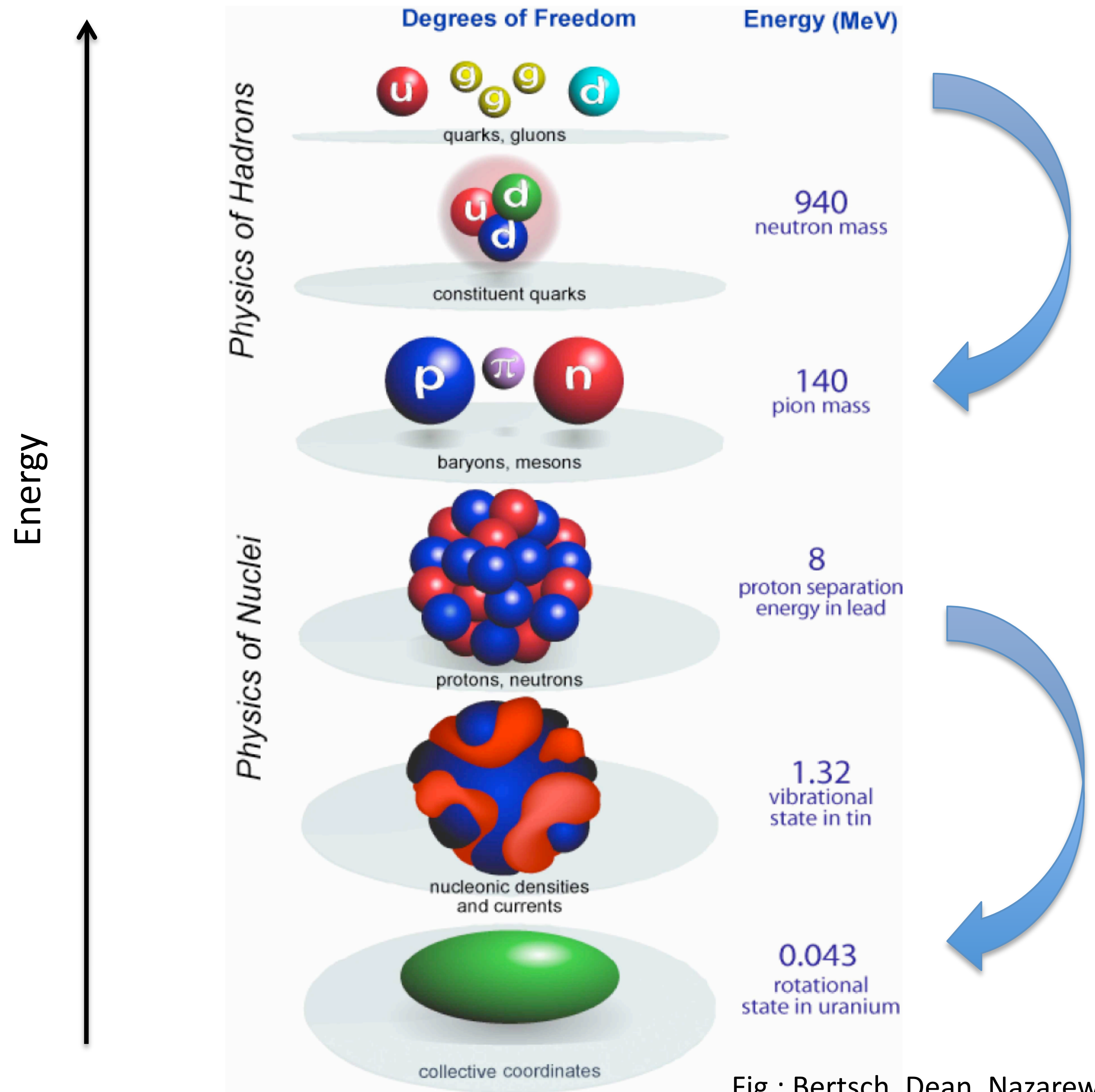
Most impressive progress (→ lecture by J. P. Lansberg)

But: first-principle computation of nuclei from QCD are still far away …

Worse: Looking at the QCD Lagrangian, it is not obvious what the low-energy QCD physics is. Neither the spontaneous breaking of chiral symmetry nor the emergence of selfbound nuclei is obvious or predicted from QCD.

(The QED Lagrangian also does not tell us about emerging phenomena such as superconductivity or crystals.) We need another approach!

Energy scales and relevant degrees of freedom



Chiral effective field theory

Effective theory of deformed nuclei

Fig.: Bertsch, Dean, Nazarewicz, SciDAC review (2007)

Effective field theory

Q: How can we economically solve a physical problem (by employing appropriate degrees of freedom)?

A: Exploit a separation of scales.

Examples:

1. Multipole expansion for the electromagnetic field.

   Q: Why / when does it work?

   A: Distance from charge distribution >> extension of charge distribution

2. Quantum chemistry employs the Coulomb potential and not QED

   Q: Why does it work?

   A: $e^+ e^-$ pair production threshold (~ 1MeV) >> chemical bonds (~ eV)

3. Nuclei are described in terms of protons and neutrons and not via quarks and gluons

   Q: Why does it work?

   A: Excitation of nucleon (~300 MeV) >> excitation energies of nuclei (~ 1MeV)

Construction of nuclear potentials via chiral EFT

Weinberg, van Kolck, Epelbaum, Machleidt, …

1. Identify the **relevant degrees of freedom** for the resolution scale of atomic nuclei: **nucleons and pions**.
2. Identify the **relevant symmetries** of low-energy QCD and investigate if and how they are broken: **spontaneously broken chiral symmetry**
3. Construct the most general Lagrangian consistent with those symmetries and the symmetry breaking.
4. Design an **organizational scheme** that can distinguish between more and less important contributions: a low-momentum expansion: **power counting**
5. Guided by the expansion, calculate Feynman diagrams to the desired accuracy for the problem under consideration.

Reviews:

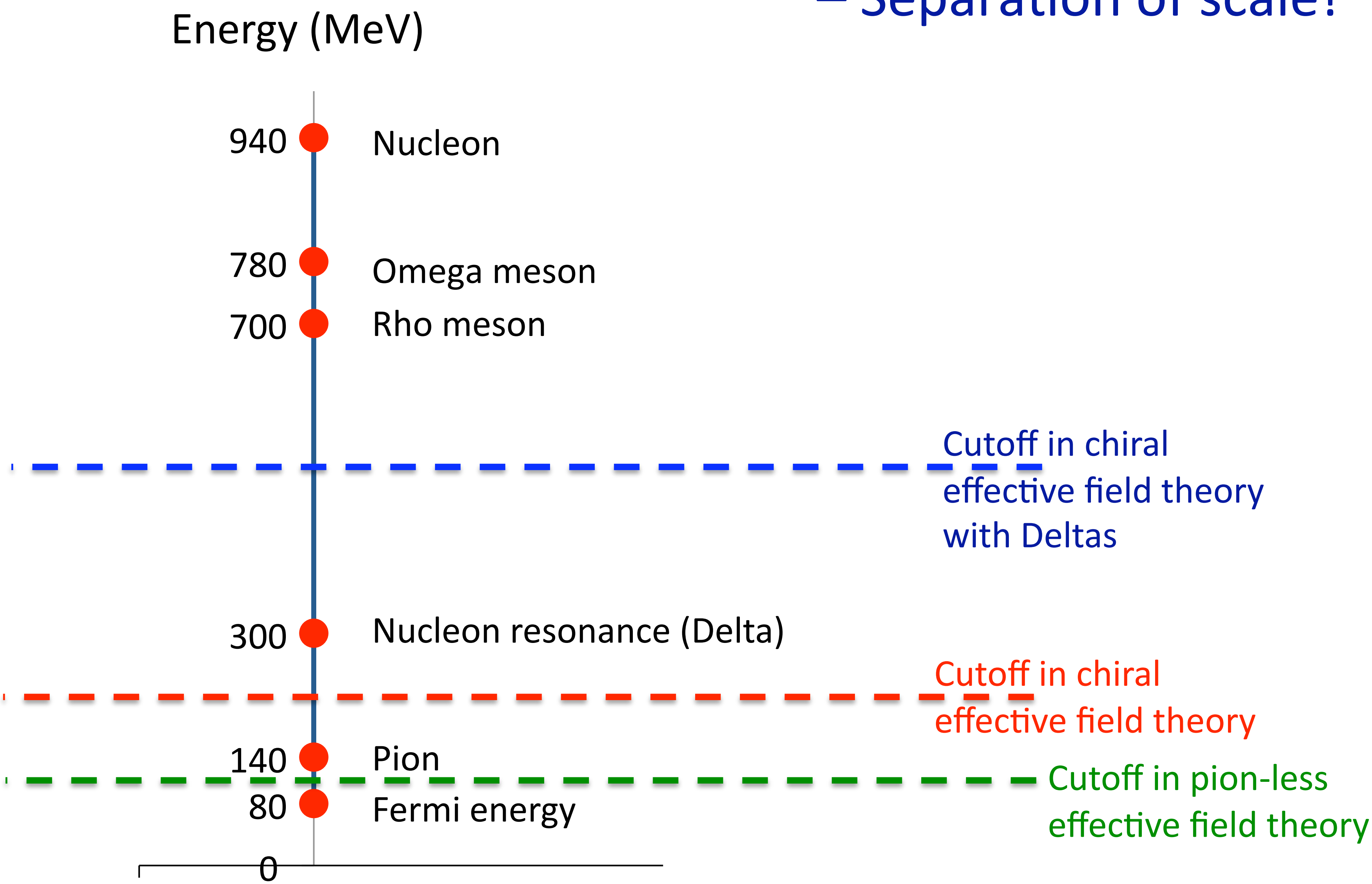
Bedaque and van Kolck, Ann. Rev. Nucl. Part. Sci. 52 (2002) 339, nucl-th/0205058.

Machleidt, arxiv:0704.0807.

Epelbaum, Hammer, Meißner, Rev. Mod. Phys. 81, 1773 (2009); arXiv:0811.1338.

1. Identify relevant degrees of freedom

– Separation of scale!



2. Identification of relevant symmetries

1. **SU(3) color symmetry** from QCD
   (Nucleons and pions are color singlets)

2. **Chiral symmetry**: Left and right-handed massless u and d quarks do not mix: $SU(2)_L \times SU(2)_R$ symmetry. Expect left-right parity doublets in nature.

Explicit breaking of chiral symmetry: u and d quarks have a small mass. Small corrections to above picture arise.

But: There are no parity doublets observed in nature!

Reason: **Spontaneous breaking of chiral symmetry** (More is different!)

- $SU(2)_L \times SU(2)_R$ symmetry spontaneously broken to $SU(2)_V$
- Pions are the Nambu-Goldstone bosons of spontaneously broken chiral symmetry
- Low-energy pion Lagrangian completely determined

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{N\pi} + \mathcal{L}_{NN}$$

Intermezzo: Spontaneous symmetry breaking

The fundamental laws of physics are invariant under rotations. How can non-spherical things (e.g. water molecules, pencils, humans) exist?

1. Non-spherical things have a ground state with nonzero spin and fixed spin projection.
2. The non-spherical things are not in their ground state
3. Macroscopic things, even in their ground states, do not need to be eigenstates of angular momentum.

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Spontaneous symmetry breaking

Def.: Spontaneous symmetry breaking happens when an arbitrarily small external perturbation yields a non-symmetric ground state.

Strictly speaking, this can only happen in macroscopic systems that have a gapless excitation spectrum.

Q1: What is the excitation energy for horizontal translational motion of a macroscopic object in this room?

A1: $E = \hbar^2/(2mL^2) \approx 10^{-70}$ Joule (Object of mass m=1kg in a box of size L=10m)

Consequence: Superpositions of practically degenerate plane wave states of the center-of-mass will yield a localized ground state because of spontaneous symmetry breaking.

Q2: What is the excitation energy for rotational motion of a macroscopic object in this room?

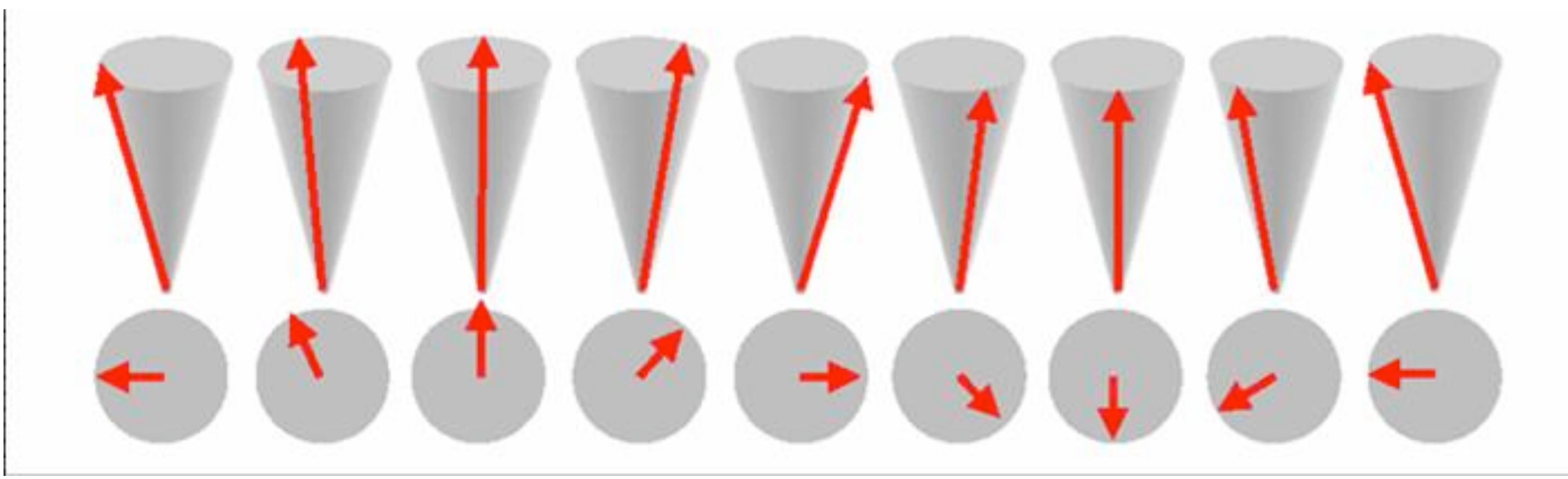
A2: $E = \hbar^2 J(J+1)/(2mR^2) \approx 10^{-64}$ Joule (Object of mass m=1kg and radius R=10cm)

Consequence: Superpositions of practically degenerate states with different angular momentum J will yield a deformed ground state because of spontaneous symmetry breaking.

Nambu-Goldstone modes are low-lying excitations in the presence of spontaneous symmetry breaking

Spontaneous symmetry breaking of rotational symmetry: ferromagnet

- Axially symmetric ground state breaks SO(3) rotational symmetry.
- Nambu-Goldstone modes generate local (i.e. position and time dependent) rotations of the spins.
- **$\exp(-i\,\psi_x(x,y,z,t)\,J_x - i\,\psi_y(x,y,z,t)\,J_y)$** with Nambu-Goldstone fields **(ψ_x, ψ_y)** and angular momentum operators **(J_x, J_y)**
- Spin waves (or magnons) are low-energy excitations with long wave length



Picture: www-llb.cea.fr

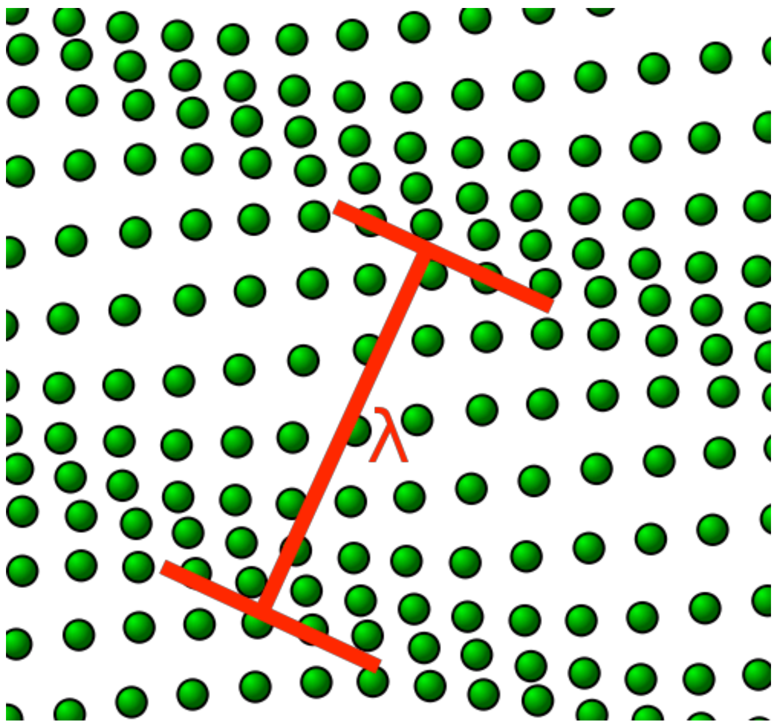
Spontaneous breaking of translational symmetry: crystal

• Crystal lattice breaks translational symmetry.

• Nambu-Goldstone modes generate local (i.e. position and time dependent) translations of the lattice points (ions).

• **exp(-i $\psi_x(x,y,z,t)\ P_x$ - i $\psi_y(x,y,z,t)\ P_y$ - i $\psi_z(x,y,z,t)\ P_z$)** with Nambu-Goldstone fields **(ψ_x, ψ_y, ψ_z)** and momentum operators **(P_x, P_y, P_z)**

• Phonons are low-energy excitations (wave length λ much larger than lattice spacing)



Source: wikipedia (Florian Marquard)

3. Construct most general Lagrangian consistent with symmetries; organizational scheme → power counting

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi} + \mathcal{L}_{N\pi} + \mathcal{L}_{NN}$$

Derivative (low-momentum) expansion indicated by superscripts

Pion-pion Lagrangian: U is SU(2) matrix parameterized by three pion fields

$$\mathcal{L}^{(2)}_{\pi\pi} = \frac{f_\pi^2}{4}\,\mathrm{tr}\left[\partial^\mu U \partial_\mu U^\dagger + m_\pi^2 (U + U^\dagger)\right]$$

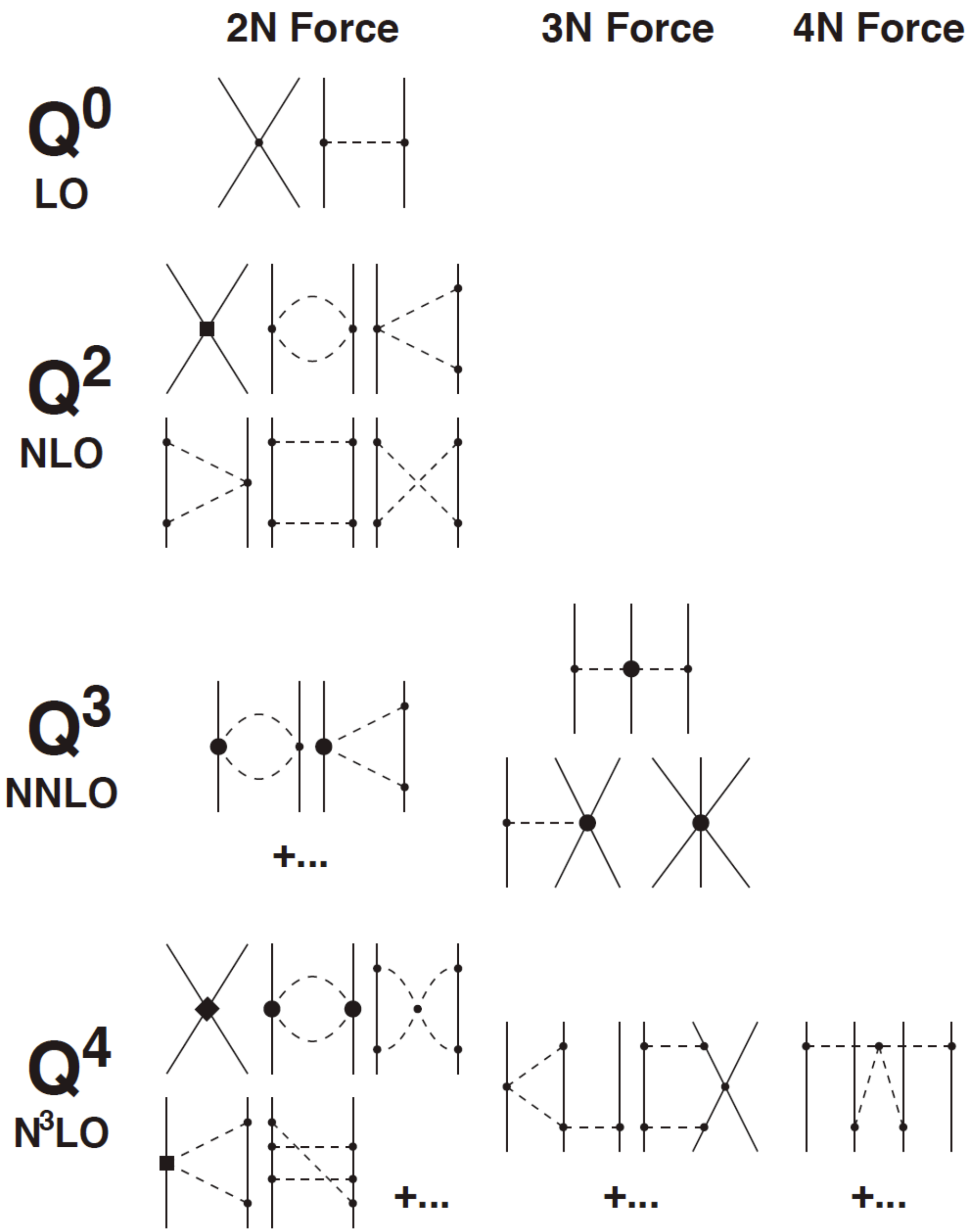
Leading order pion-nucleon Lagrangian

$$\mathcal{L}^{(1)}_{\pi N} = \bar{\Psi}\left(i\gamma^\mu D_\mu - M_N + \frac{g_A}{2}\gamma^\mu \gamma_5 u_\mu\right)\Psi$$

Leading order nucleon-nucleon Lagrangian (encodes unknown short-ranged physics)

$$\mathcal{L}^{(0)}_{NN} = -\frac{1}{2}C_S \bar{N}N\bar{N}N - \frac{1}{2}C_T \bar{N}\vec{\sigma}N\bar{N}\vec{\sigma}N$$

Effective field theory: chiral potential at order N^3LO



Nucleons: full lines
Pions: dashed lines

Features:

1. Systematic expansion of nucleon potential; small parameter (Q/Λ)
2. Low-energy constants from fit to data
3. Hierarchy of forces NN >> NNN >> NNNN

[from Machleidt arXiv:0704.0807]

Chiral nucleon-nucleon potential at leading order

One-pion exchange potential (p, p' are initial and final relative momenta)

$$V_{1\pi}(\vec{p}\,', \vec{p}) = -\frac{g_A^2}{4 f_\pi^2} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \frac{\vec{\sigma}_1 \cdot \vec{q}\,\vec{\sigma}_2 \cdot \vec{q}}{q^2 + m_\pi^2}$$

$$\vec{q} \equiv \vec{p}\,' - \vec{p}$$

Leading order contact term (encode unknown short-range physics)

$$V^{(0)}(\vec{p'}, \vec{p}) = C_S + C_T\,\vec{\sigma}_1 \cdot \vec{\sigma}_2$$

Higher-order contact terms also serve as counter terms that renormalize loop integrals.

Why contact terms?

1. Only contact terms can model really short range physics.
2. Any short-range terms (e.g. delta functions, Gaussians …) with range smaller $1/\Lambda$ would do the job, but contacts are computationally very convenient.

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How does the momentum cutoff Λ enter the EFT?

1. The construction of the chiral potential involves solving the Lippmann-Schwinger equation. Λ is the cutoff in this equation.

$$\widehat{T}(\vec{p}\,',\vec{p}) = \widehat{V}(\vec{p}\,',\vec{p}) + \int d^3p''\,\widehat{V}(\vec{p}\,',\vec{p}\,'')\,\frac{M}{p^2 - p''^2 + i\epsilon}\,\widehat{T}(\vec{p}\,'',\vec{p})$$

$$\widehat{V}(\vec{p}\,',\vec{p}) \quad \longmapsto \quad \widehat{V}(\vec{p}\,',\vec{p})\,\boldsymbol{e}^{-(p'/\Lambda)^{2n}}\;\boldsymbol{e}^{-(p/\Lambda)^{2n}}$$

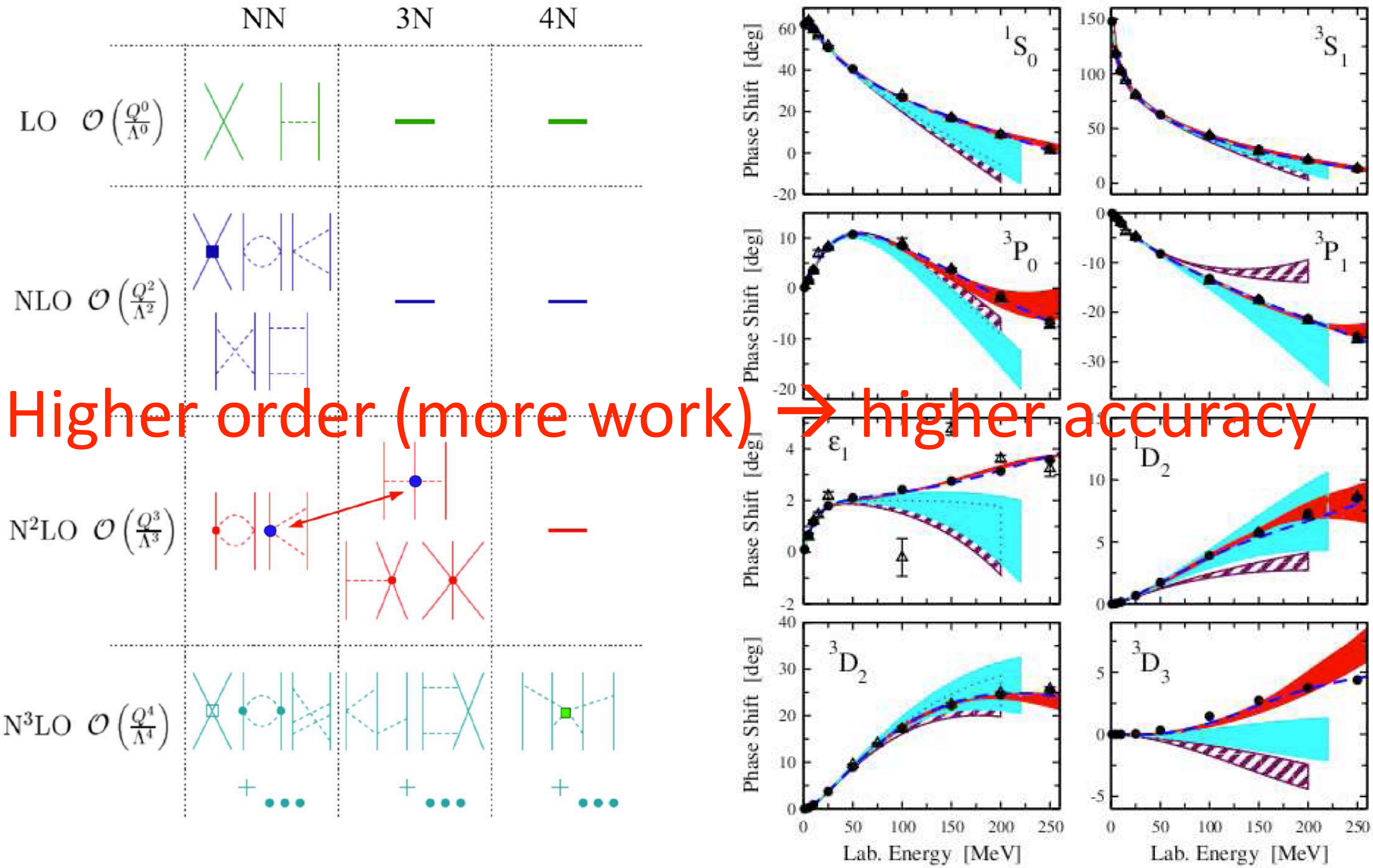
2. The loop integrals that appear beyond leading order need to be regularized. One way of regularization is by imposing a cutoff of the order of Λ.

3. As a result, the low-energy constants depend implicitly on the regularization scheme and the cutoff.

4. There are (infinitely) many different chiral potentials! Differences of potentials that employ different values for the cutoff must be of higher order.

5. Regularization schemes, and form of potentials that encode short-ranged physics (contact potential or potentials with a very short range) are at the potential builder’s discretion. **This makes the approach model independent**.

Higher order (more work) → higher accuracy

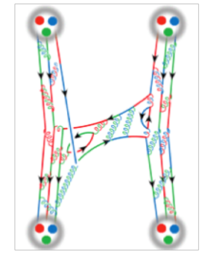


(a) (b)

From Bogner et al, arXiv:0912.3688

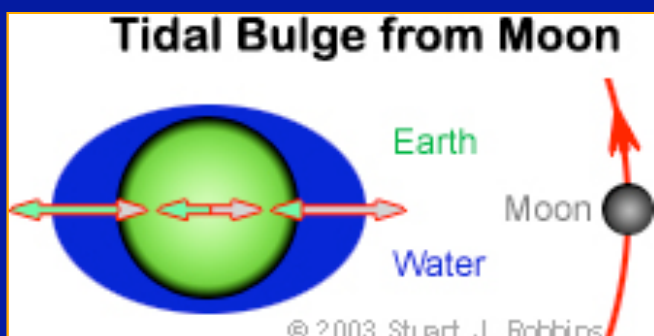
Figure 4: (a) Chiral EFT for nuclear forces. (b) Improvement in neutron-proton phase shifts shov by shaded bands from cutoff variation at NLO (dashed), N^{2}LO (light), and N^{3}LO (dark) compared extractions from experiment (points) [31]. The dashed line is from the N^{3}LO potential of Ref. [20].

Three-nucleon forces – Why?

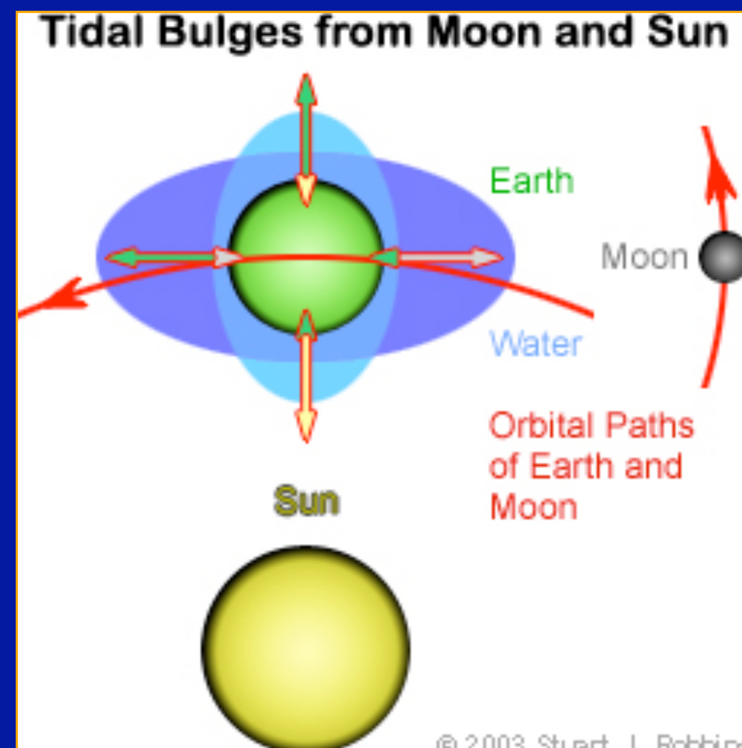


- Nucleons are not point particles (i.e. not elementary).
- We neglected some internal degrees of freedom (e.g. Δ-resonance, “polarization effects”, …), and unconstrained high-momentum modes.

Example from celestial mechanics:
Earth-Moon system: point masses and modified two-body interaction



Other tidal effects cannot be included in the two-body interaction! Three-body force unavoidable for point masses.



Renormalization group transformation: Removal of “stiff” degrees of freedom at expense of additional forces.

Three-body forces cont'd

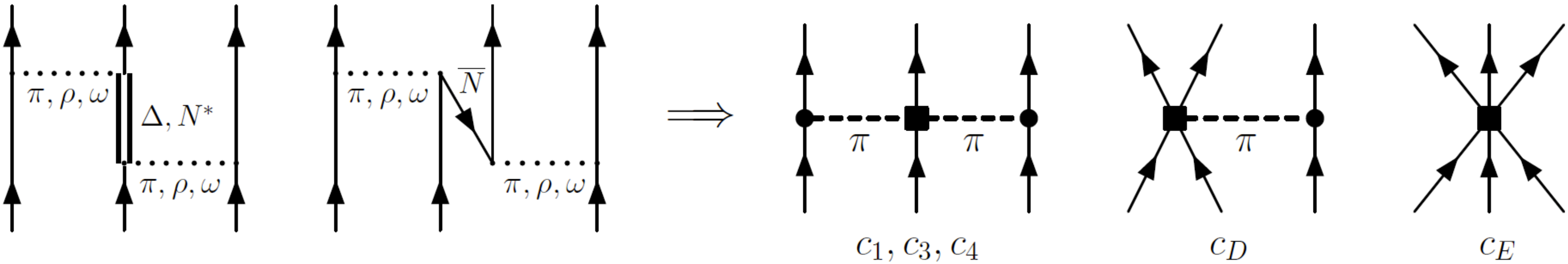


Figure 23: Eliminating degrees of freedom leads to three-body forces.

(from Bogner, Furnstahl, Schwenk, arXiv:0912.3688)

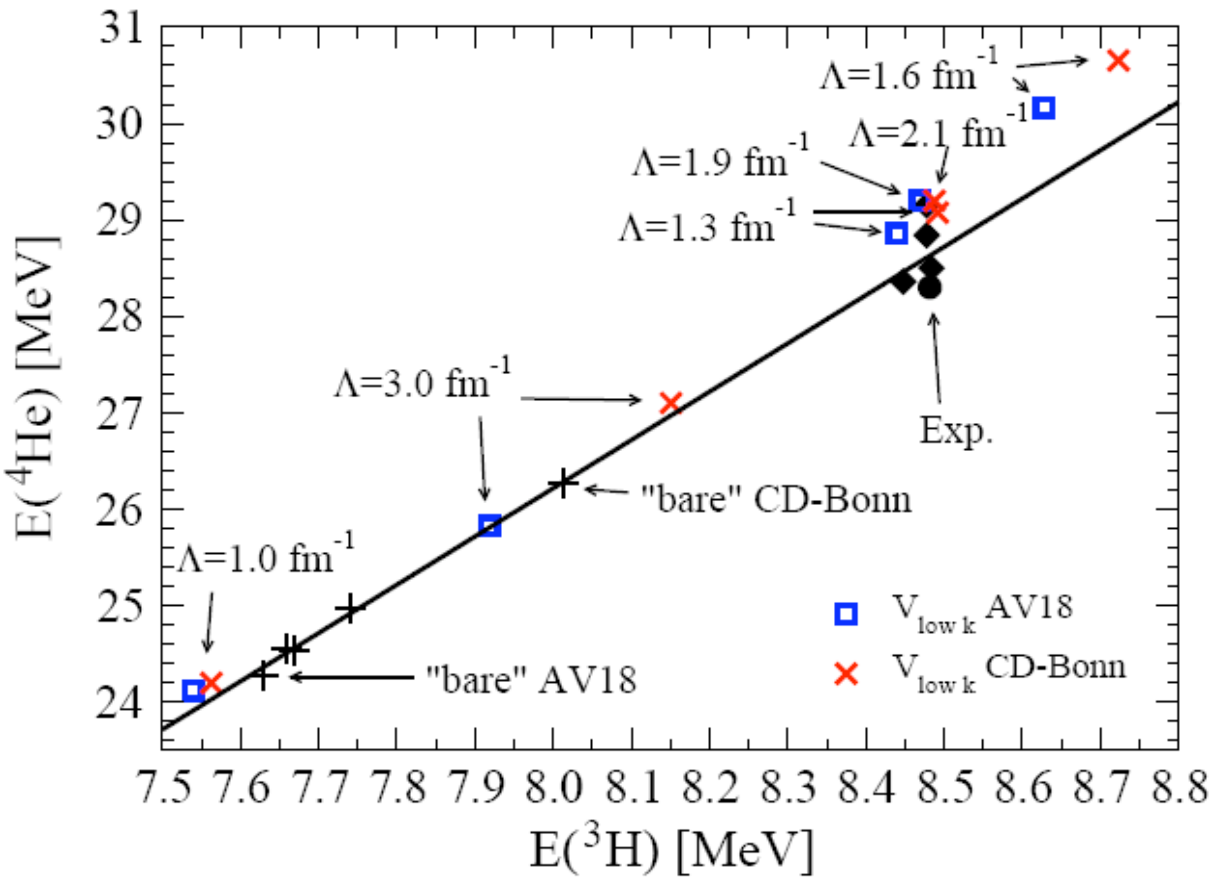
Leading three-nucleon force

1. Long-ranged two-pion term (Fujita & Miyazawa …)
2. Intermediate-ranged one-poin term
3. Short-ranged three-nucleon contact

The question is not: Do three-body forces enter the description?

The (only) question is: How large are three-body forces?

Non-uniqueness of three-nucleon forces



A. Nogga, S. K. Bogner, and A. Schwenk, Phys.Rev. C70 (2004) 061002

As cutoff Λ is varied, motion along "Tjon line".

Addition of Λ-dependent three-nucleon force yields (almost) agreement with experiment. **Q: What's missing?**

A: The complete description of ^{4}He would require four-nucleon forces!

Question: Your favorite physics friend comes to you and suggests to determine the effects of the three-body force on the structure of your favorite nucleus. You reply

1. Let's do this. This will put us on the fast track to Stockholm.
2. This is difficult to disentangle. But it can be done in a three-body system such as ^3H.
3. Which interaction are you looking at?
4. Answers 2 & 3 are correct.

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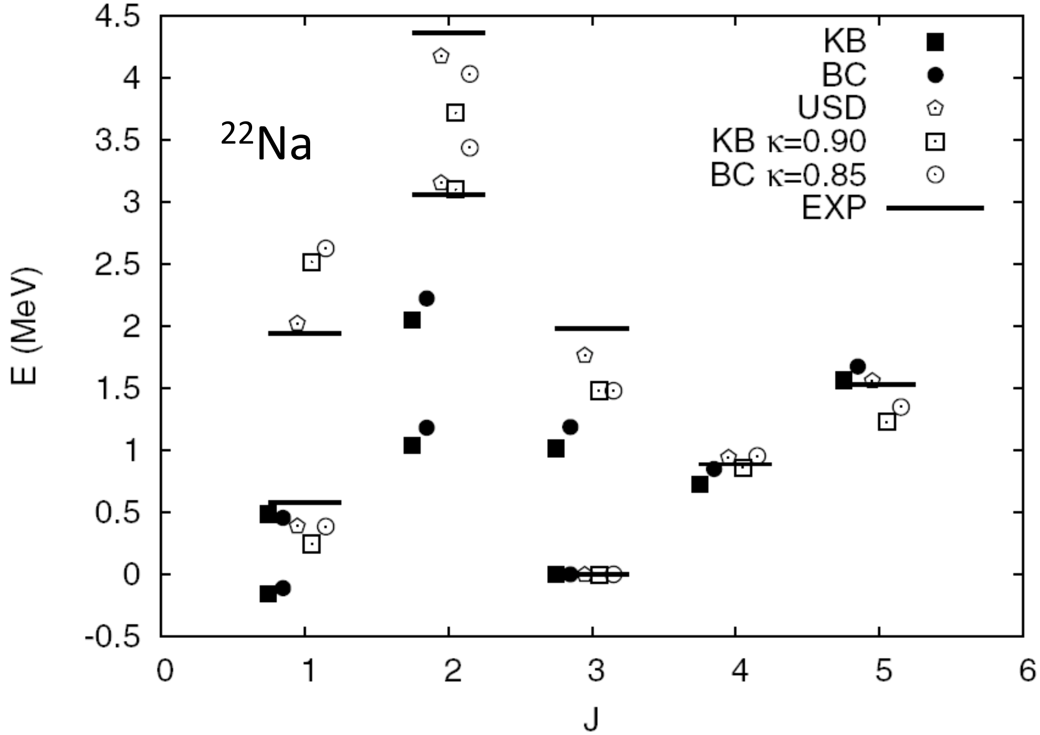
The size and form of three-body forces depends on the cutoff, and the chosen renormalization scheme. Different schemes ("implementations of the EFT at order n") yield predictions that expected to agree within the error estimate $(Q/\Lambda)^{n+1}$. Only the sum of interactions can be probed.

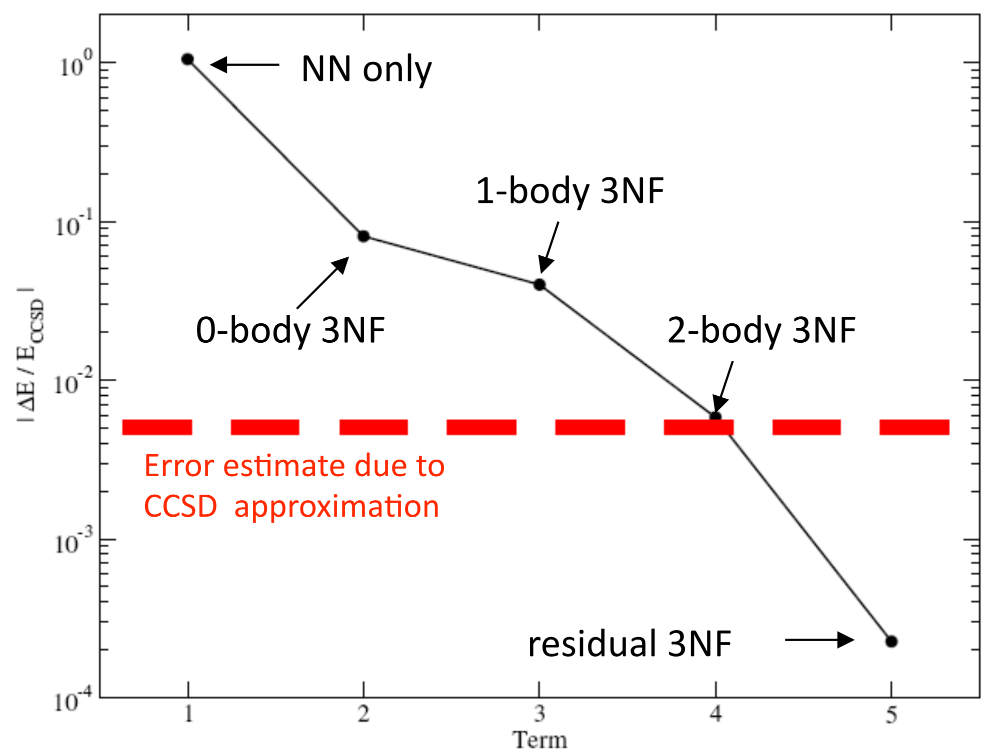
What's the role of three-nucleon forces?



Monopole shifts from 3NF as density-dependent NN force. [A.Zuker, PRL 90, 42502 (2003)]

Contributions to binding of ^{4}He. [Hagen, TP, Dean, Schwenk, Nogga, Wloch, Piecuch, PRC 76, 034302 (2007)]

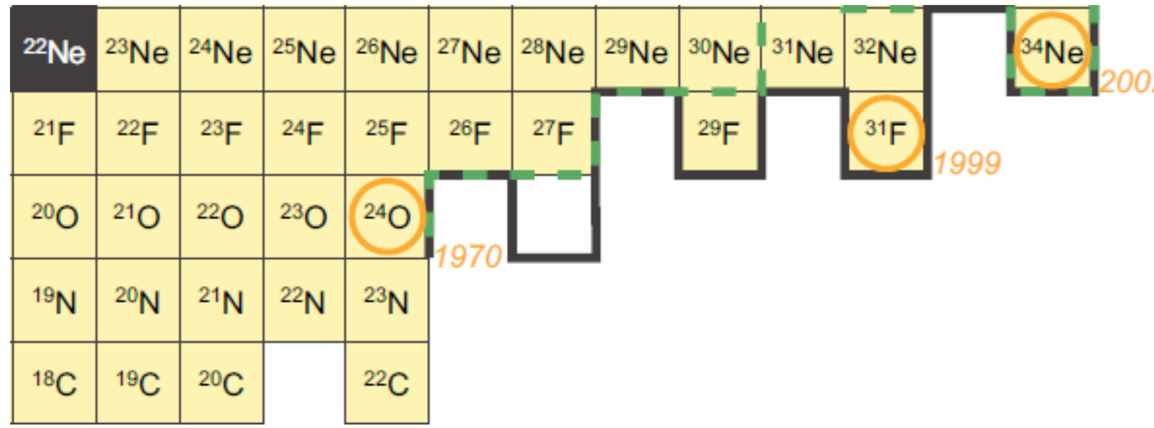
$$\hat{H}_3 = \frac{1}{6}\sum_{ijk}\langle ijk||ijk\rangle + \frac{1}{2}\sum_{ijpq}\langle ijp||ijq\rangle\{\hat{a}_p^\dagger \hat{a}_q\} + \frac{1}{4}\sum_{ipqrs}\langle ipq||irs\rangle\{\hat{a}_p^\dagger \hat{a}_q^\dagger \hat{a}_s \hat{a}_r\} + \hat{h}_3 \,,$$

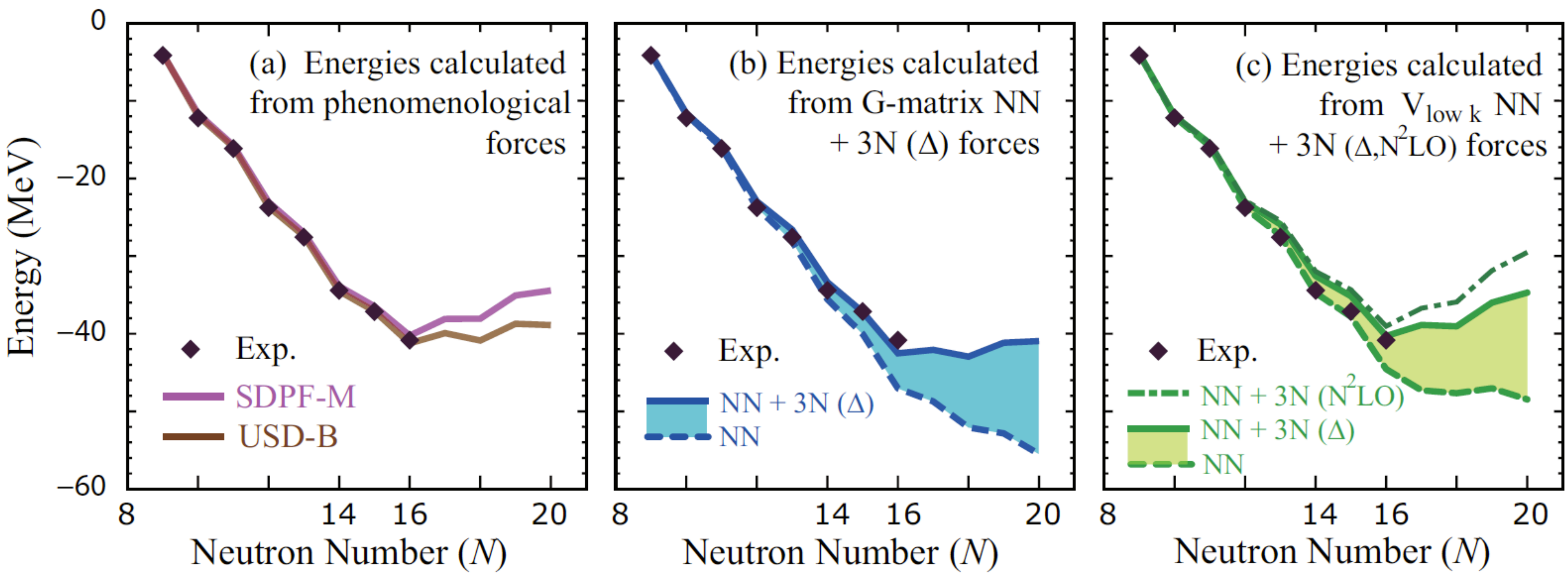


Is ^{28}O a bound nucleus?

Experimental situation

- "Last" stable oxygen isotope ^{24}O
- ^{25}O unstable (Hoffman et al 2008)
- ^{26,28}O not seen in experiments
- ^{31}F exists (adding on proton shifts drip line by 6 neutrons!?)



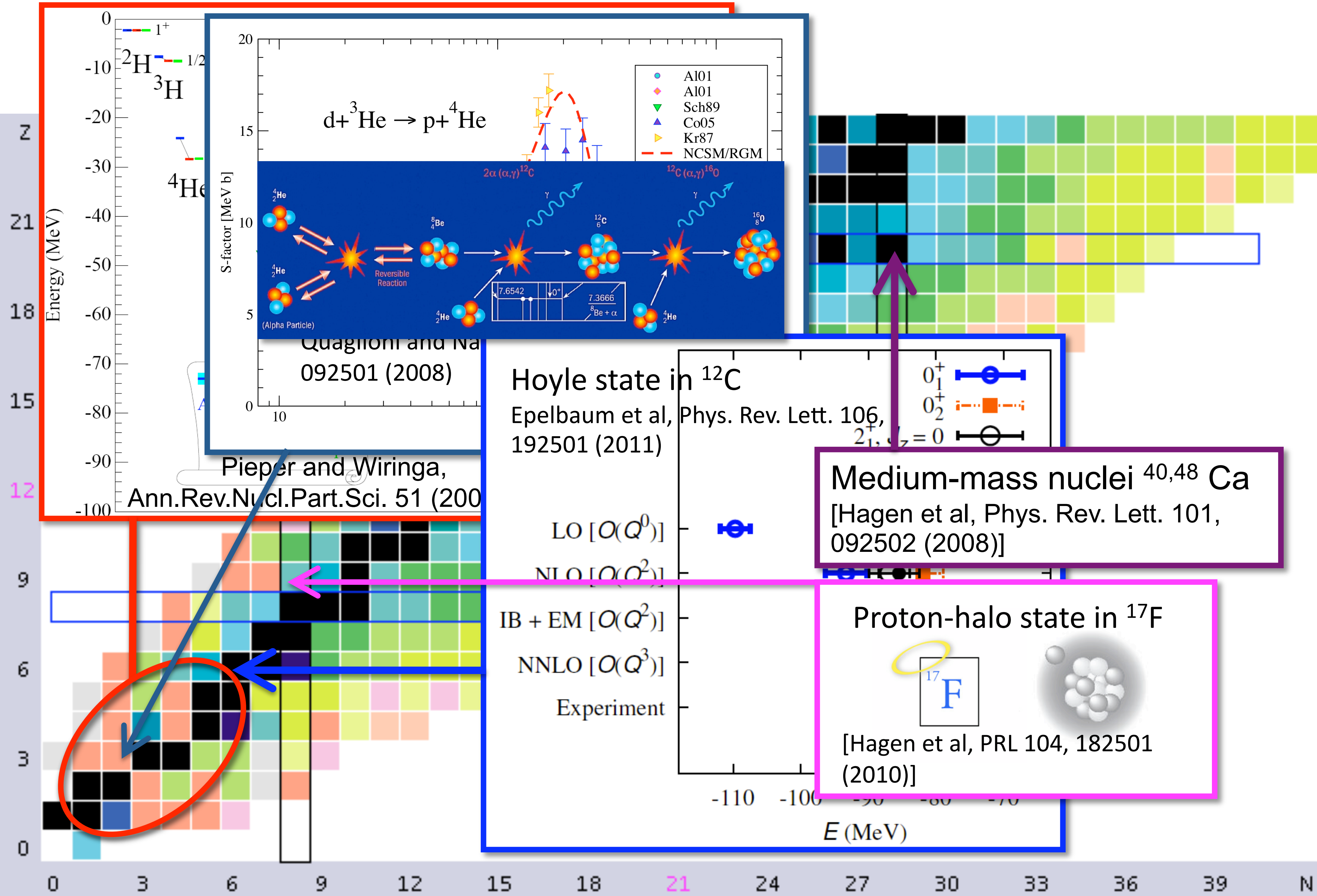


Shell model (sd shell) with monopole corrections based on three-nucleon force predicts 2nd O as last stable isotope of oxygen. [Otsuka, Suzuki, Holt, Schwenk, Akaishi, PRL (2010), arXiv: 0908.2607]

Intermission

- Systematic construction of nuclear forces within (chiral) effective field theory
- There is a recipe to follow
- Highlights: power counting, hierarchy of NN >> NNN >> NNNN forces
- Approach is model independent
- Resulting potential depends on regularization scheme and cutoff
- There are (infinitely) many good ways to implement this

Highlights: first-principle computations of nuclei



Light nuclei from a chiral interaction (N^{3}LO by Entem & Machleidt) with no-core shell model

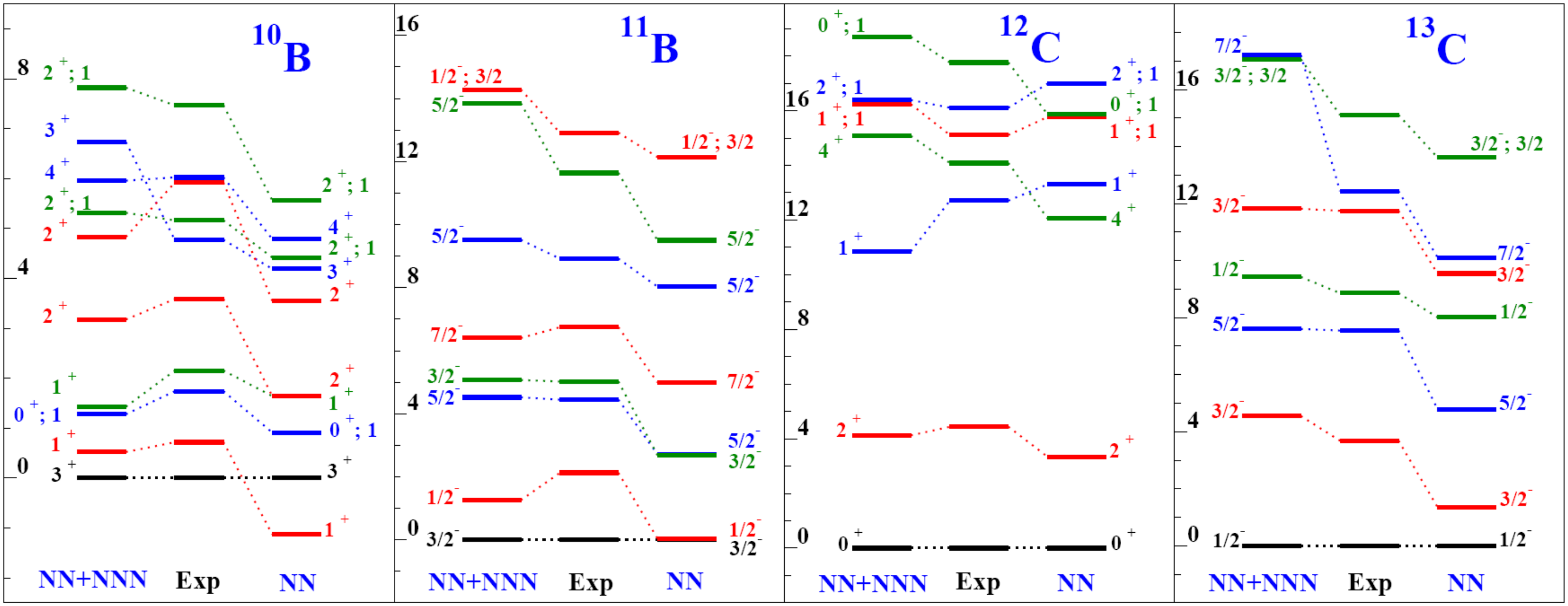
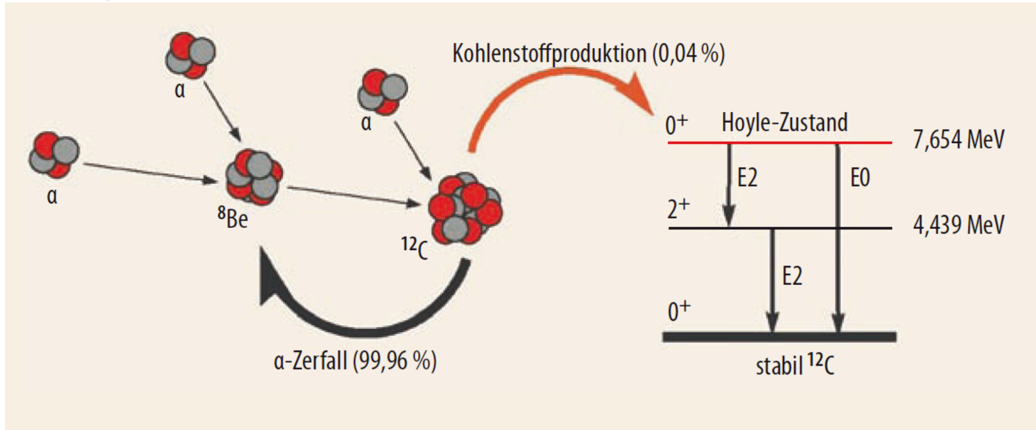


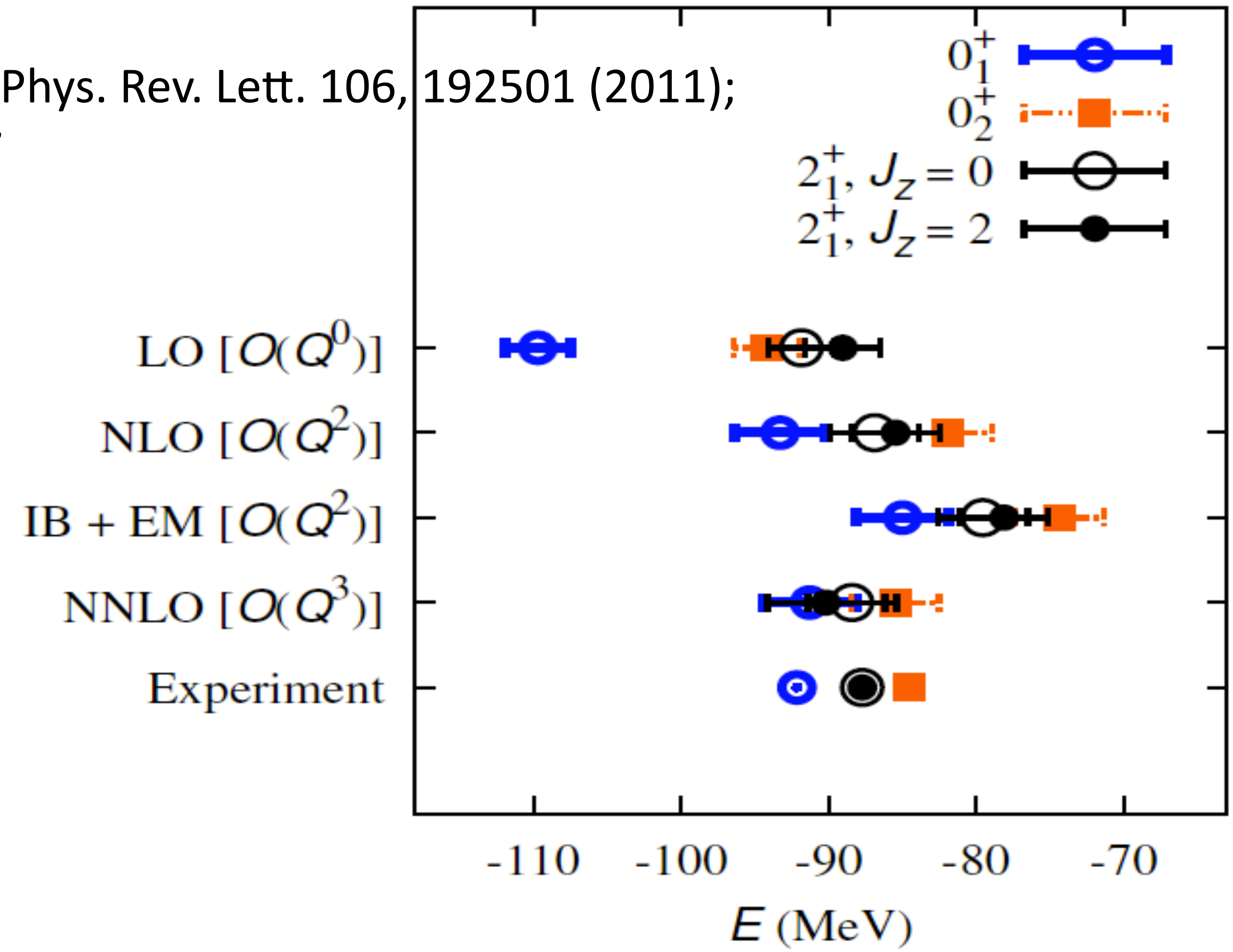
Figure 5. States dominated by p-shell configurations for ^{10}B, ^{11}B, ^{12}C, and ^{13}C calculated at $N_{\max} = 6$ using $\hbar\Omega = 15$ MeV (14 MeV for ^{10}B). Most of the eigenstates are isospin T=0 or 1/2, the isospin label is explicitly shown only for states with T=1 or 3/2. The excitation energy scales are in MeV.

P. Navratil et al., Phys. Rev. Lett. 99, 042501 (2007), nucl-th/0701038.
Review of no-core shell model: Navratil, Quaglioni, Stetcu, Barrett, arXiv:0904.0463.

^{12}C Hoyle state from lattice EFT

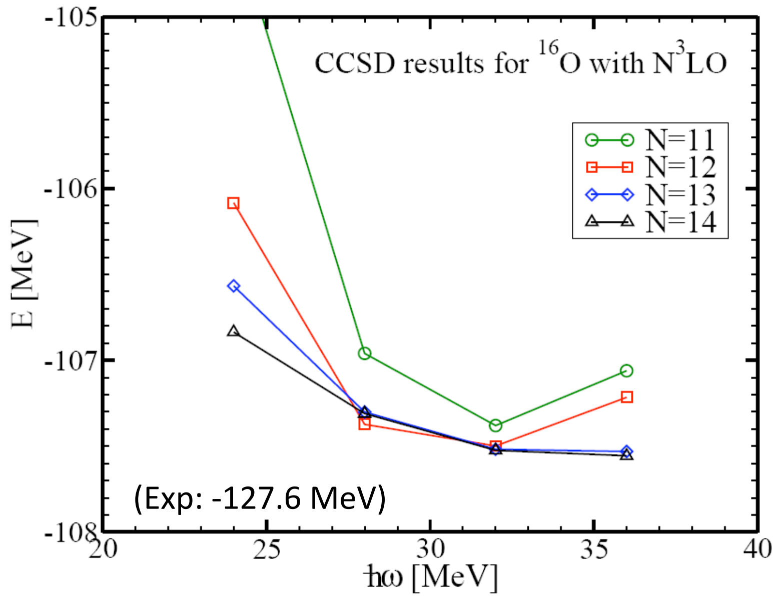


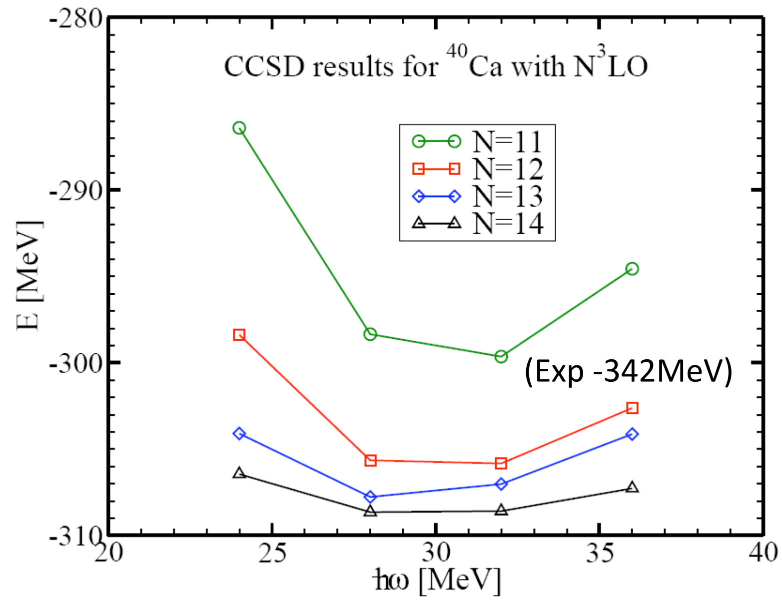
Epelbaum et al, Phys. Rev. Lett. 106, 192501 (2011); arXiv:1101.2547



Saturation properties of chiral NN interactions with coupled clusters

(Λ=500 MeV potential from Entem & Machleidt)





[Hagen, Papenbrock, Dean, Hjorth-Jensen, Phys. Rev. Lett. 101, 092502 (2008)]

Binding energy per nucleon

Nucleus	CCSD	Λ-CCSD(T)	Experiment
^{4}He	5.99	6.39	7.07
^{16}O	6.72	7.56	7.97
^{40}Ca	7.72	8.63	8.56
^{48}Ca	7.40	8.28	8.67

^{16}O by Fujii et al (0908.3376)

B/A=6.62 MeV (2 body clusters)
B/A=7.47 MeV (3 body clusters)

Three-body forces in chiral EFT expected to add 0.4MeV per nucleon?!

Estimate for model spaces and Hamiltonian matrix dimensions

Assume we want to compute the binding energy of a nucleus with mass number A in a wave function based approach. Assume that the interaction has a momentum cutoff Λ.

Q: What are the minimum requirements for the model space?

A:

1. The basis must be sufficiently extended in position space to capture a nucleus with radius $R \approx 1.2\, A^{1/3}$ fm
2. The basis must be sufficiently extended in momentum space to capture the cutoff Λ.
3. THUS: we need approximately $K=(R\Lambda/(2\pi))^3$ single-particle states (phase space volume!) In practice $K \approx (R\Lambda/2)^3 \sim \Lambda^3 A$.

Computation of oxygen: Λ=4/fm and R≈2.5fm
Thus, our model space has about $K=5^3 = 125$ single-particle states.
Matrix dimension: $D=K!/(K-A)!/A! \approx (K/A)^A \approx 8^{16} \approx 2^{48} \approx 10^{14}$.

Thus, the matrix dimension is $D=K!/((K-A)!\ A!)$, with $K \approx (R\Lambda/2)^3$

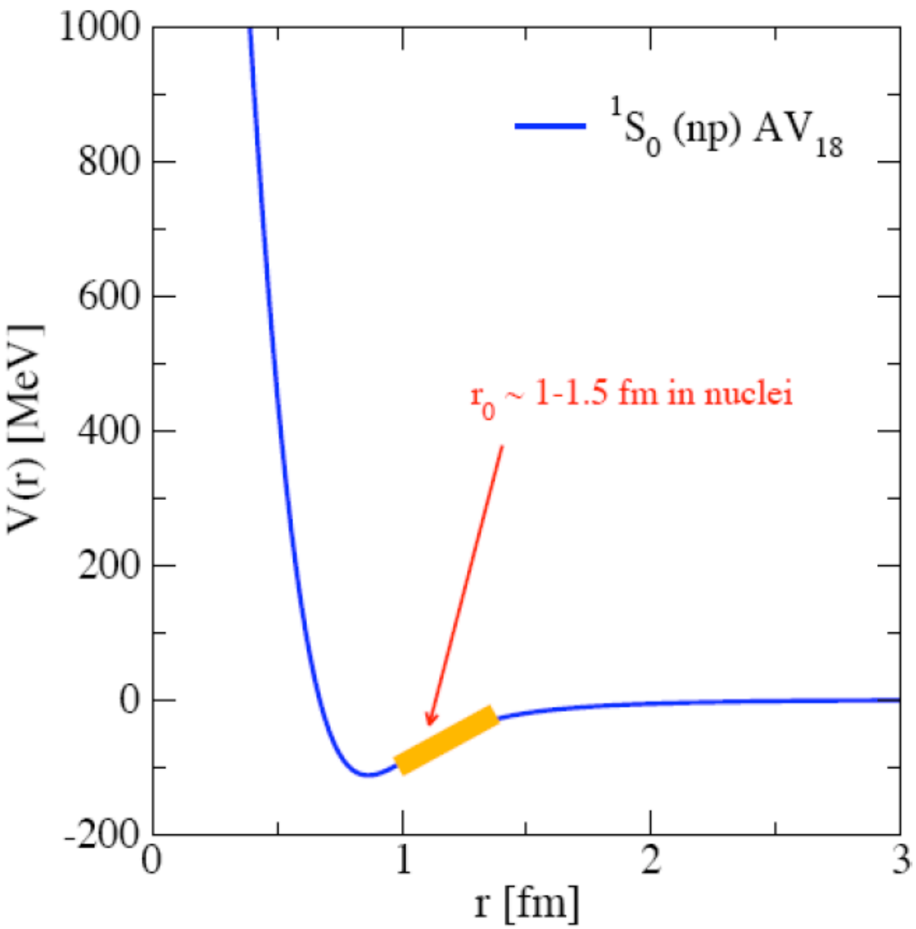
Some conclusions

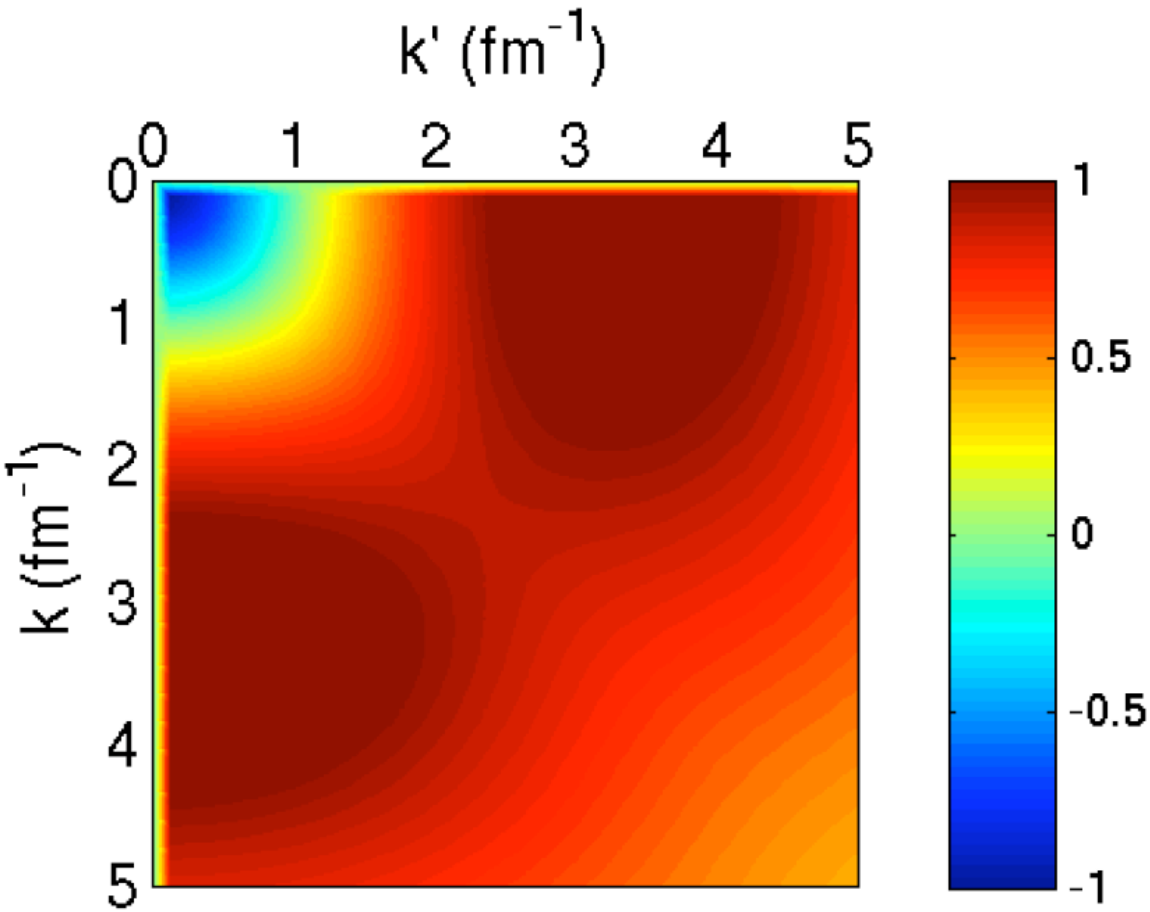
1. For "bare" chiral interactions, matrix diagonalization is possible only for light nuclei. One either needs a much more efficient method or a lower cutoff.
2. The factorial scaling with *A* is not matched by Moore's law (doubling of FLOPS about every 18 month → factor 1000 in 15 years).
3. For wave-function based methods, the most effective way to heavier nuclei is to decrease the phase-space volume $K \sim (\Lambda R)^3 \sim \Lambda^3 A$ by decreasing the cutoff.

→ Low-momentum interactions & similarity renormalization group transformations that lower the cutoff Λ.

Homework: Consider an oscillator basis. How has one choose the oscillator frequency ω and the number of oscillator shells N for a given momentum cutoff Λ and mass number A?

Momentum-dependence of phenomenological potentials





Configuration space ←→ momentum space

Hard core ←→ high-momentum modes

Bogner/Furnstahl (2007)

Similarity renormalization group (SRG) transformation

S. Glazek, K. Wilson, PRD **48** (1993) 5863; **49** (1994) 4214;
F. Wegner, Ann. Phys. **3** (1994) 77

Main idea: decouple low from high momenta via a (unitary) similarity transformation

Unitary transformation

$$\widehat{H}(s) = U(s)\widehat{H}U^\dagger(s) = U(s)\left(\widehat{T} + \widehat{V}\right)U^\dagger(s)$$

Evolution equation

$$\frac{d\widehat{H}(s)}{ds} = \left[\eta(s), \widehat{H}(s)\right] \qquad \text{with} \qquad \eta(s) \equiv \frac{dU(s)}{ds}U^\dagger(s) = -\eta^\dagger(s)$$

Choice of unitary transformation through (one does not need to construct U explicitly).

$$\eta(s) = \left[\widehat{T}, \widehat{H}(s)\right]$$

yields scale-dependent potential that becomes more and more diagonal

$$\widehat{H}(s) = \widehat{T} + \widehat{V}(s)$$

Note: Baker-Campbell-Hausdorff expansion implies that SRG of 2-body force generates many-body forces

$$e^{-\eta}\widehat{H}e^{\eta} = \widehat{H} + \left[\widehat{H}, \eta\right] + \frac{1}{2!}\left[\left[\widehat{H}, \eta\right], \eta\right] + \dots$$

SRG evolution of a chiral potential

(use cutoff $\lambda \equiv s^{-1/4}$ as evolution variable)

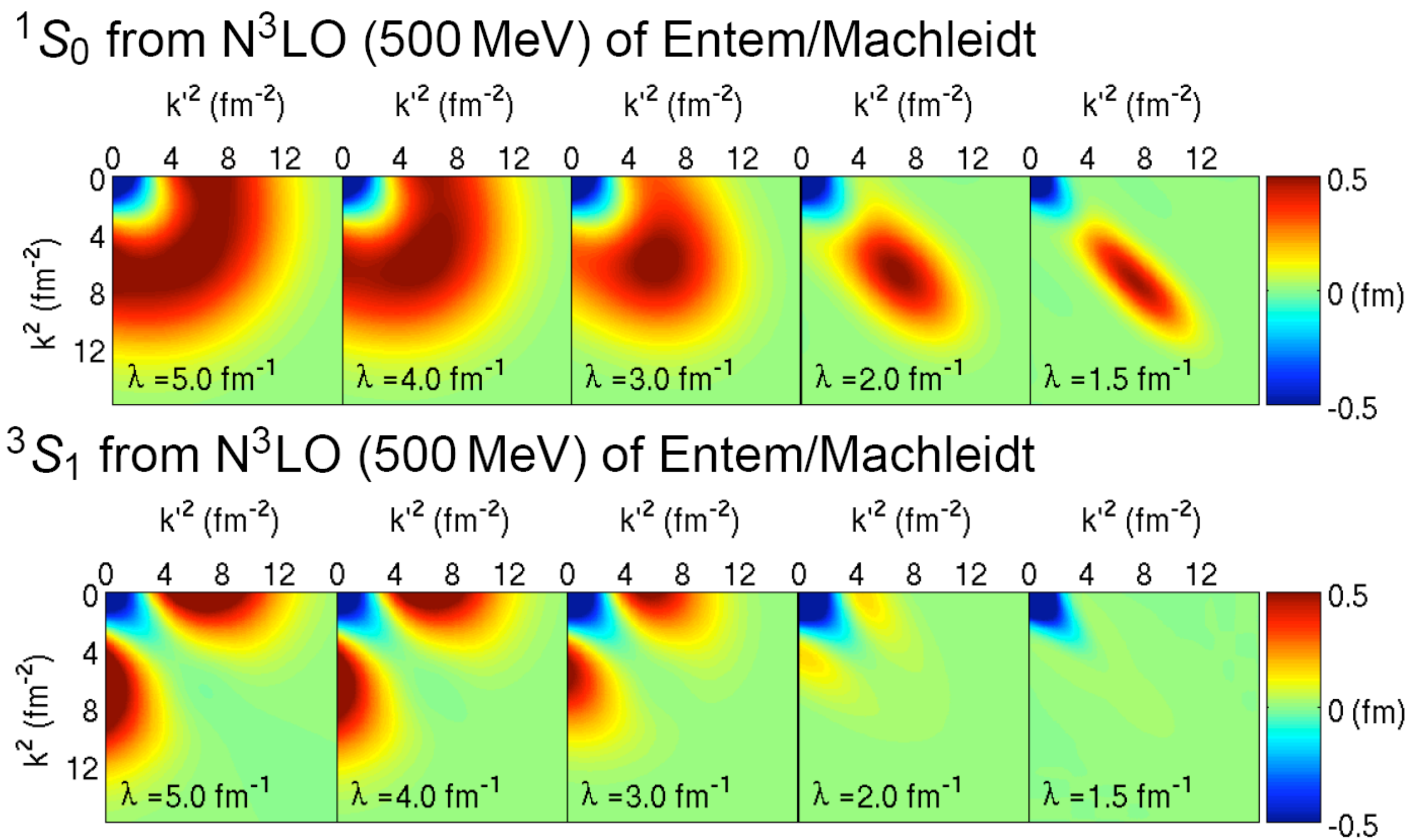
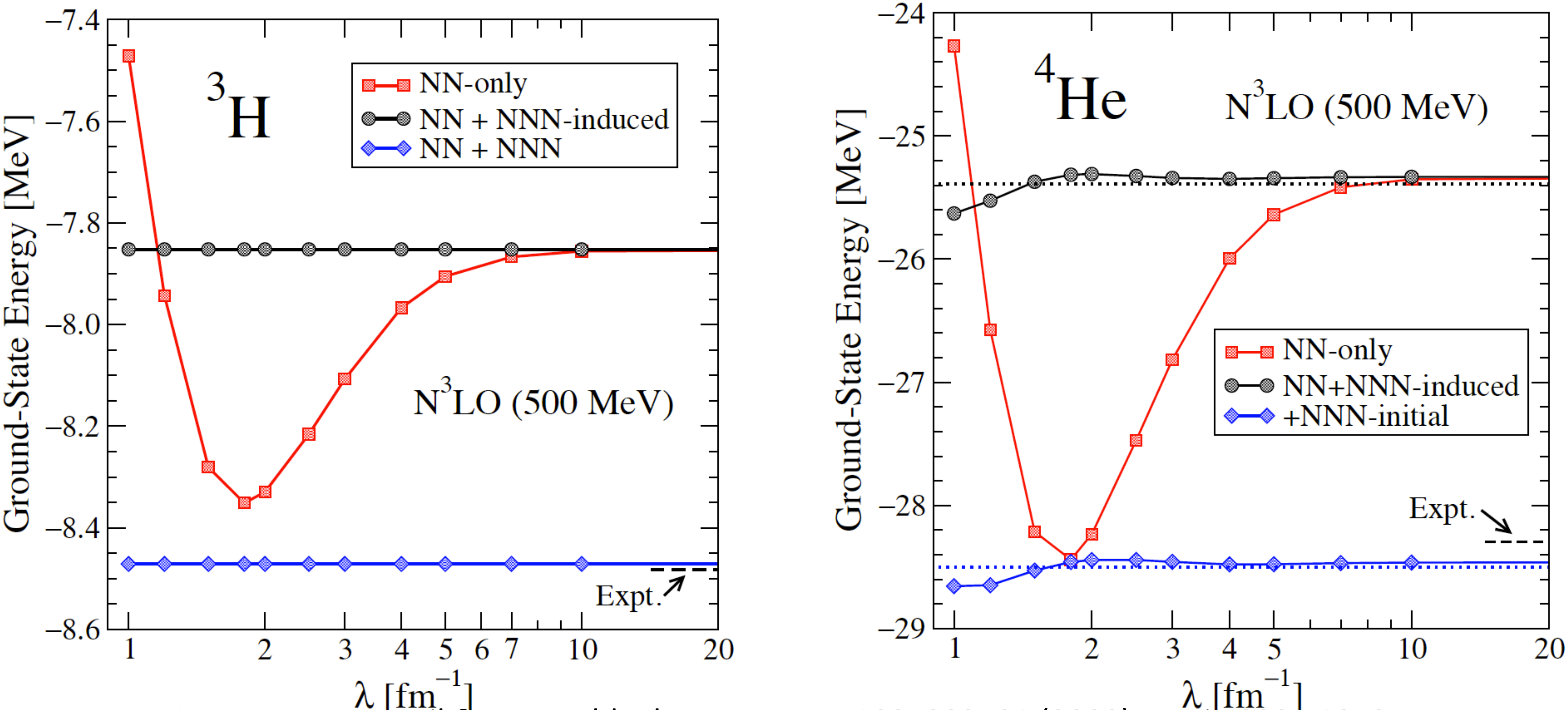


Fig.: Bogner & Furnstahl. See http://www.physics.ohio-state.edu/~ntg/srg

Solution of ^3H and ^4He with induced and initial 3NF



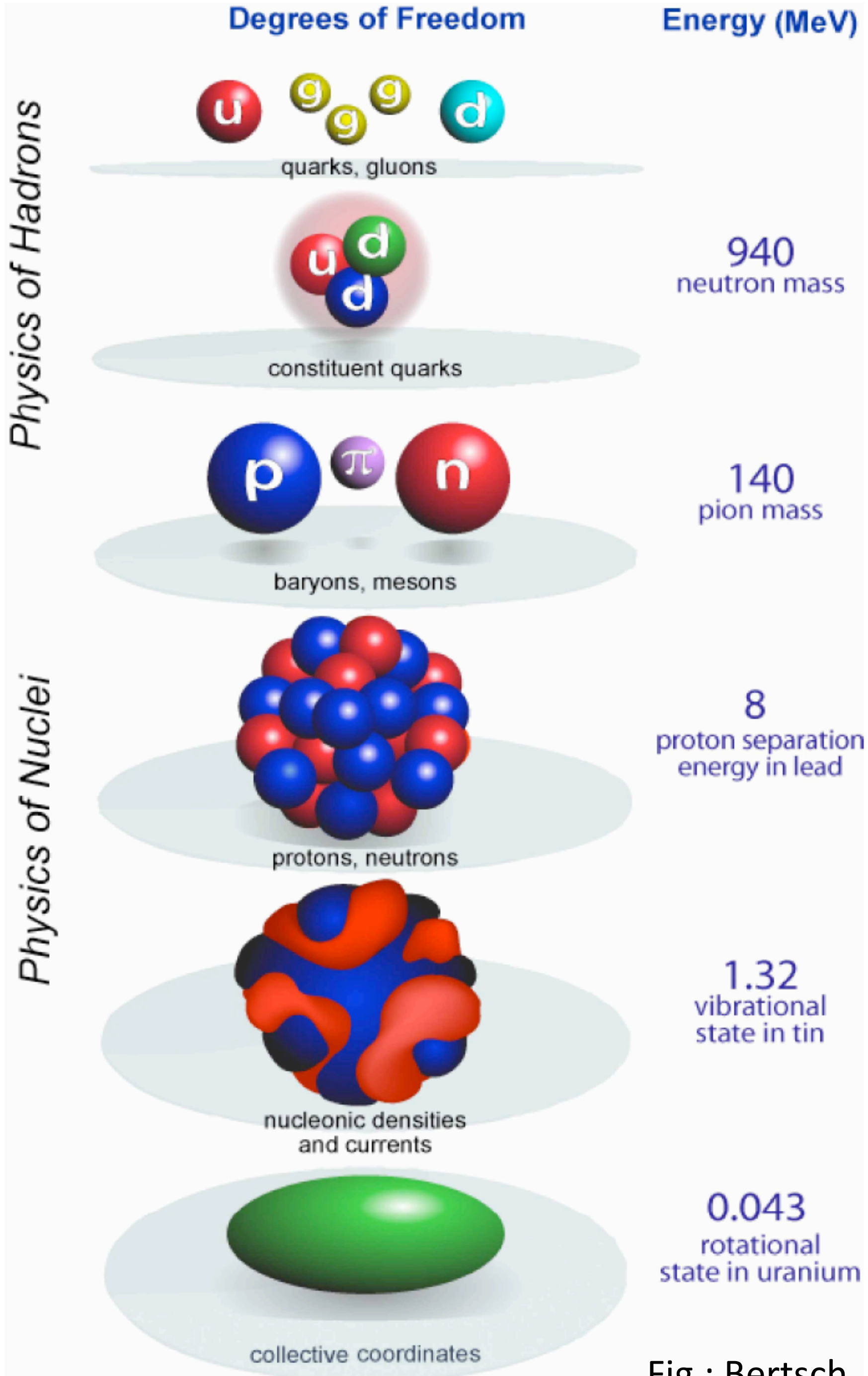
Jurgenson, Navratil & Furnstahl, Phys. Rev. Lett. 103, 082501 (2009), arXiv:0905.1873

Q: What is the effect of (omitted) 4NF and forces of even higher rank?

A: In ^4He, (short ranged) 4NF yield about 200 keV (see energies at small momentum)
Note: This is consistent with deviation from experiment!

Energy scales and relevant degrees of freedom

Energy



Chiral effective field theory

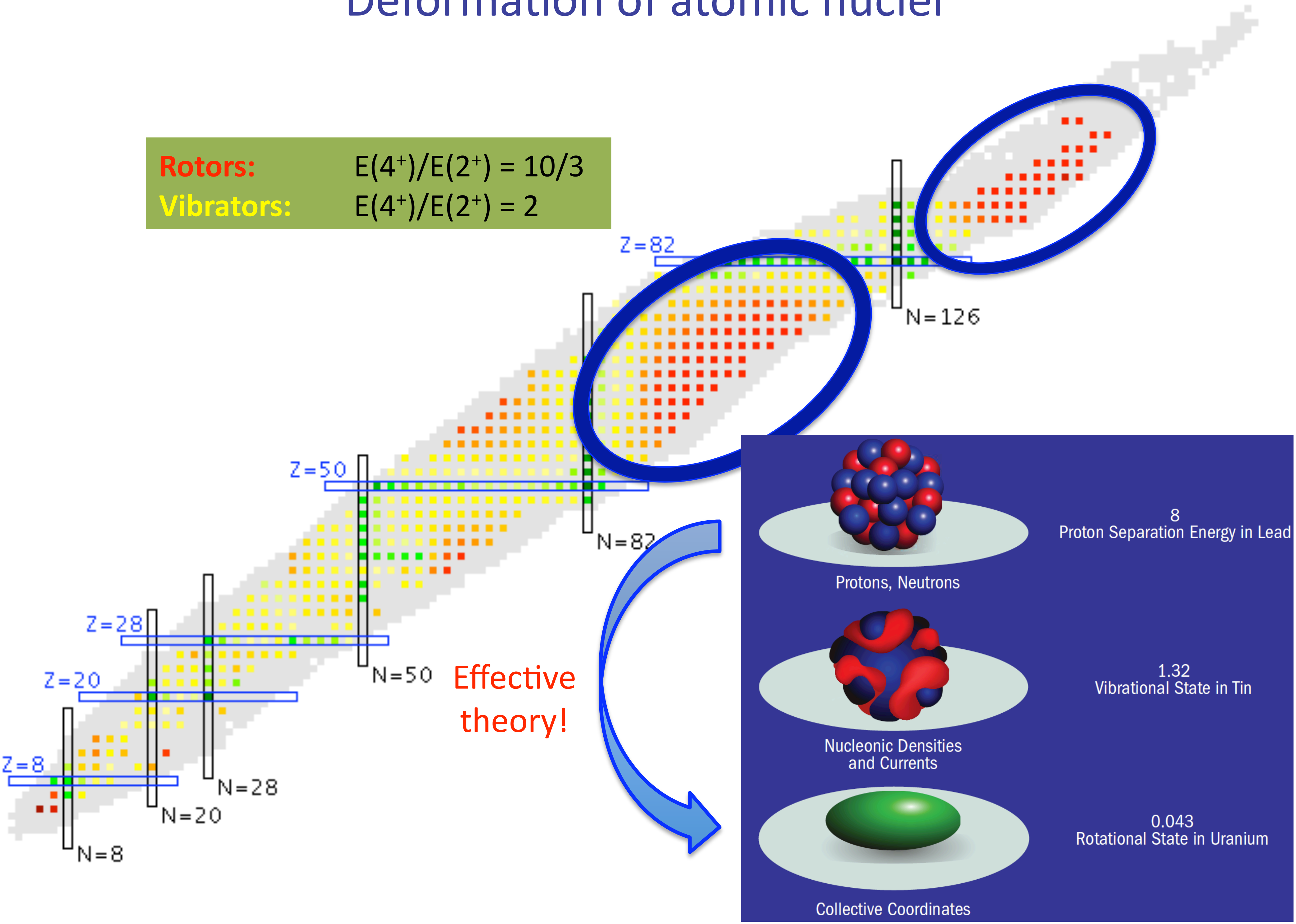
Effective theory of deformed nuclei

Fig.: Bertsch, Dean, Nazarewicz, SciDAC review (2007)

Deformation of atomic nuclei

Rotors: $E(4^+)/E(2^+) = 10/3$

Vibrators: $E(4^+)/E(2^+) = 2$



Construction of an EFT

1. Identify the **relevant degrees of freedom** for the resolution scale of interest

2. Identify the **relevant symmetries** of low-energy nuclear physics and investigate if and how they are broken

3. Construct the **most general Lagrangian** consistent with those symmetries and the symmetry breaking.

4. Design an **organizational scheme** (power counting) that can distinguish between more and less important contributions

Useful references:

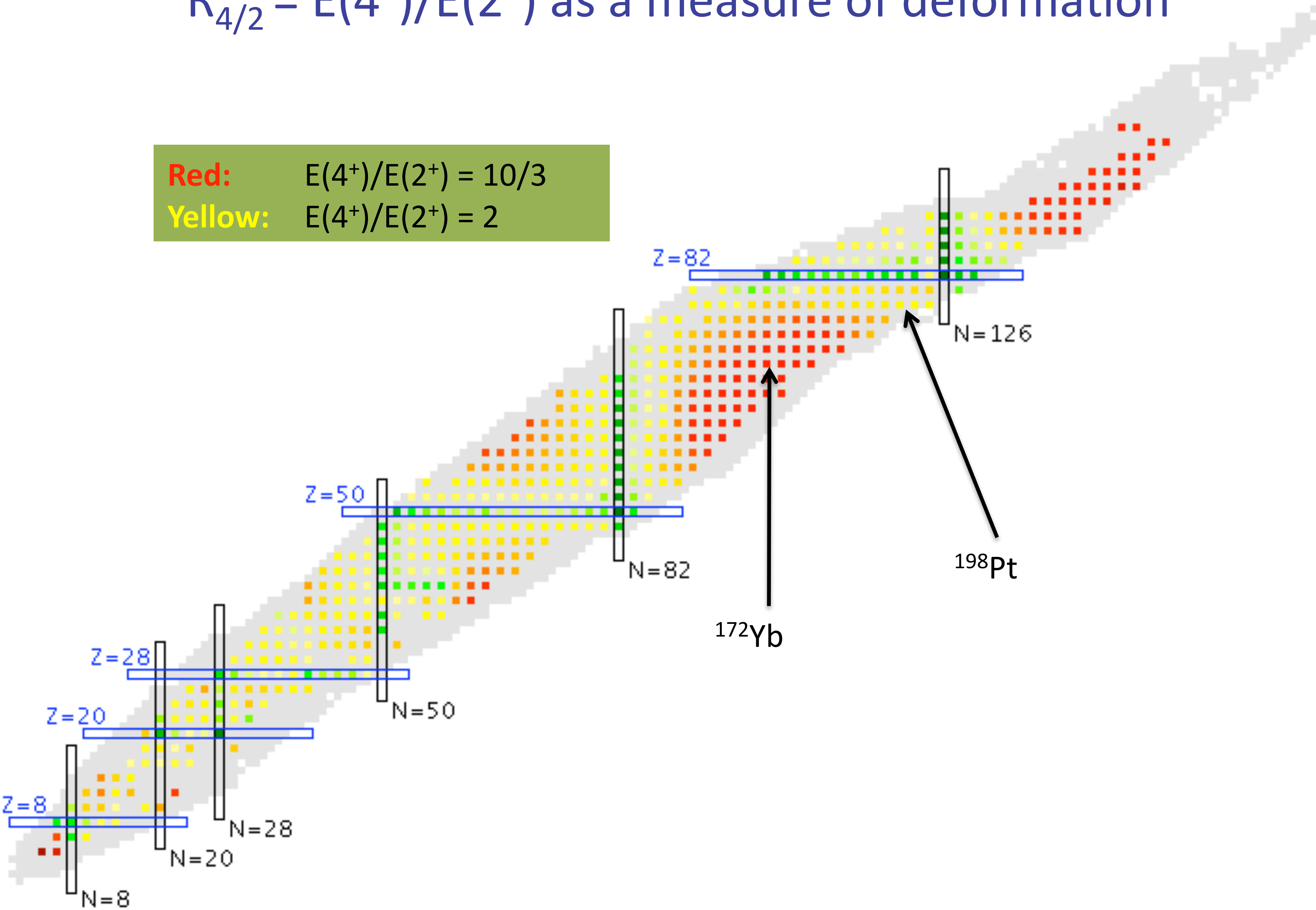
S. Weinberg, *The Quantum Theory of Fields,* Vol.II, chap. 19

H. Leutwyler, Phys. Rev. D **49** (1994) 3033, arXiv:hep-ph/9311264

C. P. Burgess, Physics Reports **330** (2000) 193

$R_{4/2} = E(4^+)/E(2^+)$ as a measure of deformation

Red: $E(4^+)/E(2^+) = 10/3$
Yellow: $E(4^+)/E(2^+) = 2$



1. Identify relevant degrees of freedom

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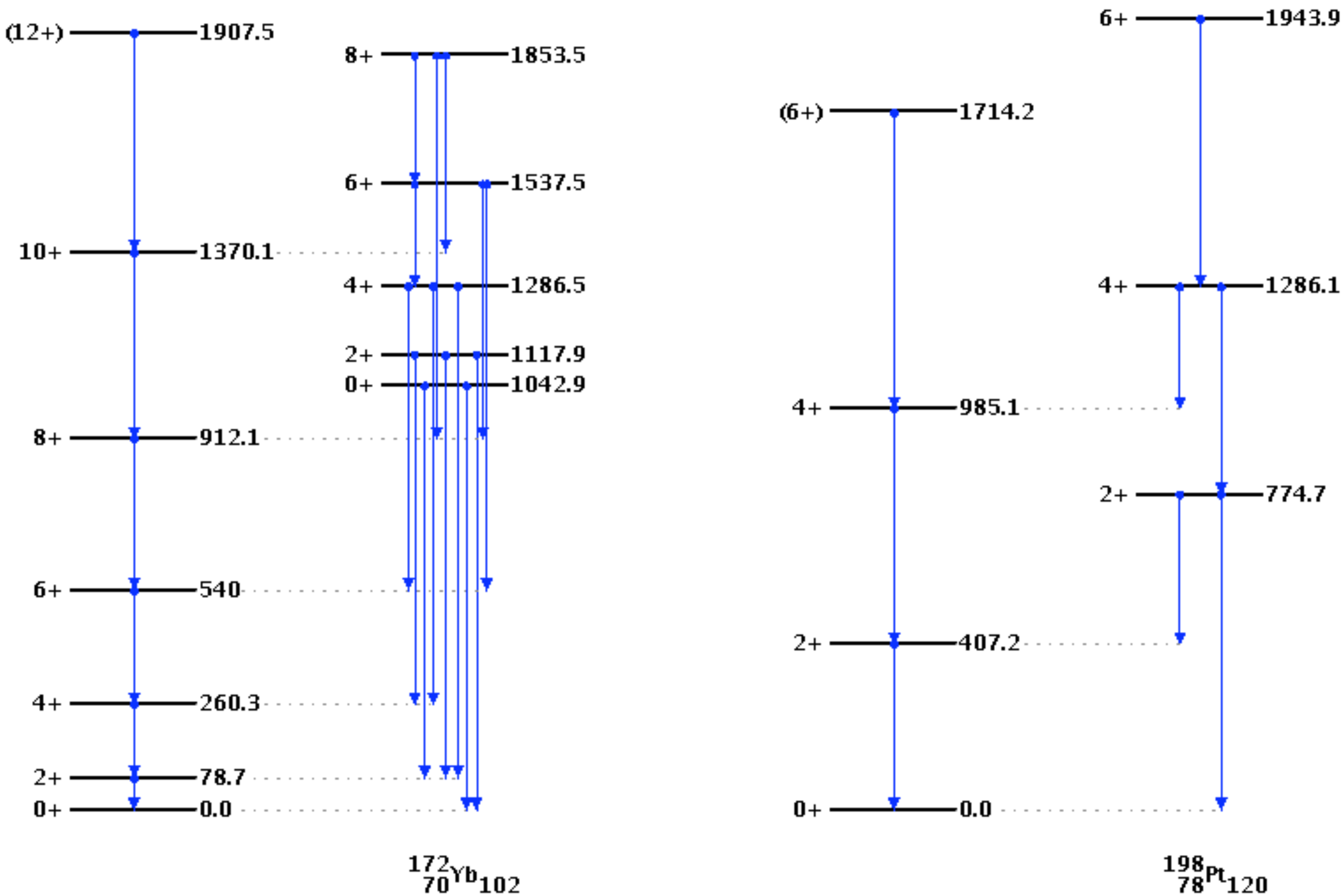
Q: What field would be able to reproduce spins and parities of low-lying states?



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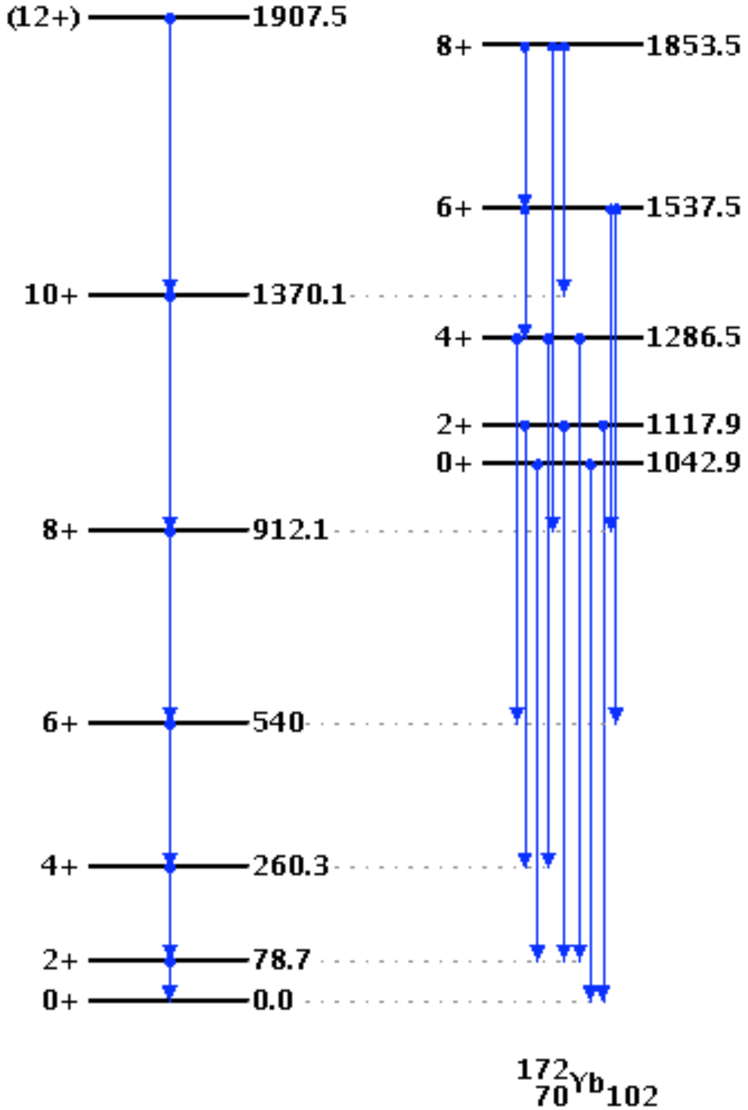

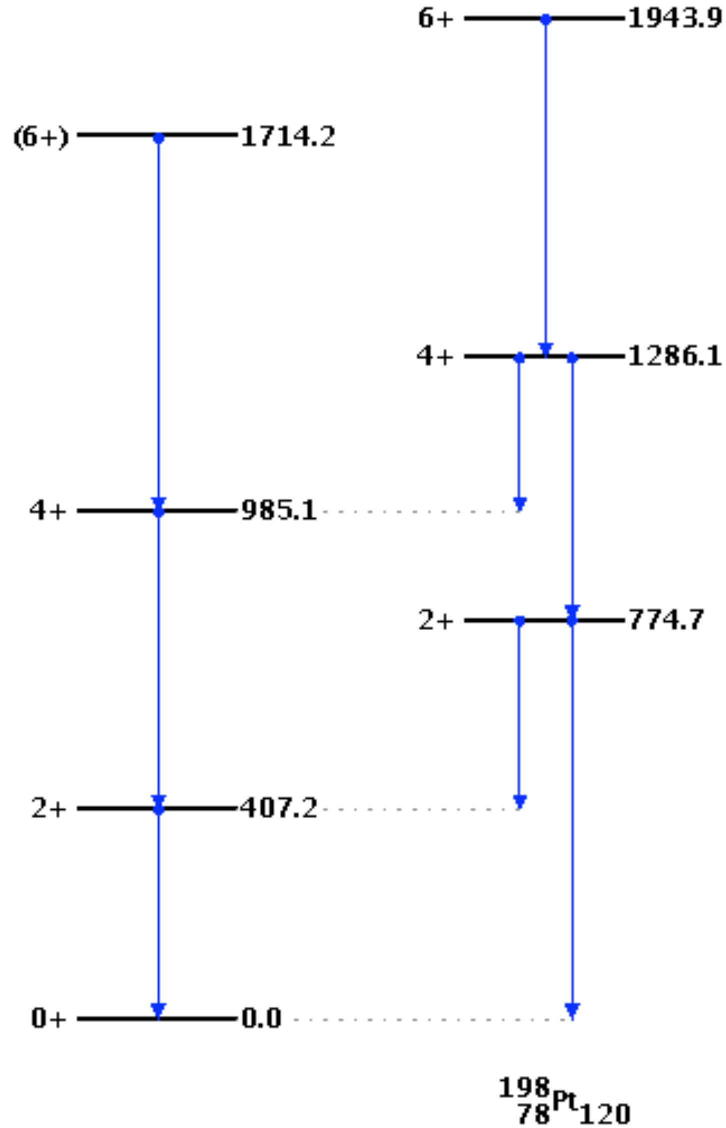

Quadrupole degrees of freedom describe spins and parity of low-energy spectra

2. Identify relevant symmetries and symmetry breaking

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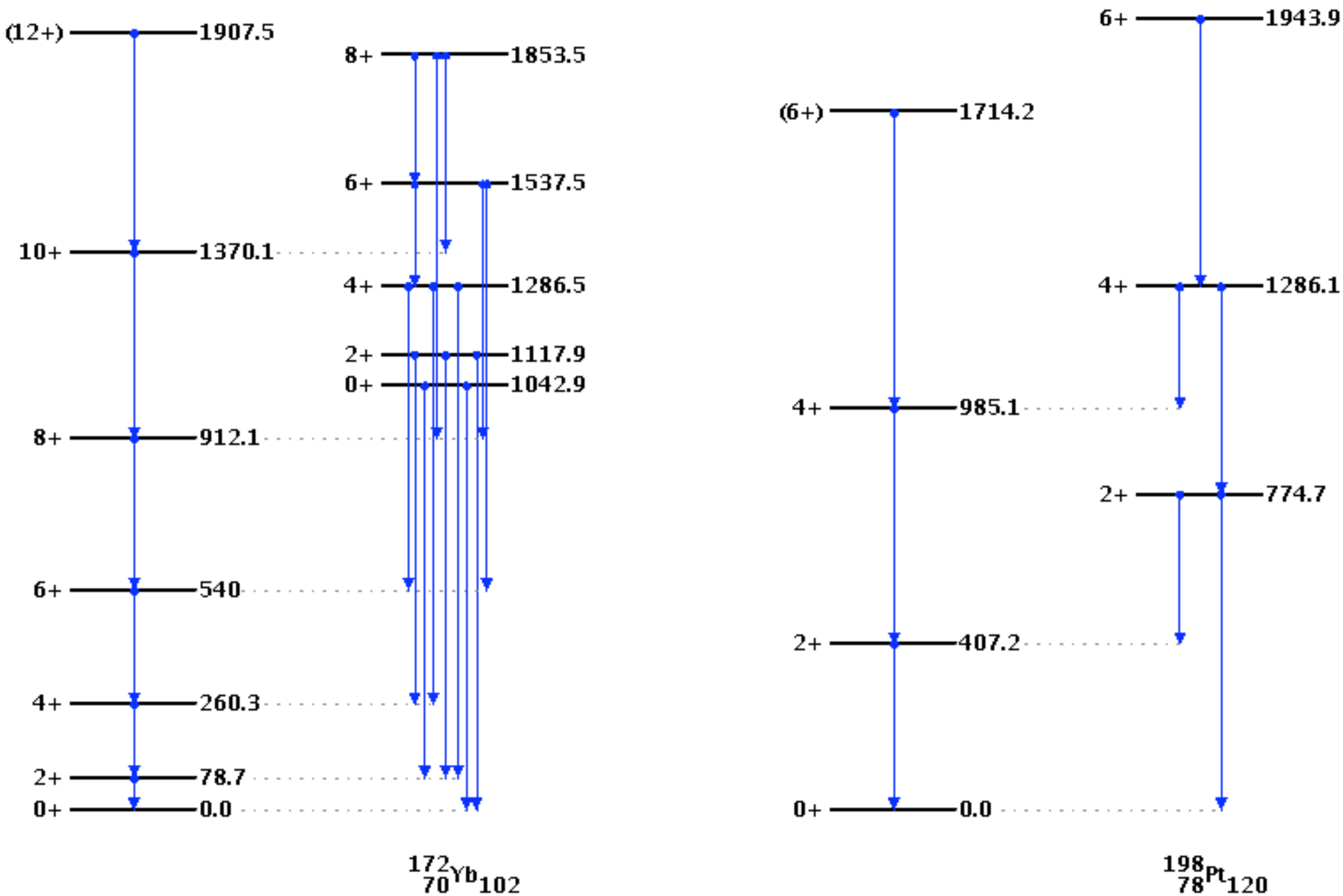
Q: What are the symmetries, and are they spontaneously broken?



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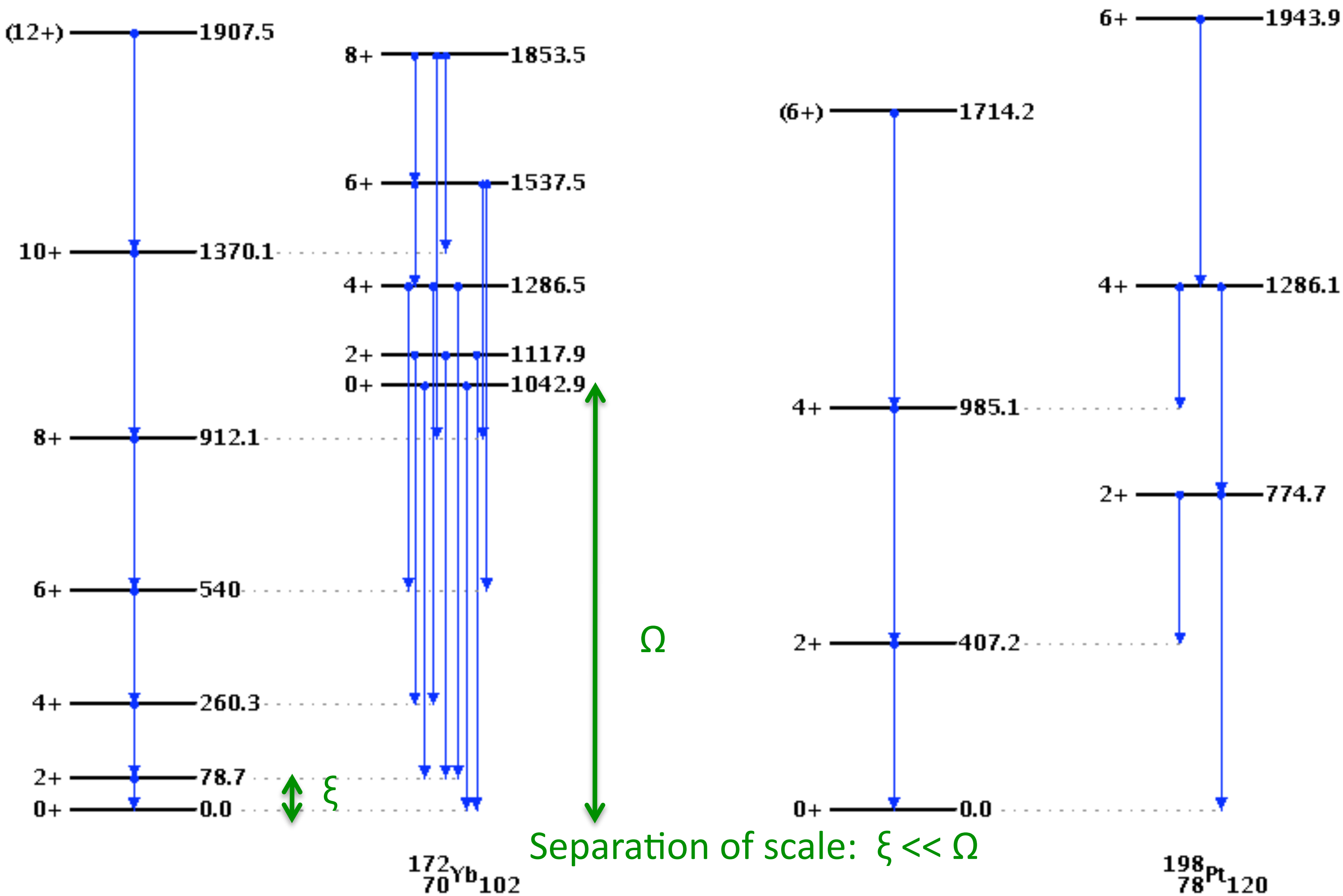

Separation of scale: $\xi \ll \Omega$

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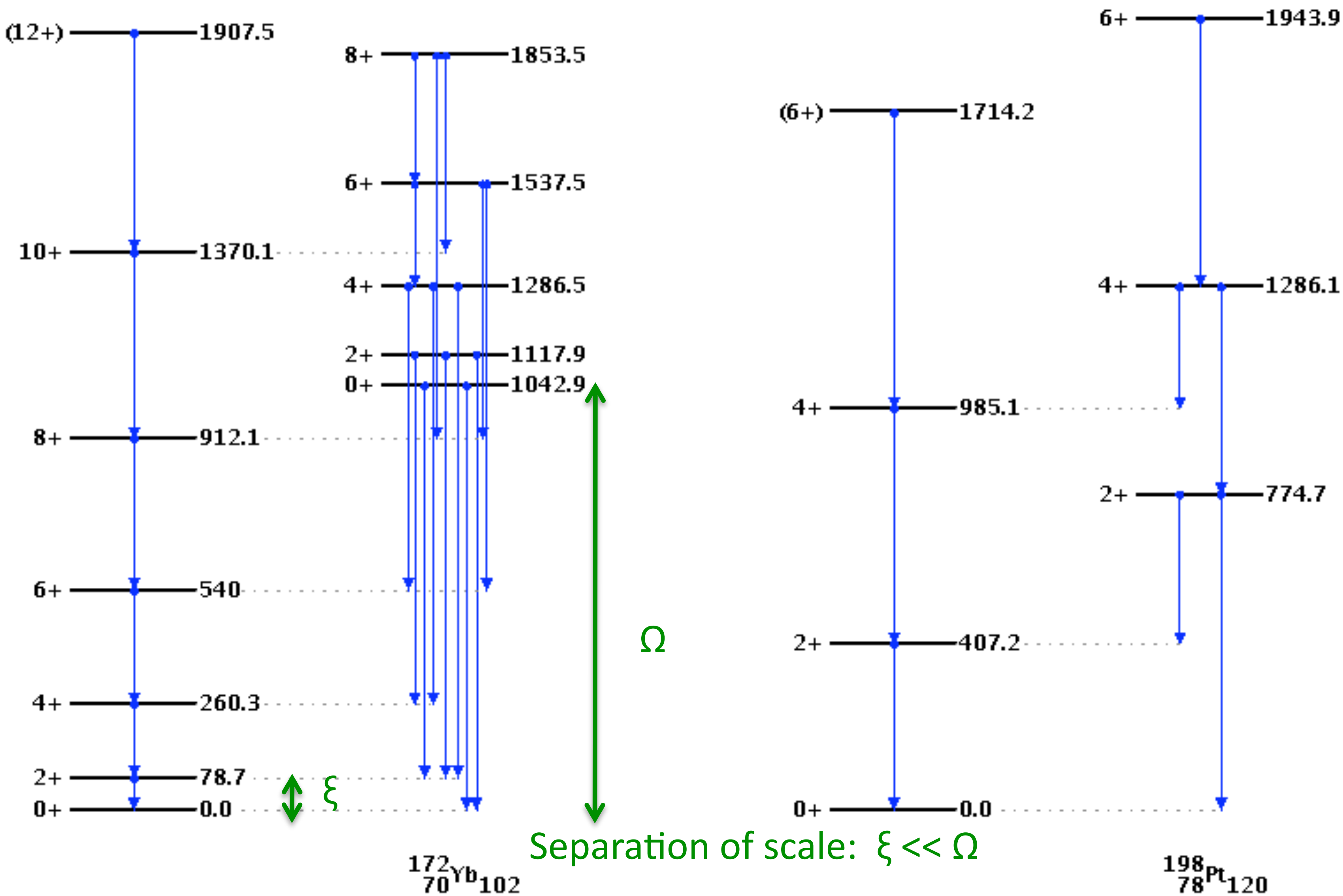
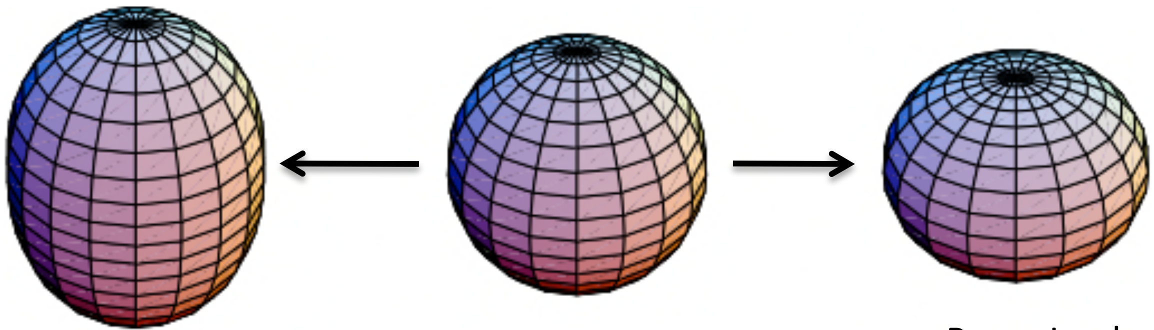


Symmetry: Rotational invariance

Very low-energy excitations (~Nambu-Goldstone modes) from spontaneous symmetry breaking

Spontaneous breaking of rotational symmetry



P. van Isacker 2008

Rotational symmetry → Axial symmetry

SO(3) → SO(2)

3 generators 1 generator

There will be 3-1=2 Nambu-Goldstone bosons

"In a general theory of rotation, symmetry plays a central role. Indeed, the very occurrence of collective rotational degrees of freedom may be said to originate in a breaking of rotational invariance, which introduces a "deformation" that makes it possible to specify an orientation of the system. Rotation represents the collective mode associated with such a spontaneous symmetry breaking (Goldstone boson)." Aage Bohr, Nobel Lecture (1975)

3. Construct the most general Hamiltonian consistent with the symmetry and the symmetry breaking

→ Nonlinear realization of (rotational) symmetry

[Weinberg 1967; Coleman, Callan, Wess & Zumino 1969]

Following this procedure is a bit technical (we might do this later). Let's first follow a simpler and geometric approach, see e.g., [H. Leutwyler, Phys. Rev. D 49 (1994) 3033, arXiv:hep-ph/9311264]

Assume ground state is invariant only under rotations around the z-axis. Rotations of that state cannot be distinguished if they differ only by a rotation around the z-axis. **Nambu-Goldstone modes parameterize the coset space SO(3)/SO(2) ~ S^2, i.e. the two-sphere**:

$$\vec{n}(\alpha,\beta) = \begin{pmatrix} \cos\alpha \sin\beta \\ \sin\alpha \sin\beta \\ \cos\beta \end{pmatrix}$$

Nonlinear realization: The unit vector n(α,β) transforms linearly under rotations, but the angles (our Nambu-Goldstone degrees of freedom) transform nonlinearly.

Physics of Nambu-Goldstone modes

Lagrangian

$$L = \frac{C_0}{2}(\partial_t \vec{n}) \cdot (\partial_t \vec{n}) = \frac{C_0}{2}(\dot{\beta}^2 + \dot{\alpha}^2 \sin^2 \beta)$$

Hamiltonian

$$H = \frac{p_\beta^2}{2C_0} + \frac{p_\alpha^2}{2C_0 \sin^2 \beta}$$

Quantization

$$\begin{aligned} p_\beta^2 &= -\frac{1}{\sin\beta}\partial_\theta \sin\beta \partial_\beta \ , \\ p_\alpha &= -i\partial_\alpha \ . \end{aligned}$$

Spectrum

$$\hat{H} Y_{lm}(\beta, \alpha) = \frac{l(l+1)}{2C_0} Y_{lm}(\beta, \alpha)$$

Rotational bands are quantized Nambu-Goldstone modes.
Low-energy constant C_0 is moment of inertia and fit to data.

4. Power counting and next-to-leading order

Let us first understand dimensional analysis

1. Low-energy scale is ξ
2. Leading-order Lagrangian $L = \frac{C_0}{2}(\partial_t \vec{n}) \cdot (\partial_t \vec{n})$ must scale as: $L \sim \xi$
3. Energy-time uncertainty implies ($\hbar=1$): $d_t \sim \xi$
4. Thus $C_0 \sim 1/\xi$

Next-to-leading order term (scalar in NG modes that we can write down)

$$L = (C_2/4)\,\left((\partial_t \vec{n}) \cdot (\partial_t \vec{n})\right)^2 \sim \xi\,(\xi/\Omega)^2 \ll \xi$$

Q1: What are it's dimensions of C_2 in powers of energy?

A1: It must have dimensions of energy^{-3} (We have two energy scales $\xi \ll \Omega$)

Q2: How should the term C_2 scale precisely? A2: ξ^{-3}, $\xi^{-2}\Omega^{-1}$, $\xi^{-1}\Omega^{-2}$, Ω^{-3} … ?

A2: $C_2/C_0 \sim$ energy^{-2} <u>and is due to omitted physics</u> at a high-energy scale Ω
Thus: $C_2/C_0 \sim \Omega^{-2}$ ("naturalness" argument)

4. Power counting and next-to-leading order

Lagrangian at next-to-leading

$$L = (C_0/2)\ (\partial_t \vec{n}) \cdot (\partial_t \vec{n})$$
$$\quad + (C_2/4)\,(\,(\partial_t \vec{n}) \cdot (\partial_t \vec{n})\,)^2$$

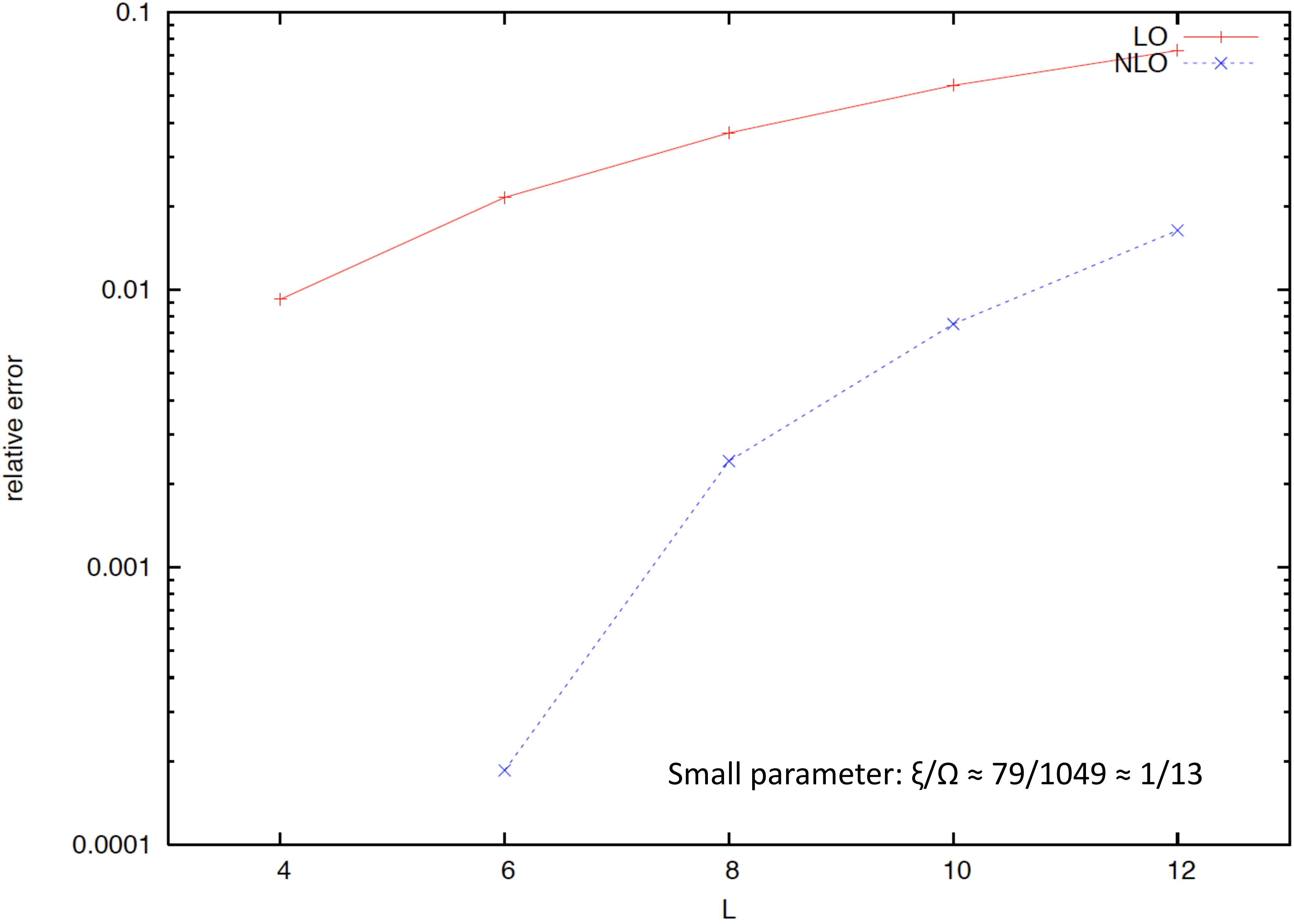
Spectrum: $J(J+1)/(2C_0) - (J(J+1))^2\,(C_2/4C_0^{\,4})$

→Bohr & Mottelson (of course!)

Q: When does the effective theory breaks down? How large can J be?

A1: energy correction << leading-order energy: $C_2/C_0^{\,3}\,J(J+1) << 1$
A2: Thus: $J << \Omega/\xi$

^{172}Yb: Relative error in LO and NLO



Nuclei with finite ground-state spins: Wess Zumino terms

A finite ground-state spin breaks time reversal invariance.
→ Consider terms that are first order in the time derivative
→ No such terms are invariant under rotations.
BUT: under rotations, E_z changes by a total derivative. Action remains essentially invariant (Wess-Zumino term)

Lagrangian
$$\begin{aligned} L_{\rm LO} &= L_{\rm LO}^{(ee)} + L_{\rm WZ} \\ &= \frac{C_0}{2}\left(\dot\beta^2 + \dot\alpha^2 \sin^2\beta\right) - q\dot\alpha\cos\beta \end{aligned}$$

Hamiltonian
$$H_{\rm LO} = \frac{p_\beta^2}{2C_0} + \frac{\left(p_\alpha + q\cos\beta\right)^2}{2C_0\sin^2\beta}$$

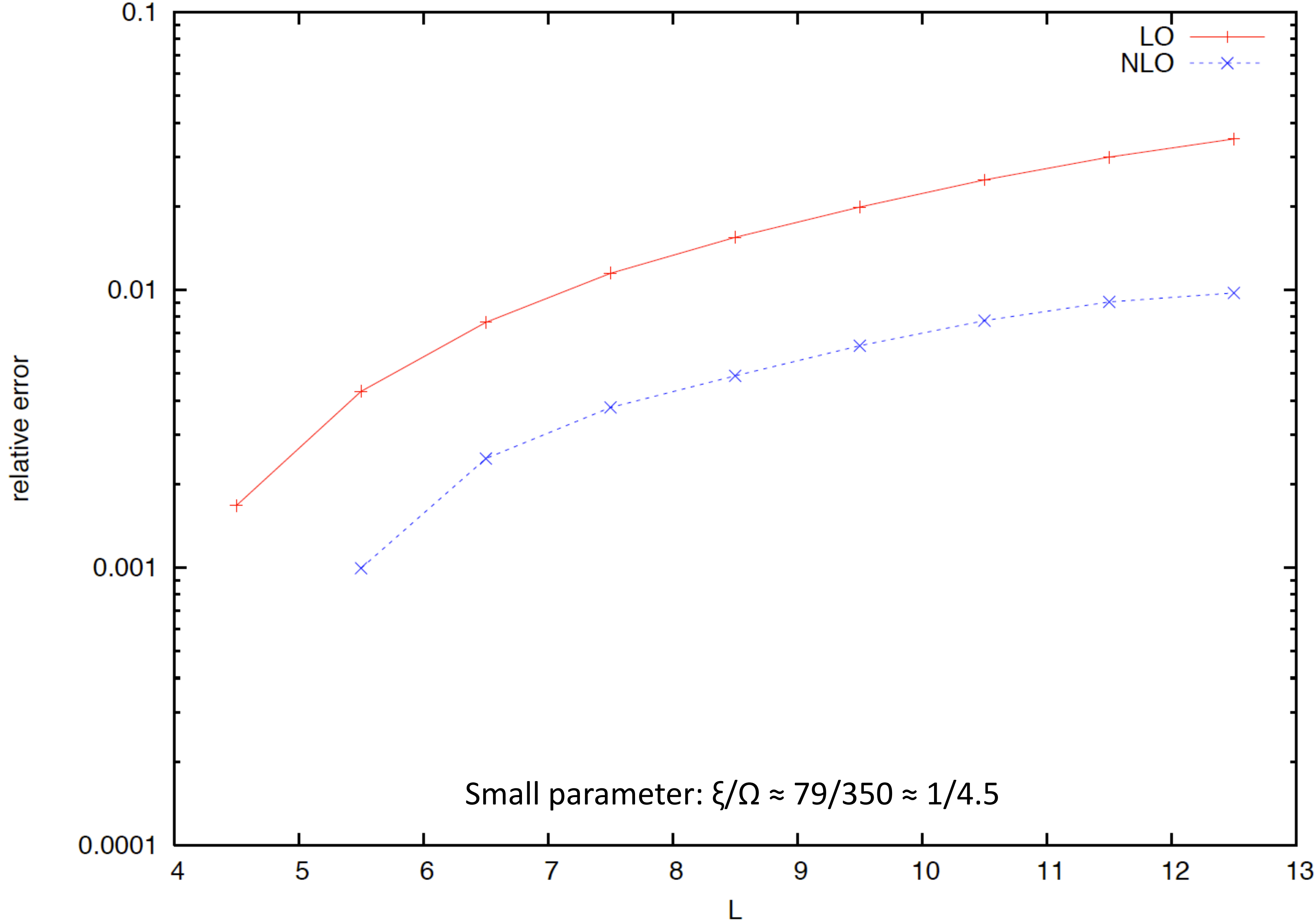
Eigenvalues and eigenfunctions (Identify q with ground-state spin!)

$$\hat H_{\rm LO}\, d^l_{mq}(\beta)e^{-i\alpha m} = E_{\rm LO}(q,l)d^l_{mq}(\beta)e^{-i\alpha m}$$

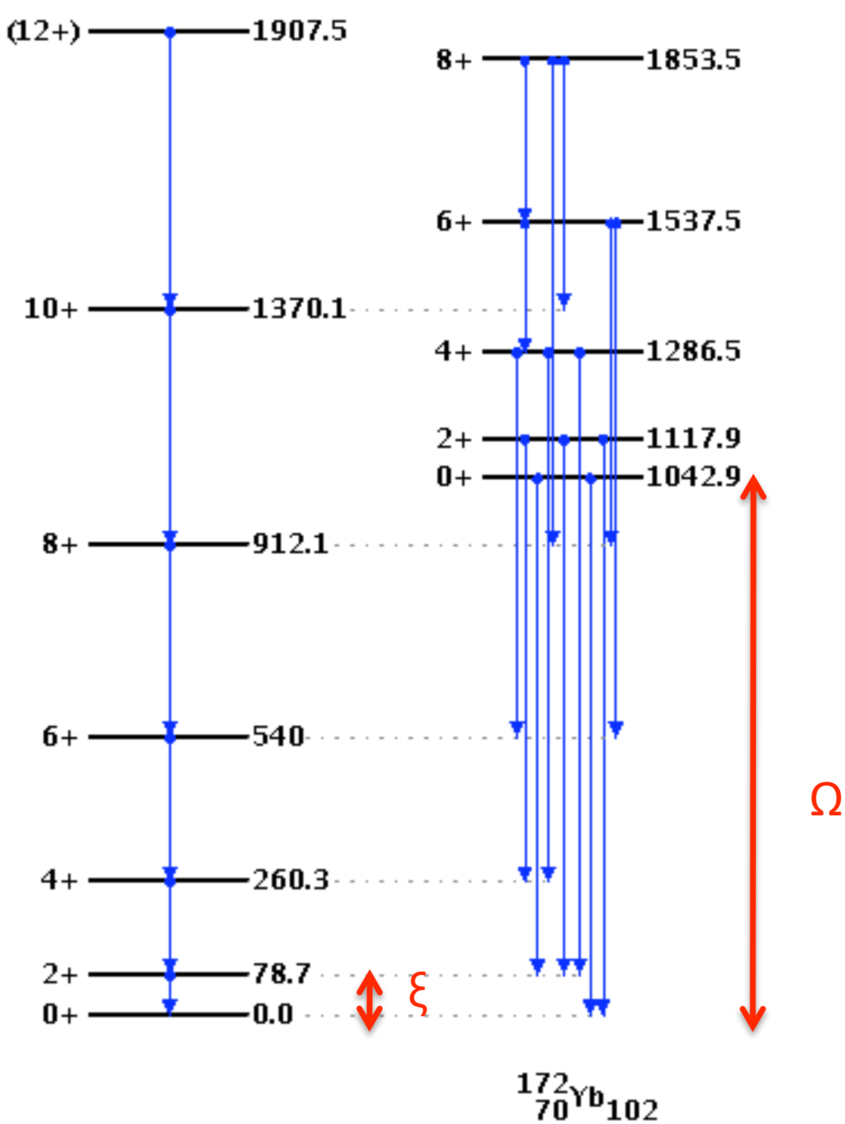
$$E_{\rm LO}(q,l) = \frac{l(l+1)-q^2}{2C_0} \qquad l = |q|, |q|+1, |q|+2, \ldots$$

$$D^l_{mq}(\alpha,\beta,\gamma) \equiv e^{-im\alpha} d^l_{mq}(\beta)e^{-iq\gamma}$$ (Wigner D functions)

^{173}Yb: Relative error in LO and NLO



Beyond NG modes: coupling to vibrations



Higher energetic degrees of freedom need to be included.

Quadrupole field exhibits spontaneous symmetry breaking.

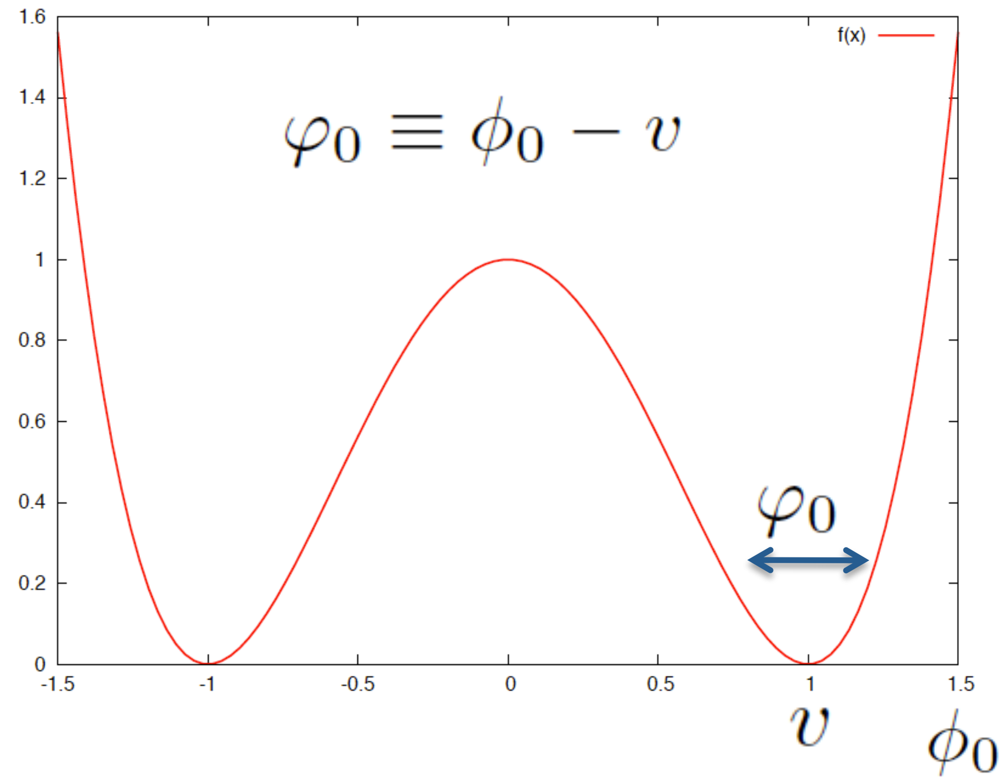
→5 DoF – 2NG = 3DoF

$$\phi = \begin{pmatrix} \phi_2 \\ 0 \\ \phi_0 \\ 0 \\ \phi_2^* \end{pmatrix}$$

Separation of scale: ξ << Ω

Couplings to vibrations: power counting

Low energy scale ξ
High energy scale Ω>>ξ



Dimensional analysis

$$
\begin{aligned}
v \sim \phi_0 \;&\sim\; \xi^{-1/2} \,, \\
\varphi_0 \sim \phi_2 \;&\sim\; \Omega^{-1/2} \,, \\
\dot{\varphi}_0 = \dot{\phi}_0 \sim \dot{\phi}_2 \;&\sim\; \Omega^{1/2} \,.
\end{aligned}
$$

Potential expanded around miniumum

$$
V_2(\phi) = \frac{\omega_0^2}{2}(\phi_0 - v)^2 + \frac{\omega_2^2}{4}|\phi_2|^2
$$

$$
V = V_2 + \sum_{k+2l>2} v_{kl}\varphi_0^k |\phi_2|^{2l}
$$

Power counting: large amplitudes $\phi_0 \approx v$ restore rotational symmetry → breakdown of EFT

$$
v_{kl}\varphi_0^k |\phi_2|^{2l} \sim \Omega \left(\frac{\xi}{\Omega}\right)^{l-1+k/2}
$$

Leading order ~O(Ω)

Lagrangian at leading order:

$$L_{\rm LO} = \frac{1}{2}\dot{\varphi_0}^2 + |\dot{\phi}_2|^2 - \frac{\omega_0^2}{2}\varphi_0^2 - \frac{\omega_2^2}{4}|\phi_2|^2$$

$$H_{\rm LO} = \frac{1}{2}p_0^2 + \frac{1}{4}\left(p_{2r}^2 + p_{2i}^2\right) + \frac{\omega_0^2}{2}\varphi_0^2 + \frac{\omega_2^2}{4}\left(\phi_{2r}^2 + \phi_{2i}^2\right)$$

Spectrum

$$E(n_0, n_2, m_l) = \omega_0\left(n_0 + \frac{1}{2}\right) + \frac{\omega_2}{2}(2n_2 + |m_l| + 1)$$

Leading order yields the band heads

Lagrangian as function of E_x, E_y, $D_t\varphi_0$, $D_t\phi_2$, ϕ_0, ϕ_2, needs to be formally invariant under SO(2) (axial symmetry only).

Next-to-leading order ~O(ξ)

Lagrangian
$$L_{\rm NLO} = L_{\rm LO} + \frac{3}{2}v^2\left(E_x^2+E_y^2\right) - 4E_z{\rm Im}\left(\dot\phi_2\phi_2^*\right) - \sum_{k+2l=3,4} v_{kl}\varphi_0^k|\phi_2|^{2l}\ .$$

Hamiltonian (kinetic energy)

$$H_{\rm NLO} = H_{\rm LO} + \frac{1}{6v^2}\left(p_\beta^2 + \frac{1}{\sin^2\beta}\left[p_\alpha^2 + 2p_\alpha l_z\cos\beta\right]\right)$$

Spectrum: a rotational band on every vibrational band head

$$E(n_0,n_2,m_l,l) = \omega_0\left(n_0+\frac{1}{2}\right) + \frac{\omega_2}{2}(2n_2+|m_l|+1) + \frac{1}{6v^2}\left(l(l+1)-(2m_l)^2\right)$$

Corrections ~ξ of band heads due to anharmonicities in the potential neglected.

In next-to-leading order, the results of the rotational-vibrational model are reproduced.

Summary
Construction of an EFT for (certain) heavy nuclei

1. Identify the relevant degrees of freedom for the resolution scale of interest:
   Quadrupole phonons
2. Identify the relevant symmetries of low-energy nuclear physics and investigate if and how they are broken:
   Spontaneously broken rotational symmetry
3. Construct the most general Lagrangian consistent with those symmetries and the symmetry breaking.
   Nonlinear realization of rotational symmetry
4. Design an organizational scheme (power counting) that can distinguish between more and less important contributions:
   Separation of scale between rotational and vibrational modes

Results: [TP, Nucl. Phys. 852, 36 (2011), arXiv:1011.5026]

1. In next-to-leading order, the results of the rotational-vibrational model are reproduced.
2. Nuclei with finite ground-state spins treated on equal footing → Wess-Zumino terms

Outlook

Enthusiastic and lively field

Intellectually stimulating problems

Moving towards a unified description of all atomic nuclei

Plenty of opportunities and challenges

Your ideas and ingenuity will shape the future!