The Nuclear Shell Model: Past, Present and Future

ALFREDO POVES
Departamento de Física Teórica and IFT, UAM-CSIC
Universidad Autónoma de Madrid (Spain)

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OUTLINE

- The nuclear A-body problem
- The Interacting Shell Model
- The effective interactions
- Collectivity

The Nuclear A-body Problem

- In the Standard Model of Nuclear Structure the elementary components are nucleons (N neutrons and Z protons, N+Z=A). The mesonic and quark degrees of freedom are integrated out
- In most cases non-relativistic kinematics is used
- The bare nucleon-nucleon (or nucleon-nucleon-nucleon) interactions are inspired by meson exchange theories or more recently by chiral perturbation theory, and must reproduce the nucleon-nucleon phase shifts, and the properties of the deuteron and other few body systems

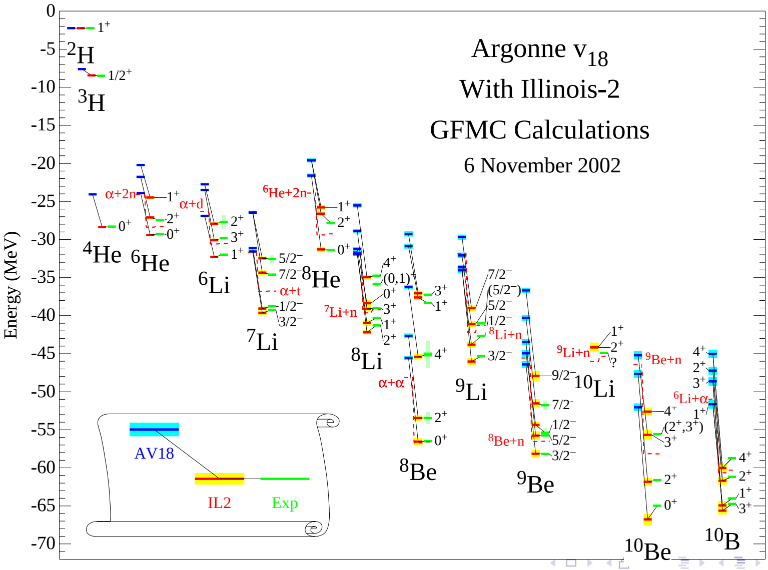
The Nuclear A-body Problem

- The challenge is to find $\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots \vec{r}_A)$ such that
- HΨ=EΨ, with
- $H=\sum_{i}^{A} T_i + \sum_{i,j}^{A} V_{2b}(\vec{r}_i, \vec{r}_j) + \sum_{i,j,k}^{A} V_{3b}(\vec{r}_i, \vec{r}_j, \vec{r}_k)$
- The knowledge of the eigenvectors Ψ and the eigenvalues E make it possible to obtain electromagnetic moments, transition rates, weak decays, cross sections, spectroscopic factors, etc.

The Nuclear A-body Problem

- The task is indeed formidable
- Only very recently and only for very light nuclei A$\leq$10 the problem has been solved ”exactly”,
- thanks to the pioneer work of Pandharipande, Wiringa and Pieper, which used variational methods (Green Function) solved by Monte Carlo techniques, GFMC
- More recently, the perturbative approach has been implemented in the framework of the No Core Shell Model (NCSM) by Barrett, Navratil, and Vary
- And even more recently, lattice gauge theory techniques have been implemented together with Chiral Perturbation Theory with very promising results in very light nuclei (Meissner, Bernard, et al. 2011)

"Ab Initio" Approaches; GFMC



"Ab Initio" Approaches

- A very important outcome of these calculations is that they show that it is compulsory to include three body forces in order to get correct solutions of the nuclear many body problem
- The GFMC and the NCSM are severely limited by the huge size of the calculations when A becomes larger than twelve
- For the rest of the chart of nuclides, approximate methods have to be used. Except for the semiclasical ones (liquid drop) and the α-cluster models, all are based on the Independent Particle Approximation

”Ab Initio” in two steps?

- Beyond the limits of applicability of the fully ”ab initio” descriptions, the methods of choice are the Interacting Shell Model and the Mean Field (and Beyond) approaches using Energy Density Functionals (aka density dependent effective interactions, like the Gogny force).
- There is nowadays renewed efforts to connect rigorously these two global methods and the bare two and three body nuclear interactions by means of the full palette of the Many Body Perturbation Methods. If this is achieved, they will deserve also the ”ab initio” label.

The Independent Particle Model

The basic idea of the IPM is to assume that, at zeroth order, the result of the complicated two body interactions among the nucleons is to produce an average self-binding potential. Mayer and Jensen (1949) proposed an spherical mean field consisting in an isotropic harmonic oscillator plus a strongly attractive spin-orbit potential and an orbit-orbit term. Later, other functional forms were adopted, *e.g.* the Woods-Saxon well

$$H = \sum_i h(\vec{r}_i)$$

$$h(r) = -V_0 + t + \frac{1}{2}m\omega^2 r^2 - V_{so}\vec{l}\cdot\vec{s} - V_B l^2$$

The Independent Particle Model

The eigenvectors of the IPM are characterized by the radial quantum number n, the orbital angular momentum l, the total angular momentum j and its z projection m. With the choice of the harmonic oscillator, the eigenvalues are:

$$\epsilon_{nljm} = -V_0 + \hbar\omega(2n + l + 3/2)$$

$$-V_{so}\frac{\hbar^2}{2}(j(j+1) - l(l+1) - 3/4) - V_B\hbar^2 l(l+1)$$

With a suitable choice of the parameters, they reproduce the magic numbers and in the large A limit also the volume, surface and (half) the symmetry terms of the semiempirical mass formula

Vocabulary

- STATE: a solution of the Schroedinger equation with a one body potential; e.g. the H.O. or the W.S. It is characterized by the quantum numbers *nljm* and the projection of the isospin t_z
- ORBIT: the ensemble of states with the same *nlj*, e.g. the 0d5/2 orbit. Its degeneracy is (2j+1)
- SHELL: an ensemble of orbits quasi-degenerated in energy, e.g. the *pf* shell
- MAGIC NUMBERS: the numbers of protons or neutrons that fill orderly a certain number of shells
- GAP: the energy difference between two shells
- SPE, single particle energies, the eigenvalues of the IPM hamiltonian
- ESPE, effective single particle energies, the eigenvalues of the monopole hamiltonian.

The Independent Particle Model

The usual procedure to generate a mean field in a system of N interacting fermions, starting from their free interaction, is the Hartree-Fock approximation, extremely successful in atomic physics. Whatever the origin of the mean field, the eigenstates of the N-body problem are Slater determinants *i.e.* anti-symmetrized products of N single particle wave functions.

The Independent Particle Model

In the nucleus, there is a catch, because the very strong short range repulsion and the tensor force make the HF approximation based upon the bare nucleon-nucleon force impracticable.

However, at low energy, the nucleus do manifest itself as a system of independent particles in many cases, and when it does not, it is due to the medium range correlations that produce strong configuration mixing and not to the short range repulsion.

The Independent Particle Model

Does the success of the shell model really "prove" that nucleons move independently in a fully occupied Fermi sea as assumed in HF approaches? In fact, the single particle motion can persist at low energies in fermion systems due to the suppression of collisions by Pauli exclusion (Pandharipande et al., RMP69(1997))

Brueckner theory takes advantage of the Pauli blocking to regularize the bare nucleon- nucleon interaction, in the form of density dependent effective interactions of use in HF calculations or G-matrices for large scale shell model calculations.

The Independent Particle Model

The wave function of the ground state of a nucleus in the IPM is the product of a Slater determinant for the Z protons that occupy the Z lowest states in the mean field and another Slater determinant for the N neutrons in the N lowest states of the mean field

In second quantization, this state can be written as:

$$|N\rangle \cdot |Z\rangle$$

with

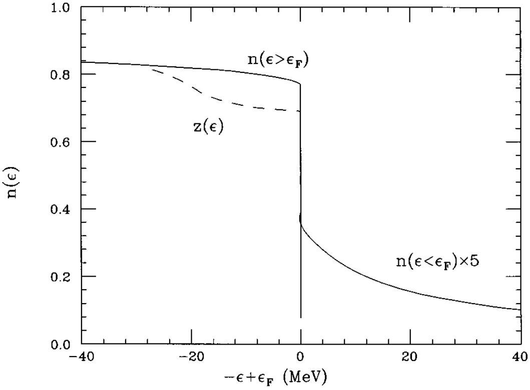
$$|N\rangle = n_1^\dagger n_2^\dagger \ldots n_N^\dagger |0\rangle$$

$$|Z\rangle = z_1^\dagger z_2^\dagger \ldots z_Z^\dagger |0\rangle$$

It is obvious that the occupied states have occupation number 1 and the empty ones occupation number 0

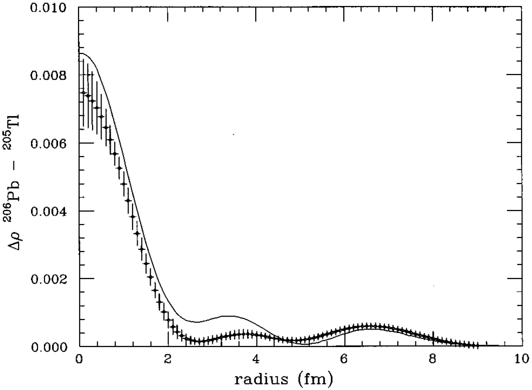
The Meaning of the Shell Model

Dilution of the Spectroscopic strength by the bare N-N interaction. Results for nuclear matter.



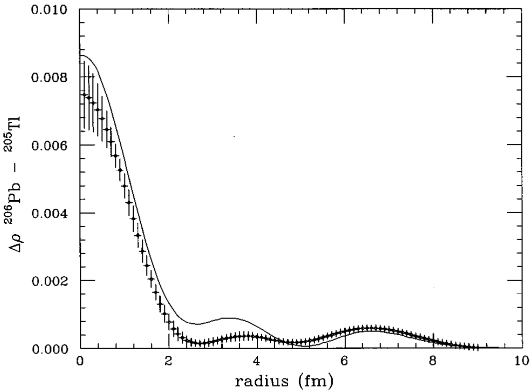
If we had a system of non interacting fermions, the figure would be a step function with occupation 1 below the Fermi level and 0 above

The Meaning of the Shell Model



In spite of that, the nuclear quasi-particles resemble extraordinarily to the mean field solutions of the IPM. This was demonstrated by the beautiful electron scattering experiment of Cavedon *et al, 1982* in which they extracted the charge density difference between ^{206}Pb and ^{205}Tl, that, in the IPM is just the square of the $2s_{1/2}$ orbit wave function.

The Meaning of the Shell Model



The shape of the $2s_{1/2}$ orbit is very well given by the mean field calculation. To make the agreement quantitative the calculated density has to be scaled down by the occupation number

To know more, Read the article " Independent particle motion and correlations in fermion systems" V. R. Pandharipande, et al., RMP 69 (1997) 981.

Beyond the IPM; The mean field way

- HF-based approaches rely on the use of density dependent interactions of different sort; Skyrme, Gogny, RMF parametrizations
- The correlations are taken into account via symmetry breaking in the mean field
- Projections before (VAP) or after (PAV) variation are enforced to restore the conserved quantum numbers
- Ideally, configuration mixing is also implemented through the GCM

Beyond the IPM; The Interacting Shell Model (ISM)

Is an approximation to the exact solution of the nuclear A-body problem using effective interactions in restricted spaces

The single particle states (i,j, k, ...), which are the solutions of the IPM, provide as well a basis in the space of the occupation numbers (Fock space). The many body wave functions are Slater determinants:

$$\Phi = a^{\dagger}_{i_1}, a^{\dagger}_{i_2}, a^{\dagger}_{i_3}, \ldots a^{\dagger}_{i_A} |0\rangle$$

We can distribute the A particles in all the possible ways in the available single particle states. This provides a complete basis in the Fock space. The number of Slater determinants will be huge but not infinite because the theory is no longer valid beyond a certain cutt-off.

A formal solution to the A-body problem

Therefore, the ”exact” solution can be expressed as a linear combination of the basis states:

$$\Psi = \sum_\alpha \Phi_\alpha$$

and the solution of the many body Schödinger equation

$$H\Psi = E\Psi$$

is transformed in the diagonalization of the matrix:

$$\langle \Phi_\alpha | H | \Phi_\beta \rangle$$

whose eigenvalues and eigenvectors provide the ”physical” energies and wave functions

Beyond the IPM; The Interacting Shell Model (ISM)

- A Shell Model calculation amounts to diagonalizing the effective nuclear hamiltonian in the basis of all the Slater determinants that can be built distributing the valence particles in a set of orbits which is called valence space. The orbits that are always full form the core.
- If we could include all the orbits in the valence space (a full No Core calculation) we should get the "exact" solution.
- The effective interactions are obtained from the bare nucleon-nucleon interaction by means of a regularization procedure aimed to soften the short range repulsion. In other words, using effective interactions we can treat the A-nucleon system in a basis of independent quasi-particles. As we reduce the valence space, the interaction has to be renormalized again in a perturbative way.

The three pillars of the shell model

- The Effective Interaction
- Valence Spaces
- Algorithms and Codes

E. Caurier, G. Martínez-Pinedo, F. Nowacki, A. Poves and A. P. Zuker. “The Shell Model as a Unified View of Nuclear Structure”, RMP 77 (2005) 427.

The Hamiltonian

$$\mathcal{H} = \sum_{r} \epsilon_r \hat{n}_r + \sum_{r\leq s, t\leq u, \Gamma} W^{\Gamma}_{rstu} Z^{+}_{rs\Gamma} \cdot Z_{tu\Gamma},$$

where Z^{+}_{Γ} (Z_{Γ}) is the coupled product of two creation (annihilation) operators and $\hat{n}_r = (a^{\dagger}_r a_r)^{00}$

$$Z^{+}_{rs\Gamma} = [a^{\dagger}_r a^{\dagger}_s]^{\Gamma}$$

where Γ is a shorthand for (J,T); r, s ..... run over the orbits of the valence space; ϵ_r are the single particle energies and W^{Γ}_{rstu} the two body matrix elements:

$$W^{\Gamma}_{rstu} = \langle j_r j_s (JT) | V | j_t j_u (JT) \rangle$$

Making the Effective Interaction Simple

Without loosing the simplicity of the Fock space representation, we can recast the two body (and three body if needed) matrix elements of any effective interaction in a way full of physical insight, following Dufour-Zuker rules

Any effective interaction can be split in two parts:

$\mathcal{H} = \mathcal{H}_m$(monopole) + $\mathcal{H}_M$(multipole).

$\mathcal{H}_m$ contains all the terms that are affected by a spherical Hartree-Fock variation, hence it is responsible for the global saturation properties and for the evolution of the spherical single particle energies.

The Monopole Hamiltonian

$$\mathcal{H}_m = \sum \epsilon_i n_i + \sum \left[\frac{1}{(1+\delta_{ij})} a_{ij}\, n_i (n_j - \delta_{ij}) \right.$$

$$\left. + \frac{1}{2} b_{ij} \left(T_i \cdot T_j - \frac{3n_i}{4} \delta_{ij} \right) \right] + \sum A_{ijk} n_i n_j n_k$$

The coefficients *a* and *b* are defined in terms of the centroïds (angular averages):

$$V_{ij}^{T} = \frac{\sum_J V_{ijij}^{JT} [J]}{\sum_J [J]}$$

as: $a_{ij} = \frac{1}{4}(3V_{ij}^1 + V_{ij}^0)$, $b_{ij} = V_{ij}^1 - V_{ij}^0$, the sums run over Pauli allowed values.

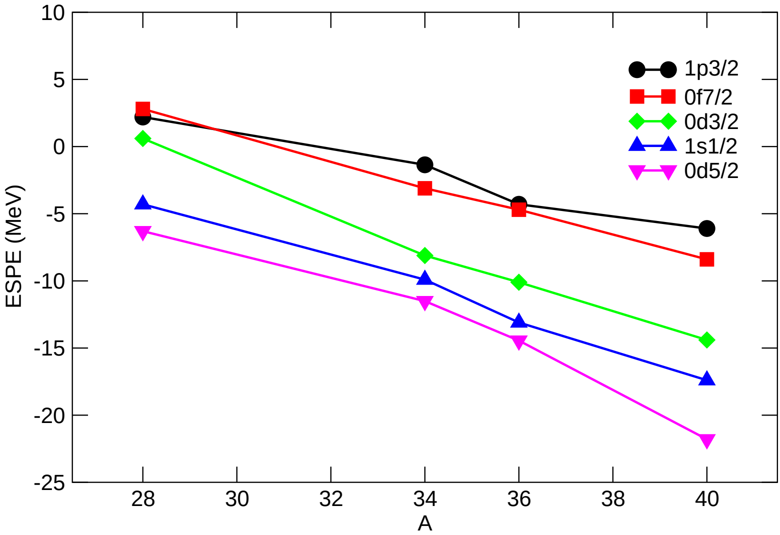
The Monopole Hamiltonian

The evolution of effective spherical single particle energies with the number of particles in the valence space is dictated by $\mathcal{H}_m$. Schematically:

$$\epsilon_j(\{n_i\}) = \epsilon_j(\{n_i = 0\}) + \sum_i a_{ij} n_i + \sum_{i,k} A_{ijk} n_i n_k$$

Even small defects in the centroids can produce large changes in the relative position of the different configurations due to the appearance of quadratic terms involving the number of particles in the different orbits.

The Drift of the Single Particle Energies: N=20



The Multipole Hamiltonian

$\mathcal{H}_M$ can be written in two representations, particle-particle and particle-hole:

$$\mathcal{H}_M = \sum_{r\leq s,t\leq u,\Gamma} W^{\Gamma}_{rstu} Z^{+}_{rs\Gamma} \cdot Z_{tu\Gamma},$$

$$\mathcal{H}_M = \sum_{rstu\Gamma} [\gamma]^{1/2} \frac{(1+\delta_{rs})^{1/2}(1+\delta_{tu})^{1/2}}{4} \omega^{\gamma}_{rtsu} (S^{\gamma}_{rt} S^{\gamma}_{su})^0,$$

where $S^{\gamma} = (a^{\dagger}a)^{\gamma}$ is the coupled product of one creation and one annihilation operator; γ is a shorthand for (γ_J,γ_T) and $[\gamma] = (2\gamma_J + 1)(2\gamma_T + 1)$

$$Z^{+}_{rs\Gamma} = [a^{\dagger}_r a^{\dagger}_s]^{\Gamma} \text{ and } S^{\gamma}_{rs} = [a^{\dagger}_r a_s]^{\gamma}$$

The Multipole Hamiltonian

The W and ω matrix elements are related by a Racah transformation

$$\omega^{\gamma}_{rtsu} = \sum_{\Gamma} (-)^{s+t-\gamma-\Gamma} \left\{ \begin{array}{ccc} r & s & \Gamma \\ u & t & \gamma \end{array} \right\} W^{\Gamma}_{rstu}[\Gamma],$$

$$W^{\Gamma}_{rstu} = \sum_{\gamma} (-)^{s+t-\gamma-\Gamma} \left\{ \begin{array}{ccc} r & s & \Gamma \\ u & t & \gamma \end{array} \right\} \omega^{\gamma}_{rtsu}[\gamma].$$

The operators $S^{\gamma=0}_{rr}$ are just the number operators for orbits r and $S^{\gamma=0}_{rr'}$ the spherical HF particle hole vertices. Both must have null coefficients if the monopole hamiltonian satisfies HF self-consistency.

The Multipole Hamiltonian

- The operator $Z^+_{rr\Gamma=0}$ creates a pair of particles coupled to J=0. The terms $W^{\Gamma}_{rrss}\ Z^+_{rr\Gamma=0} \cdot Z_{ss\Gamma=0}$ represent different kinds of pairing hamiltonians.
- The operators S^{γ}_{rs} are typical vertices of multipolarity γ. For instance, γ=(J=1,L=0,T=1) produces a $(\vec{\sigma} \cdot \vec{\sigma})$ $(\vec{\tau} \cdot \vec{\tau})$ term which is nothing but the Gamow-Teller component of the nuclear interaction
- The terms S^{γ}_{rs} γ=(J=2,T=0) are of quadrupole type r^2Y_2. They are responsible for the existence of deformed nuclei, and they are specially large and attractive when $j_r - j_s$=2 and $l_r - l_s$=2.

Universality of the Multipole Hamiltonian

A careful analysis of the effective nucleon-nucleon interaction in the nucleus, reveals that the multipole hamiltonian is universal and dominated by BCS-like isovector and isoscalar pairing plus quadrupole-quadrupole and octupole-octupole terms of very simple nature ($r^\lambda Y_\lambda \cdot r^\lambda Y_\lambda$)

Interaction	particle-particle		particle-hole		
	JT=01	JT=10	$\lambda\tau$=20	$\lambda\tau$=40	$\lambda\tau$=11
KB3	-4.75	-4.46	-2.79	-1.39	+2.46
FPD6	-5.06	-5.08	-3.11	-1.67	+3.17
GOGNY	-4.07	-5.74	-3.23	-1.77	+2.46
GXPF1	-4.18	-5.07	-2.92	-1.39	+2.47
BONNC	-4.20	-5.60	-3.33	-1.29	+2.70

The Effective Interaction

The evolution of the spherical mean field in the valence spaces remains a key issue, because we know since long that something is missing in the monopole hamiltonian derived from the realistic NN interactions, be it through a G-matrix, V_{low-k} or other options.

The need for three body forces is now confirmed. Would they be reducible to simple monopole forms? Would they solve the monopole puzzle of the ISM calculations? The preliminary results seem to point in this direction

The multipole hamiltonian does not seem to demand major changes with respect to the one derived from the realistic nucleon-nucleon potentials and this is a real blessing because it suggest that the effect of the three body interactions in the many nucleon system may be well approximated by monopole terms

The Valence Space(s)

- An ideal valence space should incorporate the most relevant degrees of freedom AND be computationally tractable
- Classical $0\hbar\omega$ valence spaces are provided by the major oscillator shells *p*, *sd* and *pf* shells
- For very neutron rich nuclei around N=28, a good choice is to take the *sd* shell for protons and the *pf* shell for neutrons.
- For very neutron rich Cr, Fe, Ni, and Zn, r_3-($0g_{9/2}, 1d_{5/2}$) for the neutrons and *pf* for protons.
- To describe the intruders around N and/or Z=20, r_2-*pf*

(note: in a major HO shell of principal quantum number **p** the orbit j=p+1/2 is called *intruder* and the remaining ones are denoted by r_p)

Algorithms and Codes

Algorithms include Direct Diagonalisation, Lanczos, Monte Carlo Shell Model, Quantum Monte Carlo Diagonalization, DMRG, etc. There are also a number of different extrapolation ansatzs

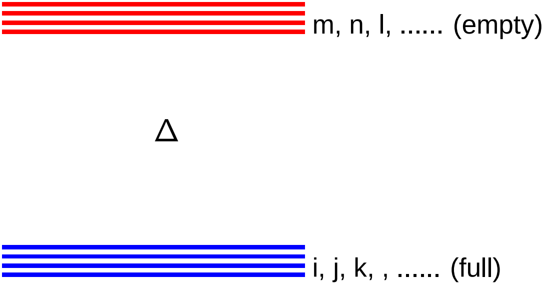
The Strasbourg-Madrid codes (Antoine, Nathan), can deal with problems involving basis of 10^{10} Slater determinants, using relatively modest computational resources. Other competitive codes in the market are OXBACH, NUSHELL and MSHELL

Collectivity in Nuclei

- For a given interaction, a many body system would or would not display coherent features at low energy depending on the structure of the mean field around the Fermi level.
- If the spherical mean field around the Fermi surface makes the pairing interaction dominant, the nucleus becomes superfluid
- If the quadrupole-quadrupole interaction is dominant the nucleus acquires permanent deformation
- In the extreme limit in which the monopole hamiltonian is negligible, the multipole interaction would produce superfluid nuclear needles.
- Magic nuclei are spherical despite the strong multipole interaction, because the large gaps in the nuclear mean field at the Fermi surface block the correlations

The meaning of collectivity

Imagine a situation such as depicted in the figure. The ground state has $J^{\pi}=0^{+}$ and, in the IPM, the lowest excited states correspond to promoting one particle from the occupied orbits to the empty ones. They are many, quasi degenerate, and should appear at an excitation energy Δ.



The meaning of collectivity

Let's take now into account the multipole hamiltonian, that, for simplicity will be of separable form, and choose as valence space the particle-hole states

$$\langle nj|V|mj\rangle = \beta_\lambda Q^\lambda_{nj} Q^\lambda_{mi}$$

The wave function can be developed in the p-h basis as:

$$\Psi = \sum C_{mi}|mi\rangle$$

The Schödinger equation HΨ=EΨ can thus be written as:

$$C_{nj}(E - \epsilon_{nj}) = \sum_{mi} \beta_\lambda C_{mi} Q^\lambda_{nj} Q^\lambda_{mi}$$

The meaning of collectivity

Then,

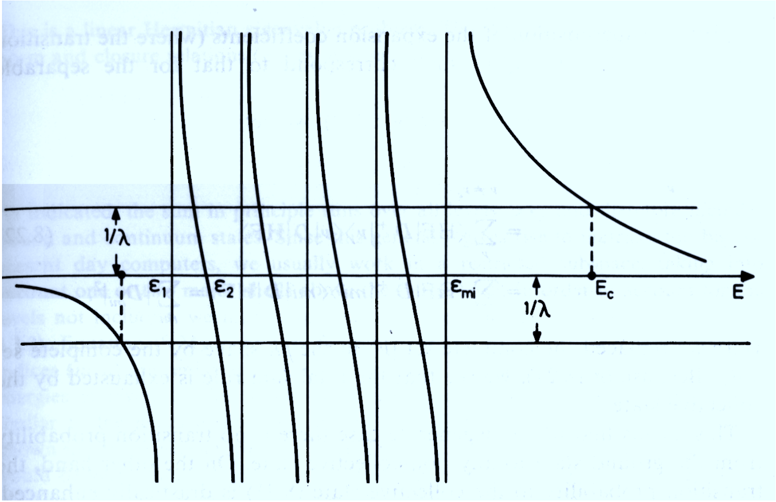
$$C_{nj} = \frac{\beta_\lambda Q^\lambda_{nj}}{E - \epsilon_{nj}} \sum_{mi} C_{mi} Q^\lambda_{mi}$$

and, trivially,

$$1 = \beta_\lambda \sum_{nj} \frac{(Q^\lambda_{nj})^2}{E - \epsilon_{nj}}$$

A graphical analysis of this equation shows that all its solutions except one are very close to the unperturbed values ϵ_{nj}, the remaining one is the lowest and it is well separated from the others.

The meaning of collectivity



The meaning of collectivity

If now we take $\epsilon_{nj} \approx \overline{\epsilon_{nj}} = \Delta$, we obtain:

$$E = \Delta + \beta_\lambda \sum_{nj} (Q^\lambda_{nj})^2$$

If the interaction is attractive, $\beta_\lambda < 0$, the lowest state gains an energy which is proportional to β_λ, the strength of the multipole interaction, and to the coherent sum of the squared one body matrix elements of the one body multipole operators between the particle and hole orbits in the space. This mechanism of coherence explains the appearance of vibrational states in the nucleus and represents the basic microscopic description of the nuclear "phonons".

The meaning of collectivity

Because the couplings β_λ are constant except for a global scaling, the onset of collectivity requires the presence of several quasi degenerate orbits above and below the Fermi level. In addition, these orbits must have large matrix elements with the multipole operator of interest

The wave function of the coherent (collective) (phonon) state has the following form:

$$\Psi_c(J=\lambda) = \frac{\sum_{nj} Q^\lambda_{nj} |nj\rangle}{\sum_{nj} (Q^\lambda_{nj})^2}$$

The meaning of collectivity

The coherent state is coherent with the transition operator Q^λ because the probability of its Eλ decay to the 0^+ ground state is very much enhanced

$$B(E\lambda) \sim |\langle 0^+|Q^\lambda|\Psi_c(J=\lambda)\rangle|^2 = \sum_{nj}(Q^\lambda_{nj})^2$$

Which should be much larger than the single particle limit (many WU). Notice that a large value of the $B(E\lambda)$ does not imply necessarily the existence of permanent deformation in the ground sate.

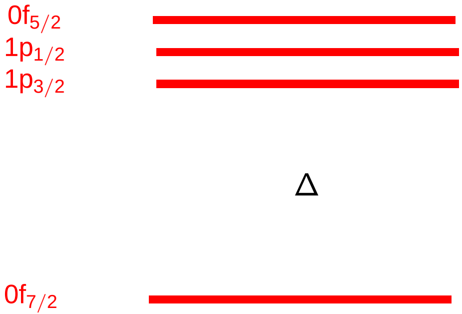
Shape transition; Deformed nuclei

Notice that nothing prevents that:

$$|\beta_\lambda \sum_{nj} (Q^{\lambda}_{nj})^2| > \Delta$$

In this case the vibrational phonon is more bound than the ground state and the model is no longer valid. What happens is that a phase transition from the vibrational to the rotational regime takes place as the nucleus acquires permanent deformation of multipolarity λ. The separation between filled and empty shells does not hold any more and both have to be treated at the same footing.

The case of ^{48}Cr



This would be the case if we tried to treat ^{48}Cr as a vibrator. The energy of the one phonon 2^+ state would be lower than the ground state one.

The “flaws” of the standard SM description

- Quadrupole effective charges are needed if the valence space does not contain $2\hbar\omega$ 1-particle 1-hole excitations. (But their value is universal and rather well understood)
- Spin operators are quenched by another universal factor which relates to the regularization of the interaction (also known as short range correlation). Indeed, BMF approaches share this shortcoming
- Not all the regions of the nuclear chart are amenable to a SM description yet

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ALFREDO POVES
Departamento de Física Teórica and IFT, UAM-CSIC
Universidad Autónoma de Madrid (Spain)

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OUTLINE

- Spherical mean field and Correlations
- The physics of very neutron rich nuclei at N=20 and N=28

Monopole anomalies and Multipole universality

- The different facets of the nuclear dynamics depend on the balance of the two main components of the nuclear hamiltonian; the Monopole which produces the effective spherical mean field and the Multipole responsible for the correlations
- Large scale shell model calculations have unveiled the monopole anomalies of the two-body realistic interactions, *i.e* that they tend to produce effective single particle energies which are not compatible with the experimental data and which, if used without modifications, produce spectroscopic catastrophes

Monopole anomalies and Multipole universality

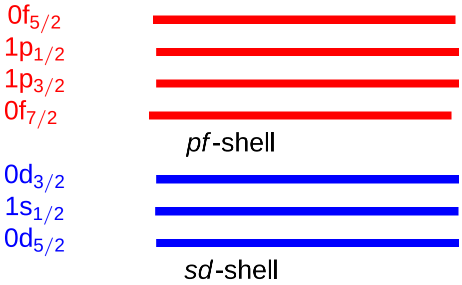
- Already in the mid 70's Pasquini and Zuker showed that the Kuo Brown interaction could not produce neither a magic ^{48}Ca nor a magic ^{56}Ni. In this last case it made a nearly perfect rotor instead. A few monopole corrections (mainly T=1) restored high quality spectroscopy
- Holt, Otsuka and Schwenk have recently shown that the monopole component of the three body force may cure the monopole anomalies relevant for ^{28}O and ^{48}Ca .
- The Multipole Hamiltonian of the realistic two body interactions (dominated by L=0 pairings, quadrupole and octupole) does not seem to require any substantial modification and is "universal" in the sense that all the interactions produce equivalent multipole hamiltonians

The fate of magic closures

- Magic numbers are associated to energy gaps in the spherical mean field. Therefore, to promote particles above the Fermi level costs energy.
- However, some intruder configurations can overwhelm their loss of monopole energy with their huge gain in correlation energy.
- Several examples of this phenomenon exist in stable magic nuclei in the form of coexisting spherical, deformed and superdeformed states in a very narrow energy range, Nuclear Allotropy? In the case of ^{40}Ca they have described in tha spherical shell model framework

The valence space *sd*-*pf*

The valence space of two major shells

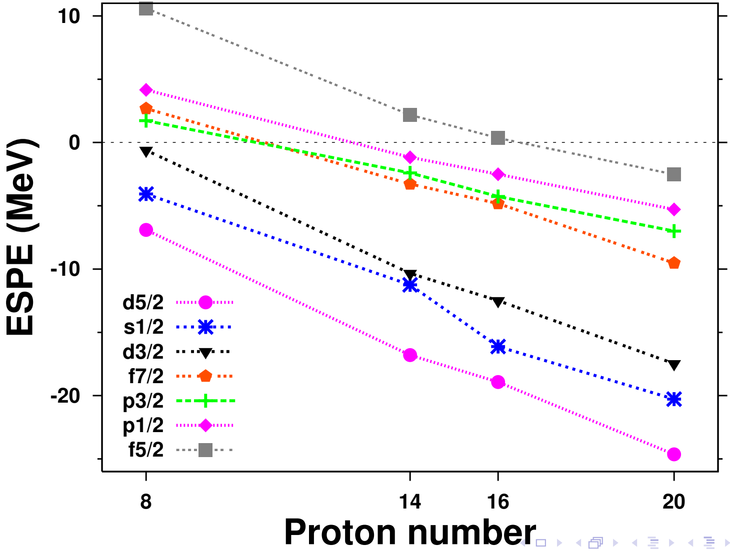


can encompass the physics of nuclei between ^{18}O and ^{64}Ge including the "islands of inversion" which appear at N=20 (around ^{31}Na) and N=28 (around ^{42}Si) as well as the excited superdeformed bands of N=Z magic nuclei like ^{40}Ca. And, indeed, using essentially a single effective interaction, SDPF-U.

N=20 far from stability

- The region around ^{31}Na provides a beautiful example of intruder dominance in the ground states, known experimentally since long (Detraz, Thibault, Guillemaud, Klotz, Walter) .
- Early shell model calculations (Poves and Retamosa (87), Warburton, Becker and Brown (90)) pointed out the role of deformed intruder configurations 2p-2h neutron excitations from the *sd* to the *pf*-shell and started the study of the boundaries of the so called "island of inversion" and the properties of its inhabitants.
- Similar mechanisms produce the other known "islands of inversion" centered in ^{11}Li (N=8), ^{42}Si (N=28), and ^{64}Cr (N=40)

What have we learnt about the Effective Single Particle Energies for N=20



Why an island of inversion/deformation? ^{34}Si

- Four protons away from doubly magic ^{40}Ca, ^{34}Si is a new doubly magic nucleus: the proton Z=14 and the neutron N=20 gaps seem to reinforce each other.
- According to the ESPE's. to promote two *sd*-shell neutrons to the *pf*-shell should cost at least 9 MeV. However after mixing at fixed $0\hbar\omega$ and $2\hbar\omega$, the 2p-2h 0^+ state is only 2 MeV above the 0p-0h 0^+. In a full 0-2-4-6$\hbar\omega$ calculation it appears very close to the experimental value of 2.72 MeV (Grèvy et al. 2011) Notice the huge difference in correlation energy between the two configurations

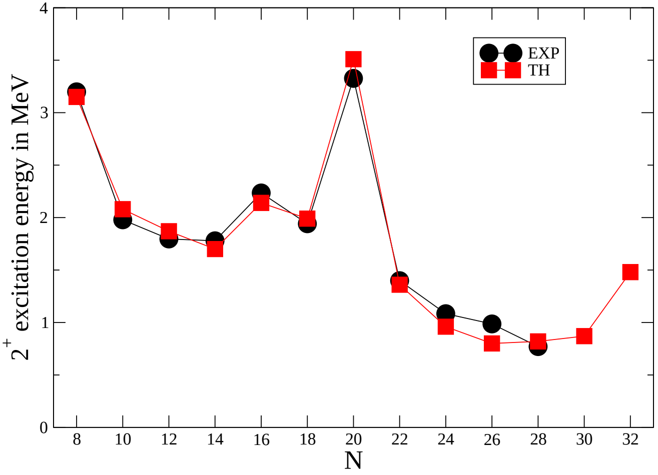
Why an island of inversion/deformation? ^{32}Mg

- To go even more neutron rich, one needs to remove protons from the $0d_{5/2}$ orbit.
- This causes a reduction of the N=20 neutron gap and a small increase of proton collectivity.
- Both effects conspire in the sudden appearance of an Island of Inversion in which Deformed Intruder states become ground states, as in ^{32}Mg, ^{31}Na and ^{30}Ne.

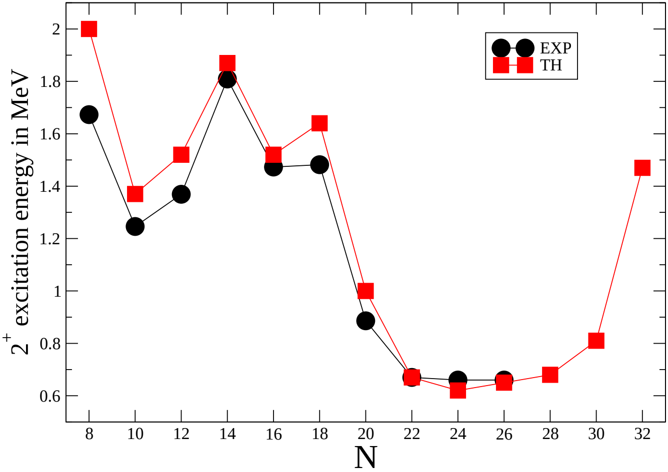
Why an island of inversion/deformation? ^{32}Mg

- According to the ESPE's, to promote two *sd*-shell neutrons to the *pf*-shell in ^{32}Mg should cost at least 7 MeV. However, after mixing at fixed 0$\hbar\omega$ and 2$\hbar\omega$, the 2p-2h 0^+ state is 1 MeV below the 0p-0h 0^+. The transition has taken place !!! In a full 0-2-4-6$\hbar\omega$ calculation it appears very close, albeit somewhat higher than the experimental value of 1.06 MeV (Van Duppen et al. 2011)
- ^{31}Na and ^{30}Ne behave in a similar way. In ^{29}F and ^{28}0 the ESPE gap is washed out and the role of correlations is less important

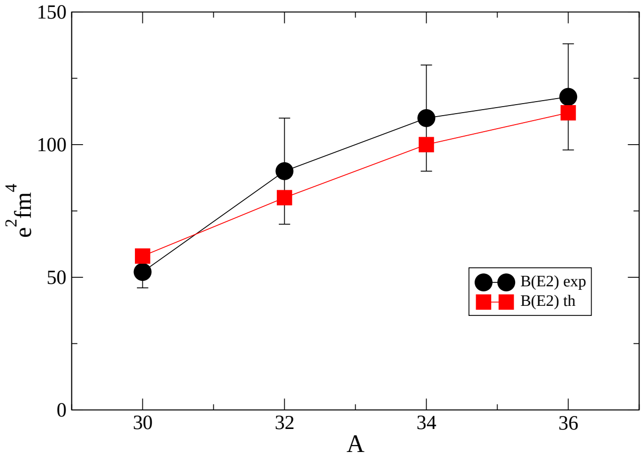
The Silicon isotopes from the proton to the neutron dripline; SDPF-U interaction



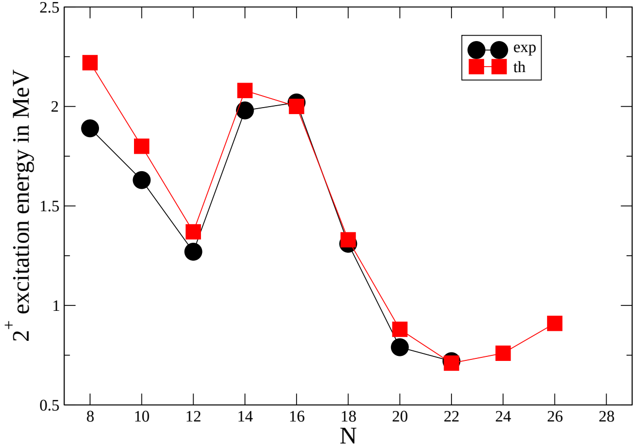
The Magnesium isotopes from the proton to the neutron dripline; SDPF-U interaction



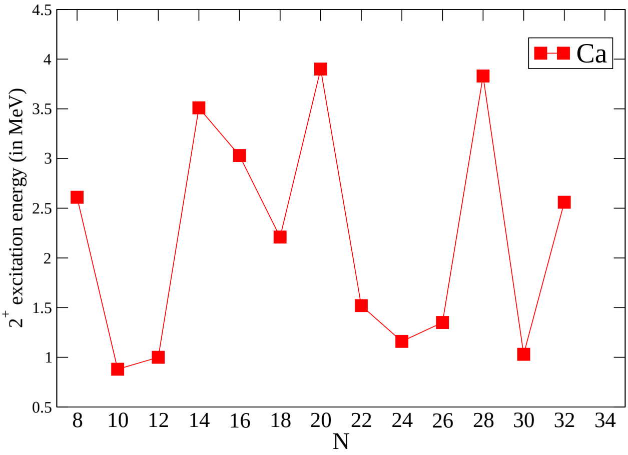
The Magnesium isotopes; B(E2)'s SDPF-U int.



The Neon isotopes; SDPF-U int.



The Calcium isotopes, exp.



Conclusions

- State of the art Shell Model calculations encompassing two major oscillator shells make it possible to describe complete series of isotopes from the proton to the neutron drip lines
- They can also cross the ”islands of inversion” at N=8, 20, and 28