Review of mean-field theory

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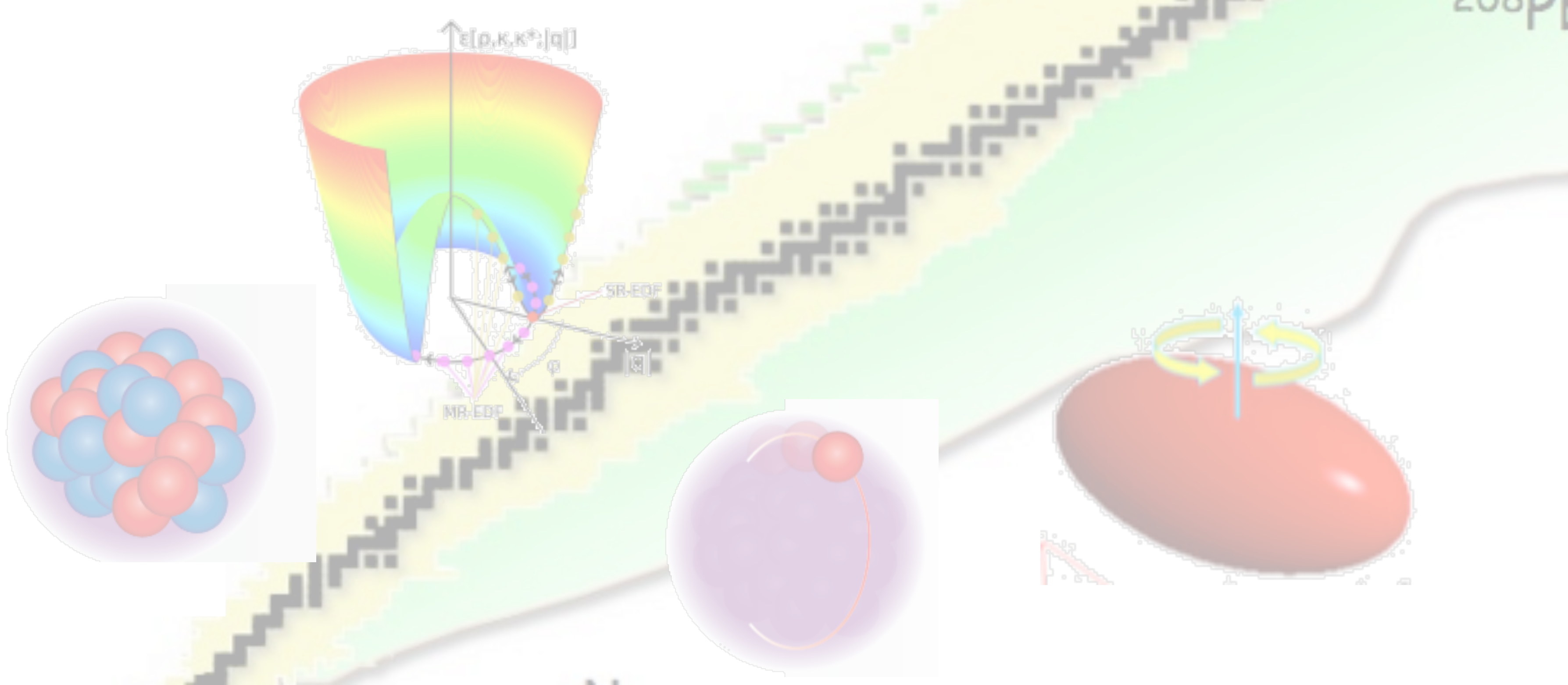
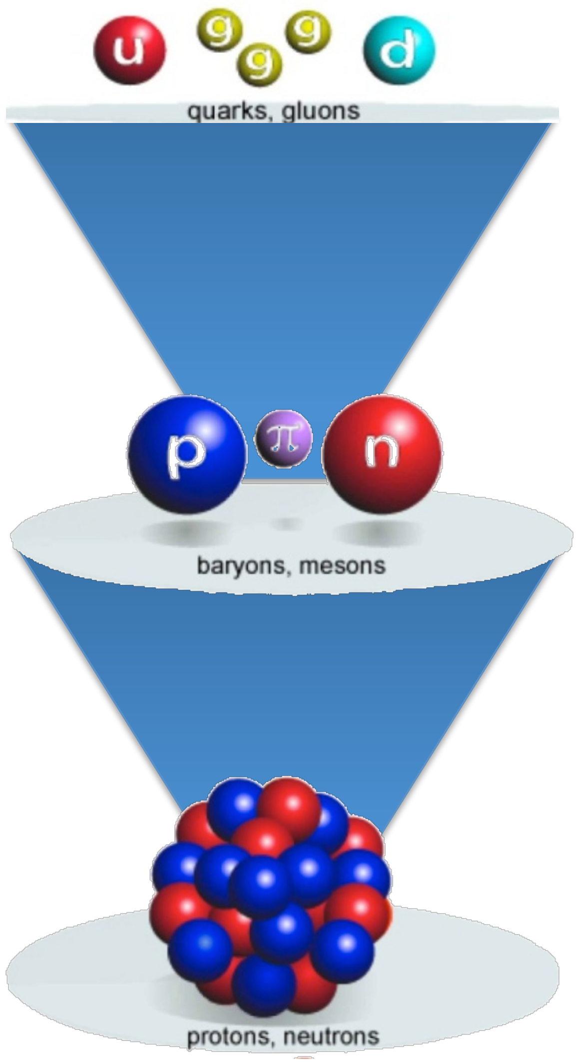
GANIL-Caen

International Joliot-Curie School 2011



Complex and simple aspects of nuclei

generalities



General remarks

- Nucleons are made of quarks
- The energy to excite quarks degrees of freedom (DOF) is much higher than the energy to excite nucleon DOF

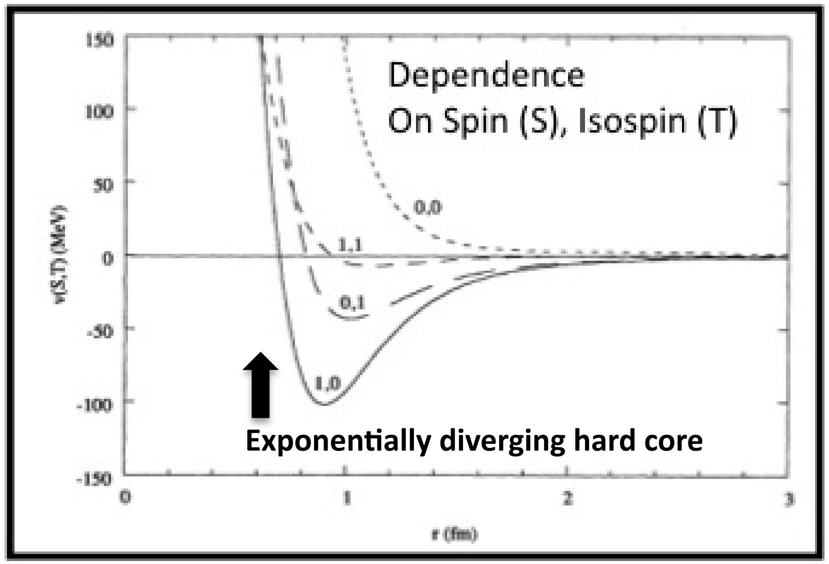
➡ Nucleons are considered as elementary (point-like) Particles (low energy nucl. Phys. Point of view)

Nucleons are quantum interacting objects

$$\phi_{\text{nucleon}} \equiv \phi(\mathbf{r}, \sigma, \tau)$$

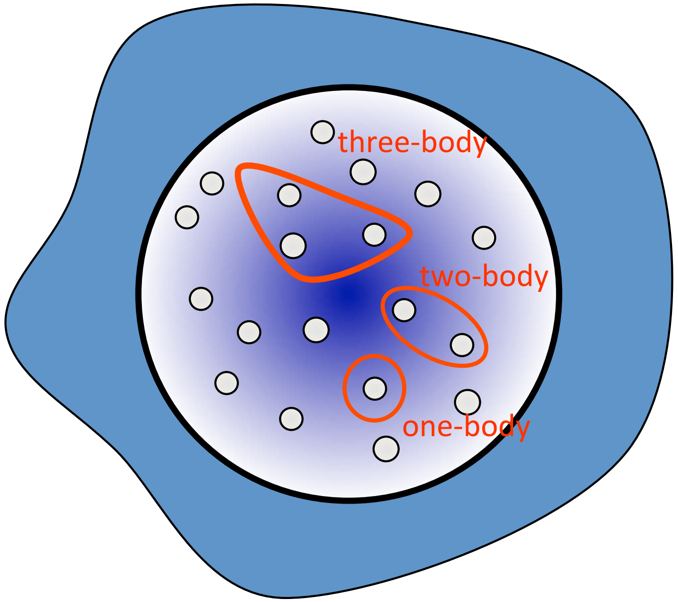
$\sigma = \uparrow, \downarrow$ spin

$\tau = n, p$ isospin



Wiringa, Rev. Mod. Phys. 1993

+ Coulomb field



From one to many nucleons

For one nucleon:

$$\phi_{\rm nucleon} \equiv \phi(\mathbf{r}, \sigma, \tau)$$

For a N nucleon system:

$$\Psi(r_1, \cdots, r_{12}, \cdots, r_{123}, \cdots)$$

***spin, isospin omitted**

→ [N! × size of nucleon space] DOF …

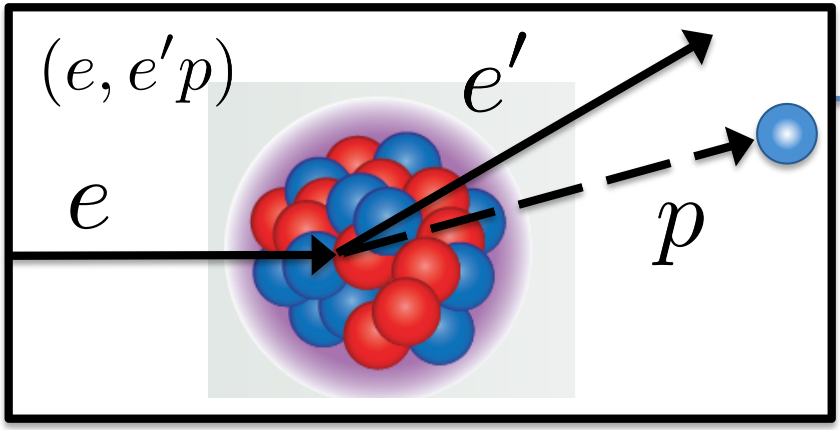
→ Nuclei are among the most complex quantum mesoscopic systems

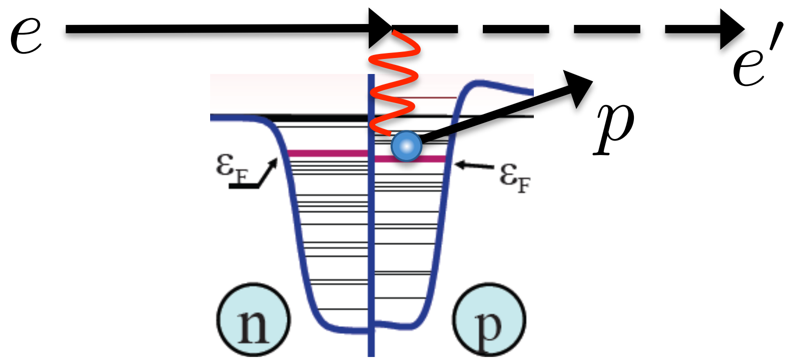
→ Exact (ab-initio) description can be made only in rare cases.

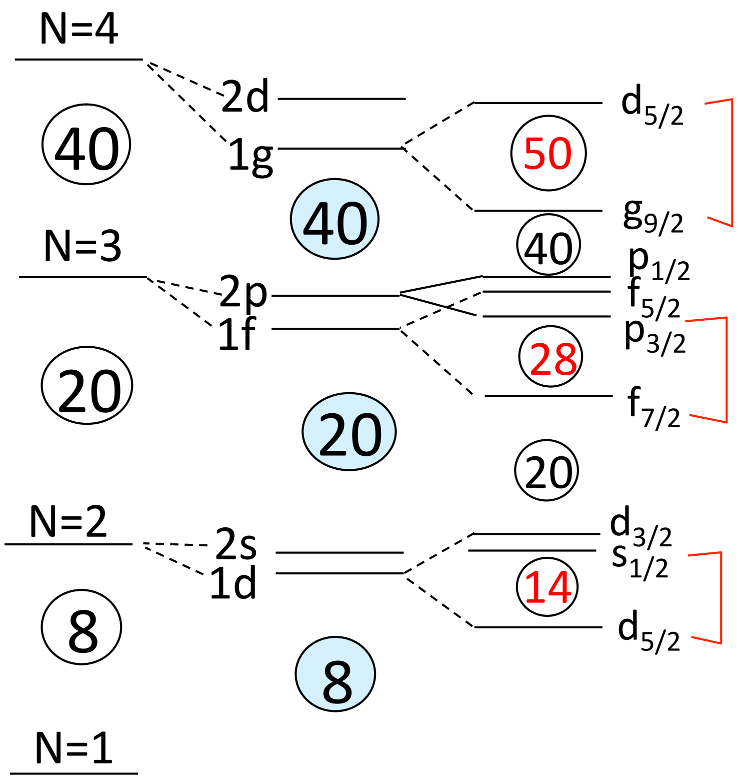
(see T. Papenbrock lecture)

Complex and simple aspects of nuclei

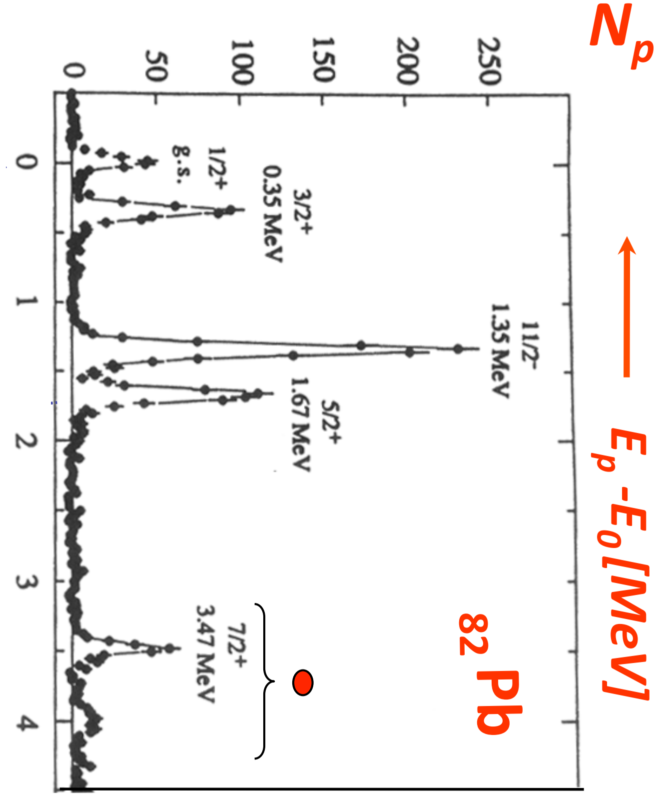
Observation: emergence of single-particle aspects





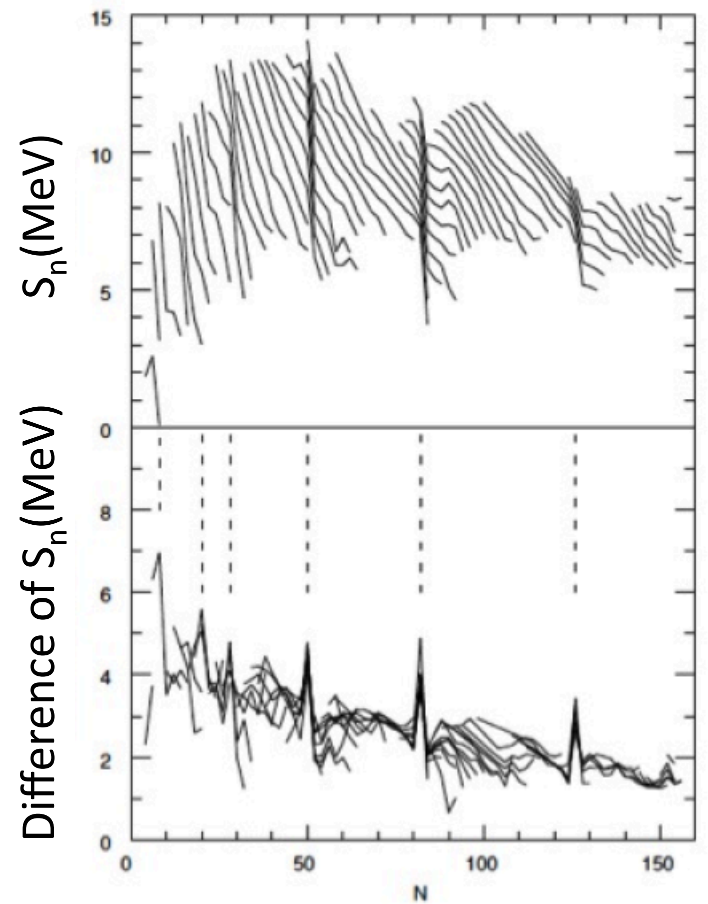


H.O. + L^2 + $\vec{L}.\vec{S}$



(Courtesy O. Sorlin, EJC2009)

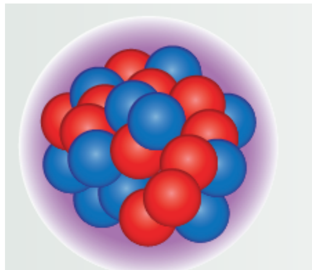
Complex and simple aspects of nuclei



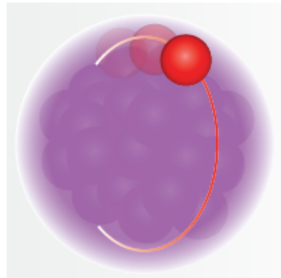
Observation: Separation energy, magic numbers,…

Separation energy: $S_n(A,Z) = B(A,Z) - B(A-1,Z)$

Nucleons behaves like independent particles in a single-particle field !!!



$\Psi(r_1, \cdots, r_{12}, \cdots, r_{123}, \cdots)$



$\Psi = \mathcal{A}(\varphi_1(r_1), \cdots \varphi_N(r_N))$

$$\left\{ \frac{\hat{\mathrm{p}}^2}{2m} + V(\hat{\mathrm{r}}) + f(\mathrm{r})\vec{l}.\vec{s} \right\} |\varphi_\alpha\rangle = \varepsilon_\alpha |\varphi_\alpha\rangle$$

Example : Wood Saxon potential $V(r) = \frac{V_0}{1+\exp(\frac{r-R_0}{a})}$

Summary : first dilemma

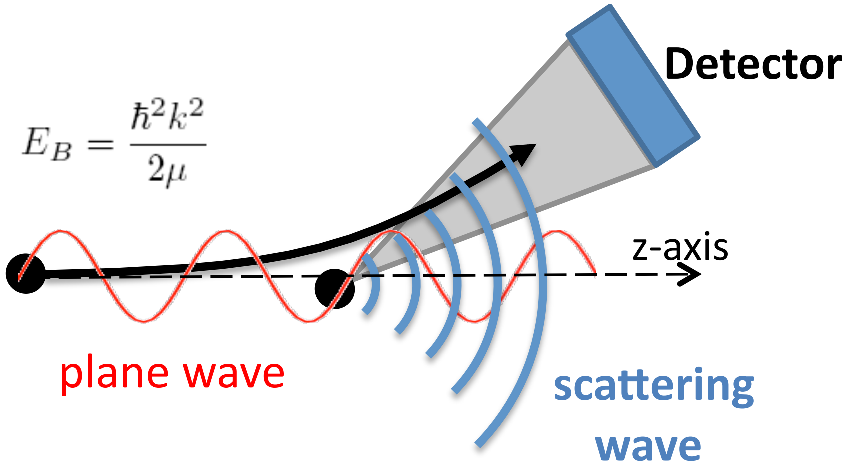
Complexity

Nuclei are quantum mesoscopic objects composed of fermions interacting through a strong highly repulsive interaction at short range

Simplicity

Many aspects of nuclei can be fairly well understood assuming that nucleons behaves like independent particles in an external one-body field

Scattering theory, medium effects, …



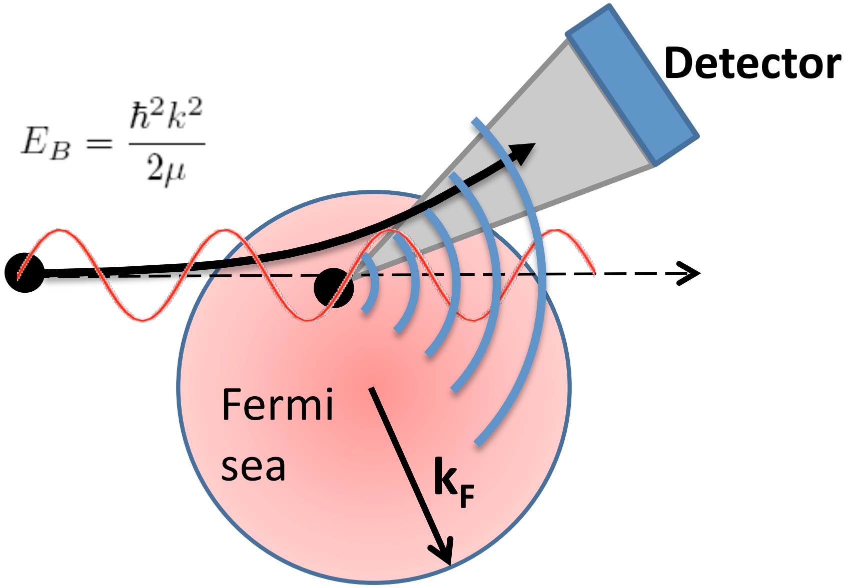
$$\Psi_{\rm scat}(\mathbf{r}) \overset{\mathbf{r}\to\infty}{\longrightarrow} e^{ikz} + f(\theta,\varphi)\frac{e^{ikr}}{r}$$

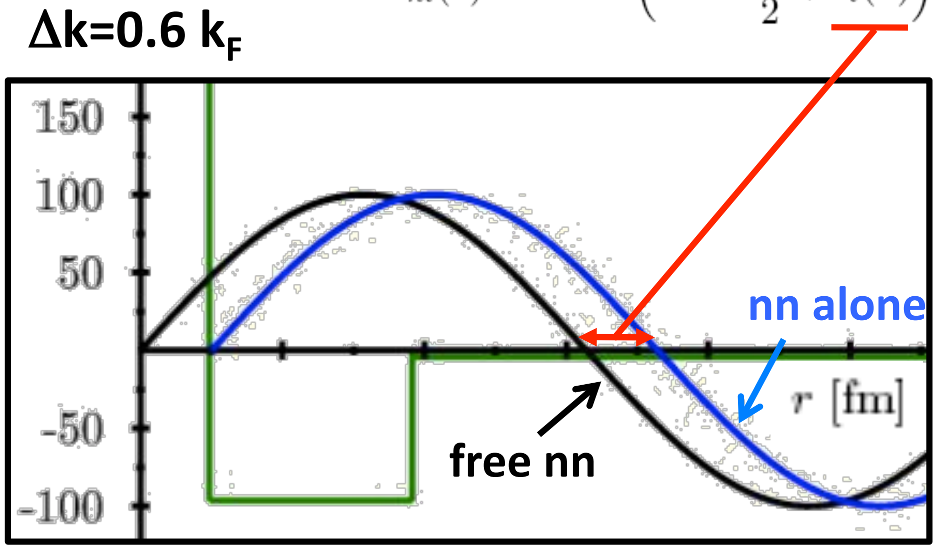
$$\sigma_k(\theta,\varphi) \;=\; |f(\theta,\varphi)|^2$$

Spherical symmetric potential:

$$\Psi_{\rm scat}(\mathbf{r}) \;=\; \sum c_{nlm}\frac{u_{nl}(r)}{r}Y_{lm}(\theta,\varphi)$$

$$u_{nl}(r) \overset{\mathbf{r}\to\infty}{\sim} \sin\left(kr - l\frac{\pi}{2} + \delta_l(k)\right)$$

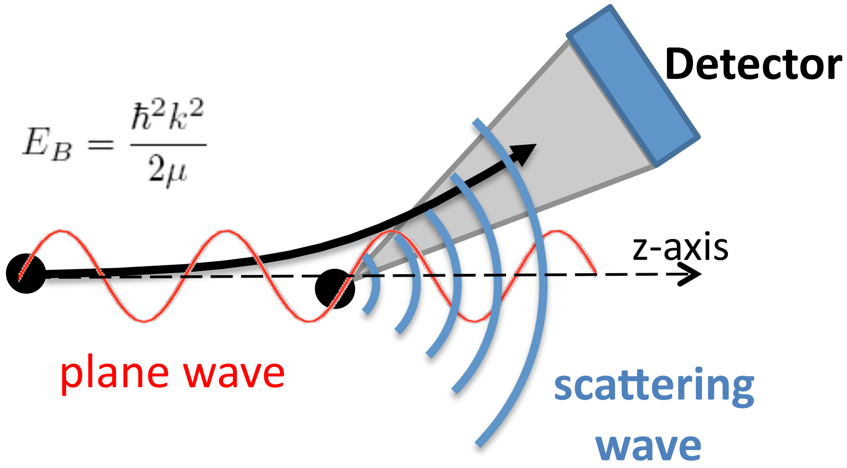




Gomes, Walecka and Weisskopf, Ann. Phys. (1958)

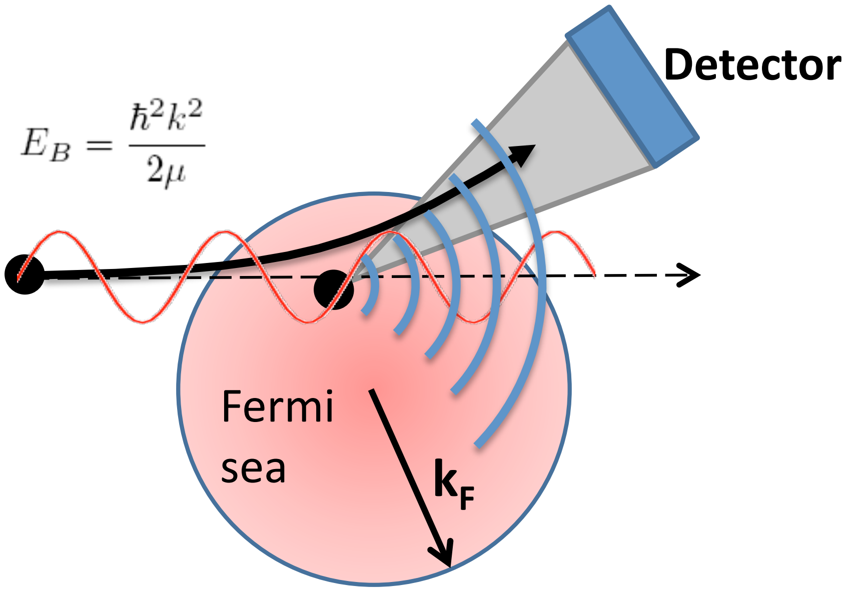
Why the independent particle works:

Scattering theory, medium effects, …



$$\Psi_{\rm scat}(\mathbf{r}) \xrightarrow{\mathbf{r}\to\infty} e^{ikz} + f(\theta,\varphi)\frac{e^{ikr}}{r}$$

$$\sigma_k(\theta,\varphi) = |f(\theta,\varphi)|^2$$ $\Rightarrow$ λ Mean-free path

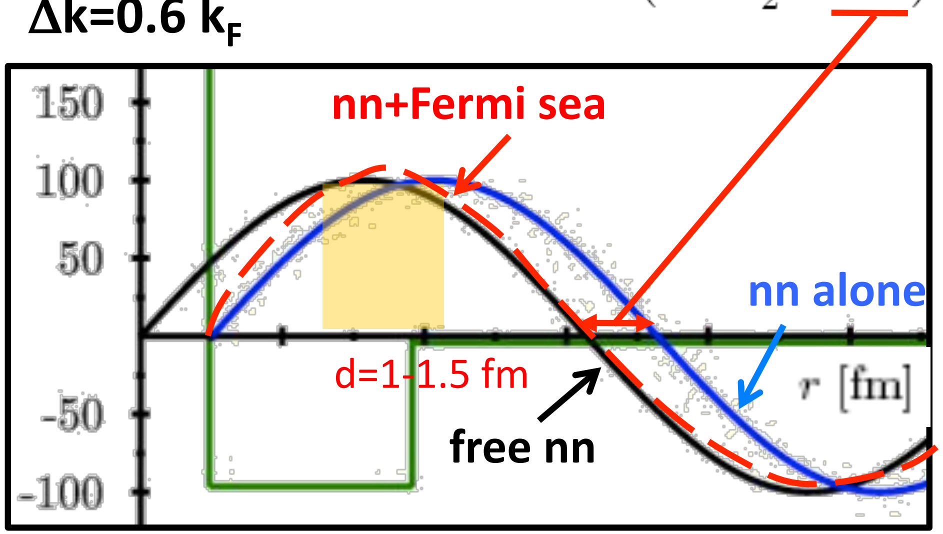


Spherical symmetric potential:

$$\Psi_{\rm scat}(\mathbf{r}) = \sum c_{nlm}\frac{u_{nl}(r)}{r} Y_{lm}(\theta,\varphi)$$

$$u_{nl}(r) \overset{\mathbf{r}\to\infty}{\sim} \sin\left(kr - l\frac{\pi}{2} + \delta_l(k)\right)$$

$\Delta k = 0.6\ k_F$

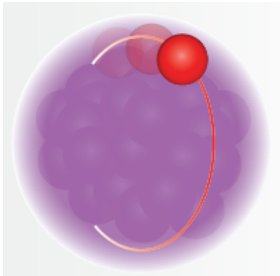


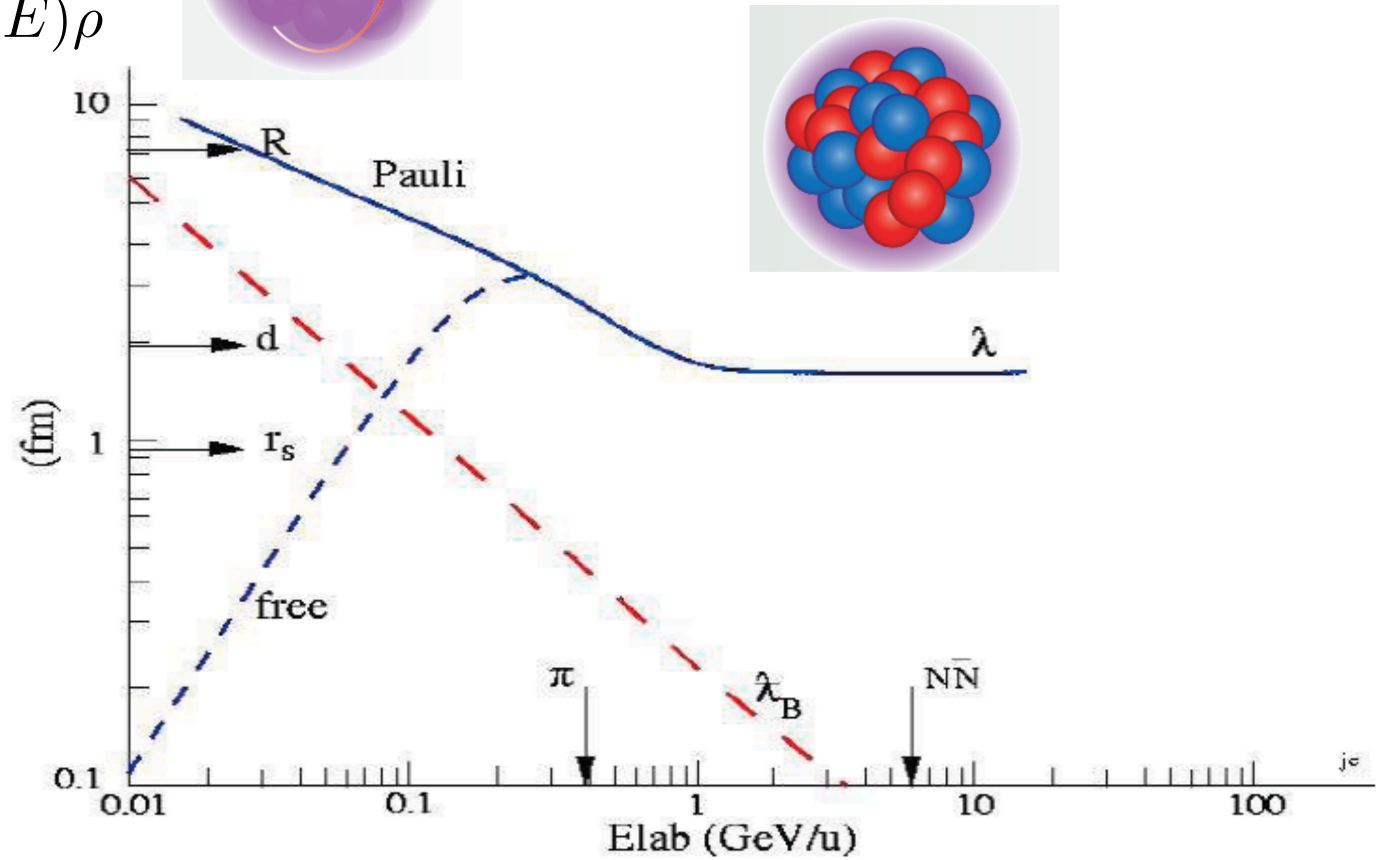
Gomes, Walecka and Weisskopf, Ann. Phys. (1958)

In medium Mean-Free Path

λ Mean-free path

$$\lambda(E) \propto \frac{1}{\sigma(E)\rho}$$





Cugnon EJC2007

Basic aspects of Hartree-Fock theory

① Start from $\hat{H}$

And assume that $\Psi = \mathcal{A}(\varphi_1(r_1), \cdots \varphi_N(r_N))$

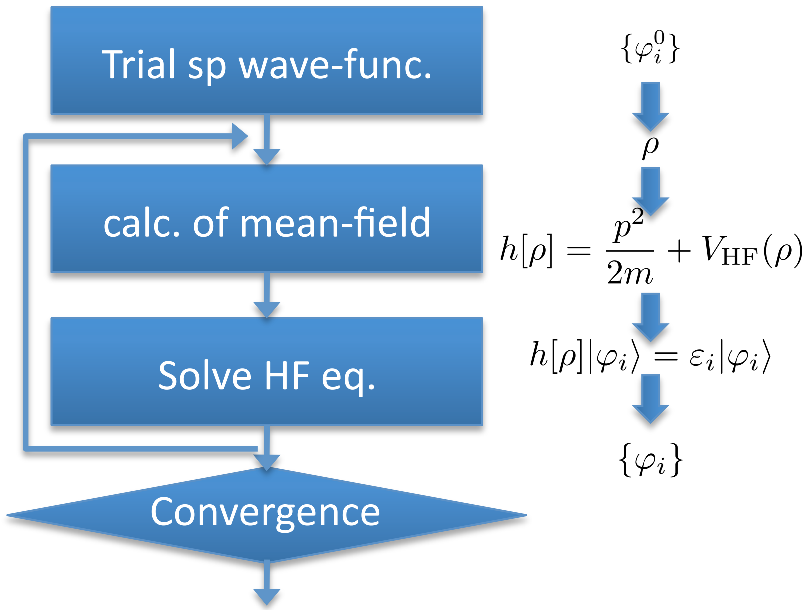
② Minimize the action $\delta\left(\langle\Psi|\hat{H}|\Psi\rangle - E\langle\Psi|\Psi\rangle\right) = 0$

with respect to single-particle deg. of freedom.

➡ $h[\rho]|\varphi_i\rangle = \varepsilon_i|\varphi_i\rangle$

with $\rho = \sum_i |\varphi_i\rangle\langle\varphi_i|$

and $h[\rho] = \dfrac{\partial E}{\partial \rho}$

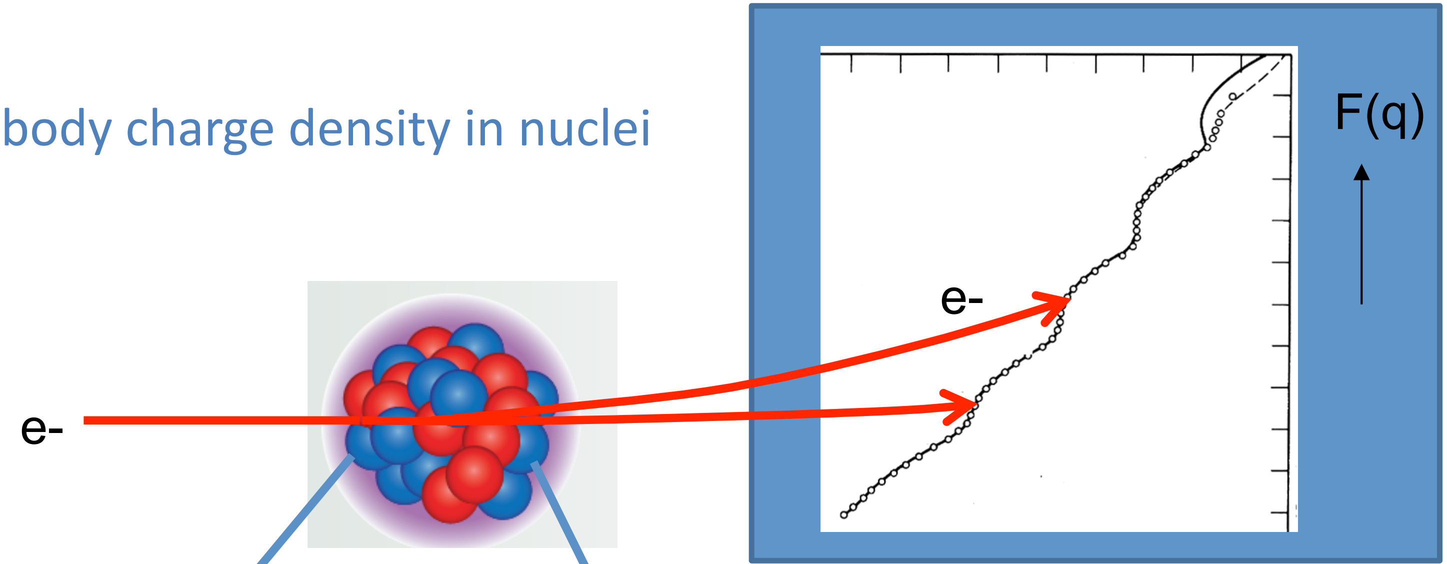


③ Solve the HF equation self-consistently

One-body density, Infinite nuclear matter, Saturation property

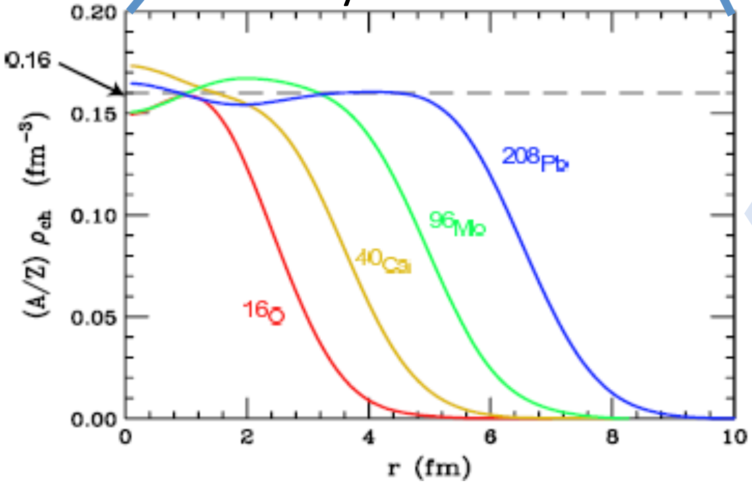
(e, e')

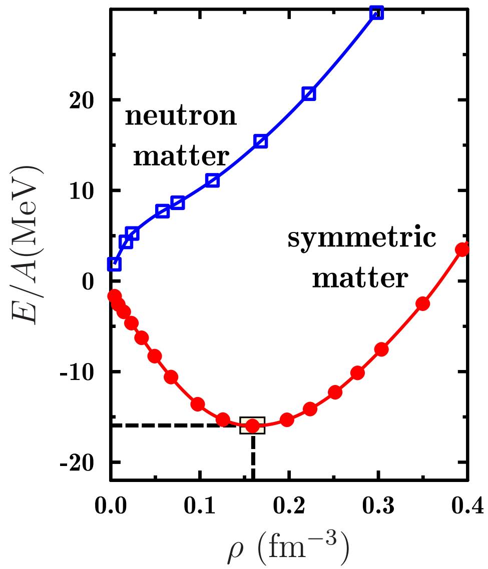
One-body charge density in nuclei



Large transferred momentum **q** provides shape of the central density distribution

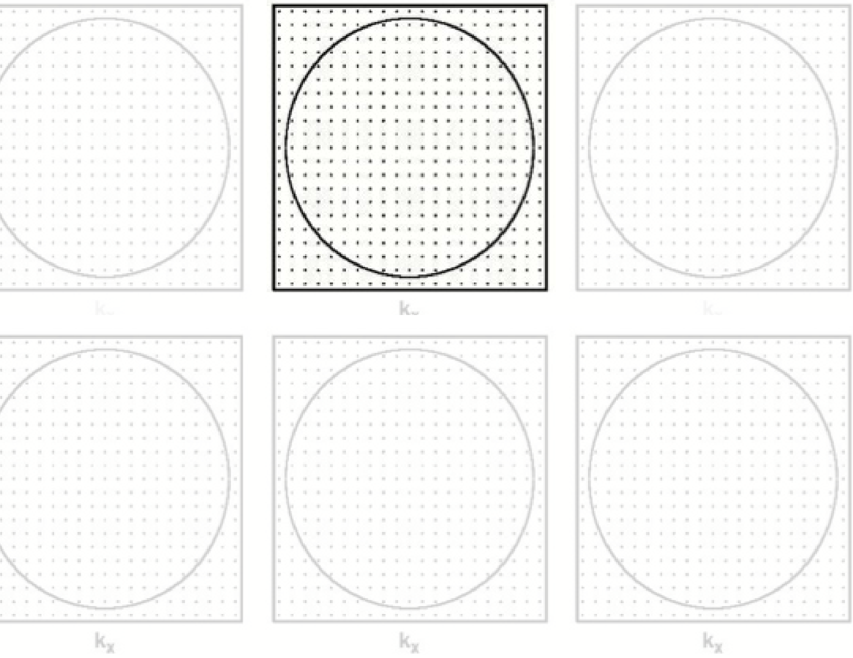
Density in the nucleus





The second dilemma: failure of the Hartree-Fock theory

The infinite nuclear matter case



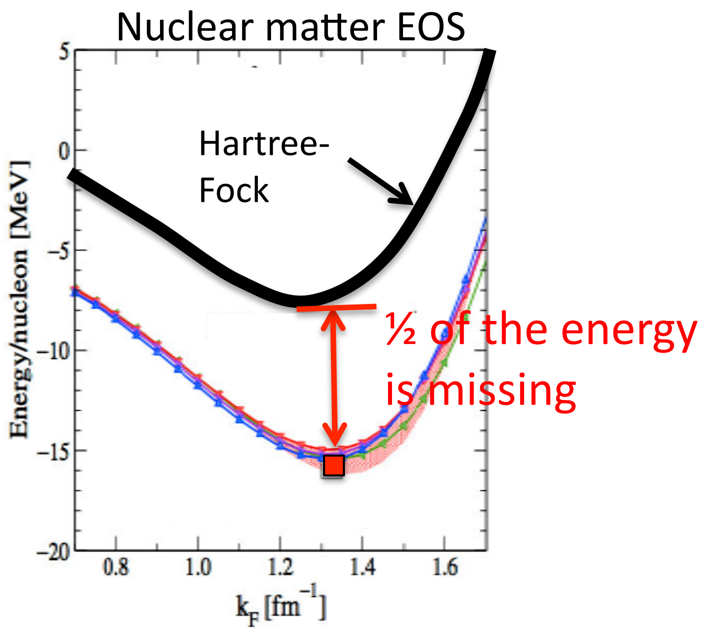
Single-particle state $|\mathbf{k}\sigma\tau\rangle$

$$\begin{aligned}\langle \mathbf{r}|\mathbf{p}\rangle &= \frac{1}{V^{1/2}} e^{i\mathbf{pr}/\hbar} \\ &= \frac{1}{V^{1/2}} e^{e^{i(k_x x + k_y y + k_z z)}}\end{aligned}$$

$$k_i = \frac{2\pi n_i}{L}, \quad i = x, y, z, \quad n_i = 0, \ \pm 1, \ \pm 2, \cdots$$

Density

$$\rho_{\sigma,\tau} = \frac{V}{(2\pi)^3} \int_{\mathbf{k}} d^3\mathbf{k} |\mathbf{k}\sigma\tau\rangle n_{\sigma\tau}(\mathbf{k}) \langle \mathbf{k}\sigma\tau|$$



Bogner, et al, NPA 763 (2005).

The second dilemma: failure of the Hartree-Fock theory

Second Dilemma

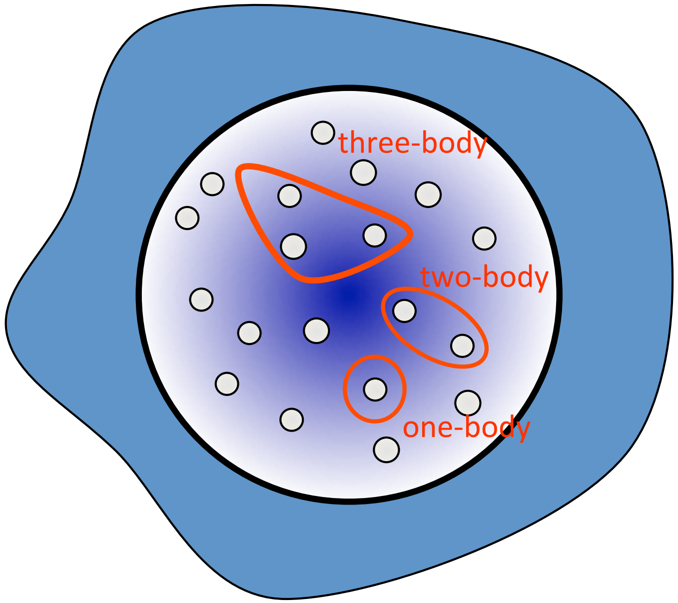
Simplicity

Many aspects of nuclei can be fairly well understood assuming that nucleons behaves like independent particles in an external one-body field

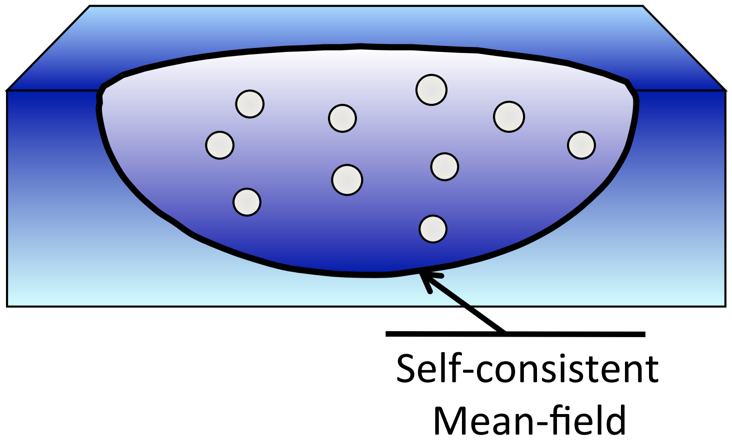
Complexity

The natural approach to map a many-boby problem into a one-body theory (HF) does not work in nuclear physics

→ The Energy Density Functional approach



Mean-field:
(DFT/EDF)



Complex many-body states:

$$\Psi(r_1, \cdots, r_{12}, \cdots, r_{123}, \cdots)$$

Independent particles or quasi-particle states

Parameters of the functional are directly adjusted on data

Link to underlying bare Hamiltonian is lost

Basic aspects of Functional approach to nuclear systems

EDF has taken different names:

Skyrme/Gogny Hartree-Fock,
Nuclear Density Functional Theory (DFT)
Energy Density Functional (EDF)
Relativistic Mean-Field …

What is Density Functional Theory ?

In its simplest form in DFT, the exact ground state of an N-body problem can be replaced by the minimization of an energy functional of the local one-body density ρ(r).
At the minimum, the energy and the density corresponds to the exact energy

$$\langle \Psi | \hat{H} | \Psi \rangle \Rightarrow \mathcal{E}[\rho(r)]$$

Hohenberg, Kohn, Phys. Rev. (1965)

Extensions:

- Introduction of auxiliary state $\Psi = \mathcal{A}(\varphi_1(r_1), \cdots \varphi_N(r_N))$

  $\{\varphi_i\} \rightarrow \rho(r) \rightarrow \mathcal{E} \rightarrow U_{KS}(r) \rightarrow \{\varphi_i\} \ldots$

  Kohn, Sham, Phys. Rev. (1965) $\rho(r) = \sum_i |\varphi_i(r)|^2$

  $h[\rho]|\varphi_i\rangle = \varepsilon_i|\varphi_i\rangle$ with $h[\rho] = \dfrac{\partial \mathcal{E}}{\partial \rho}$

- GGA, Meta GGA, …

  $\mathcal{E} = \mathcal{E}[\rho(r), \nabla\rho, \Delta\rho, \tau, \ldots]$

  A primer to DFT, Lectures notes in Physics 620 (2003)

- DFT with pairing

  $\mathcal{E} = \mathcal{E}[\rho, \kappa \ldots]$

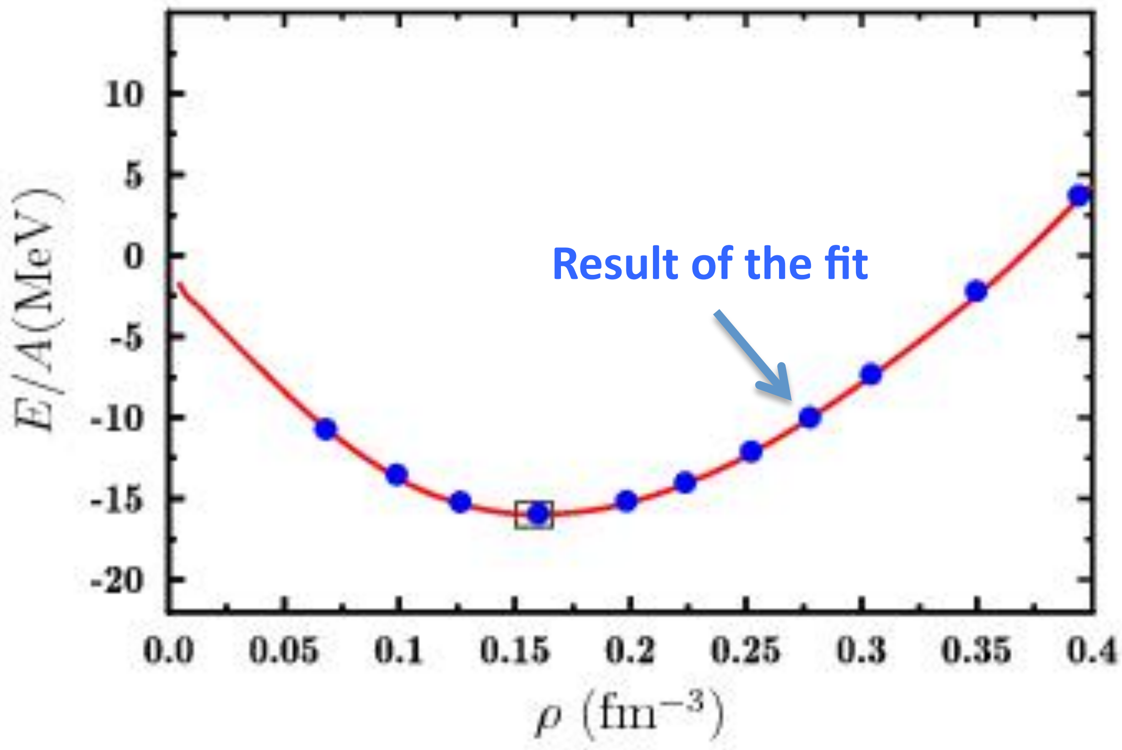
  Oliveira et al, PRL (1988)

- Time-dependent DFT

  Runge, Gross PRL (1984)

(and much more …)

EDF from a simple perspective



Exercise : fit the curve with

$$E = \left\langle \frac{p^2}{2m} \right\rangle + U[\rho]$$

In nuclear matter:

$$\left\langle \frac{p^2}{2m} \right\rangle = \frac{3}{5}\left(\frac{3\pi^2}{2}\right)^{2/3} \rho^{5/3}$$

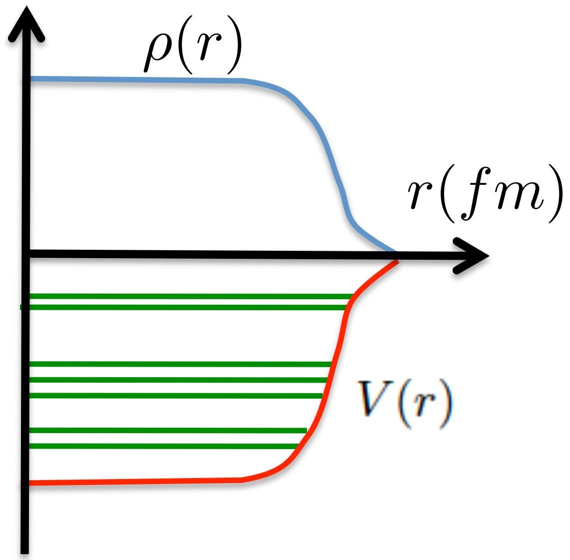
Fit with

$$U[\rho] = \sum_n c_n \rho^n$$

- **An excellent fit is obtained**
- **Coefficients contains many-body physics**

Example 1 : empirical construction of EDF for finite nuclei

Observation



$$\rho(r) = \frac{\rho_0}{1+\exp(\frac{r-R_0}{a})}$$

$$V(r) = \frac{V_0}{1+\exp(\frac{r-R_0}{a})}$$

$$V(r) = \frac{V_0}{\rho_0}\rho(r) = c_0\rho(r)$$

$\rho_0 \simeq 0.16 \text{ fm}^{-3}$ $\quad V_0 \simeq 50 \text{ MeV}$

$$c_0 \simeq 300 \text{ MeV.fm}^{-3}$$

From potential to energy: $V(r) \propto c_0\rho(r)$ $\quad\Rightarrow\quad$ $\mathcal{E}(r) \propto \frac{1}{2}c_0 \int \rho^2(r)dr$

Improving the functional: Spin-orbit

$$V_{\text{S.O.}} \propto l.s\nabla V(r)$$
$$= c_0 l.s\nabla\rho(r)$$

Coulomb, …

Introducing the effective interaction

$$v(\mathbf{r}_1-\mathbf{r}_2) = c_0\delta(\mathbf{r}_1-\mathbf{r}_2) \quad\Rightarrow\quad U_{\text{H}}(\mathbf{r}) = c_0\int d\mathbf{r}'\delta(\mathbf{r}-\mathbf{r}')\rho(\mathbf{r}') = c_0\rho(\mathbf{r})$$

Nuclear Energy Density Functional based on effective interaction

Illustration with the Skyrme Functional

Vautherin, Brink, PRC (1972)

$$
\begin{aligned}
v(\mathbf{r}_1-\mathbf{r}_2) \; &= t_0\,(1+x_0\hat{P}_\sigma)\;\delta(\mathbf{r}) && \text{central term}\\
&+\tfrac{1}{2}\,t_1\,(1+x_1\hat{P}_\sigma)\left[\mathbf{P}'^2\;\delta(\mathbf{r})+\delta(\mathbf{r})\;\mathbf{P}^2\right] && \\
&+t_2\;(1+x_2\hat{P}_\sigma)\,\mathbf{P}'\cdot\delta(\mathbf{r})\;\mathbf{P} && \text{non local terms}\\
&+iW_0\sigma.\left[\mathbf{P}'\times\delta(\mathbf{r})\;\mathbf{P}\right] && \text{spin orbit term}\\
&+\tfrac{1}{6}\,t_3\,(1+x_3\hat{P}_\sigma)\;\rho^\alpha(\mathbf{R})\;\delta(\mathbf{r}) && \text{density dependent term}
\end{aligned}
$$

Nuclear Energy Density Functional based on effective interaction

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Vautherin, Brink, PRC (1972)

$$
\begin{aligned}
v(\mathbf{r}_1-\mathbf{r}_2) \; &= t_0\,(1+x_0\hat{P}_\sigma)\,\delta(\mathbf{r}) \\
&+\tfrac{1}{2}\,t_1\,(1+x_1\hat{P}_\sigma)\left[\mathbf{P}'^2\,\delta(\mathbf{r})+\delta(\mathbf{r})\,\mathbf{P}^2\right] \\
&+t_2\,(1+x_2\hat{P}_\sigma)\,\mathbf{P}'\cdot\delta(\mathbf{r})\,\mathbf{P} \\
&+iW_0\sigma.\left[\mathbf{P}'\times\delta(\mathbf{r})\,\mathbf{P}\right] \\
&+\tfrac{1}{6}\,t_3\,(1+x_3\hat{P}_\sigma)\,\rho^\alpha(\mathbf{R})\,\delta(\mathbf{r})
\end{aligned}
$$

$$\mathcal{E} \;=\; \langle\Psi|H(\rho)|\Psi\rangle = \int \mathcal{H}(r)d^3\mathbf{r}$$

$$
\begin{aligned}
\mathcal{H} &= \mathcal{K}+\mathcal{H}_0+\mathcal{H}_3+\mathcal{H}_{\rm eff} \\
&\quad +\mathcal{H}_{\rm fin}+\mathcal{H}_{\rm so}+\mathcal{H}_{\rm sg}+\mathcal{H}_{\rm Coul}
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{H}_0 &= \tfrac{1}{4}t_0\left[(2+x_0)\rho^2-(2x_0+1)(\rho_p^2+\rho_n^2)\right] \\
\mathcal{H}_3 &= \tfrac{1}{24}t_3\rho^\alpha\left[(2+x_3)\rho^2-(2x_3+1)(\rho_p^2+\rho_n^2)\right] \\
\mathcal{H}_{\rm eff} &= \tfrac{1}{8}\left[t_1(2+x_1)+t_2(2+x_2)\right]\tau\rho \\
&+ \tfrac{1}{8}\left[t_2(2x_2+1)-t_1(2x_2+1)\right](\tau_p\rho_p+\tau_n\rho_n) \\
\mathcal{H}_{\rm fin} &= \tfrac{1}{32}\left[3t_1(2+x_1)-t_2(2+x_2)\right](\nabla\rho)^2 \\
&- \tfrac{1}{32}\left[3t_1(2x_1+1)+t_2(2x_2+1)\right]\left[(\nabla\rho_p)^2+(\nabla\rho_n)^2\right] \\
\mathcal{H}_{\rm so} &= \tfrac{1}{2}W_0\left[\mathbf{J}.\nabla\rho+\mathbf{J}_p.\nabla\rho_p+\mathbf{J}_n.\nabla\rho_n\right] \\
\mathcal{H}_{\rm sg} &= -\tfrac{1}{16}(t_1x_1+t_2x_2)\mathbf{J}^2+\tfrac{1}{16}(t_1-t_2)\left[\mathbf{J}_p^2+\mathbf{J}_n^2\right]
\end{aligned}
$$

Functional of $\rho, \rho_n, \rho_p, \tau, \tau_n, \tau_p, \mathbf{J}, \ldots$

Around 10-14 parameters to be adjusted

Nuclear Energy Density Functional based on effective interaction

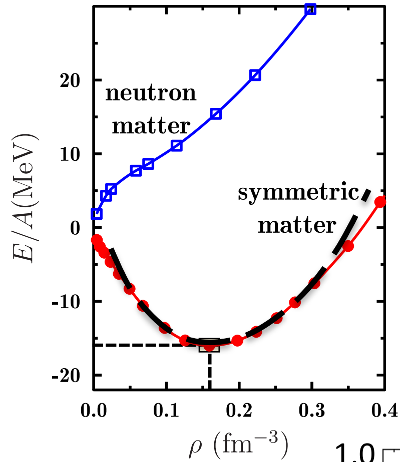
Constraining the functional

Vautherin, Brink, PRC (1972)

$$
\begin{aligned}
v(\mathbf{r}_1-\mathbf{r}_2) &= t_0\,(1+x_0\hat{P}_\sigma)\,\delta(\mathbf{r}) \\
&+\tfrac{1}{2}\,t_1\,(1+x_1\hat{P}_\sigma)\left[\mathbf{P}'^2\,\delta(\mathbf{r})+\delta(\mathbf{r})\,\mathbf{P}^2\right] \\
&+t_2\,(1+x_2\hat{P}_\sigma)\,\mathbf{P}'\cdot\delta(\mathbf{r})\,\mathbf{P} \\
&+iW_0\sigma.\left[\mathbf{P}'\times\delta(\mathbf{r})\,\mathbf{P}\right] \\
&+\tfrac{1}{6}\,t_3\,(1+x_3\hat{P}_\sigma)\,\rho^\alpha(\mathbf{R})\,\delta(\mathbf{r})
\end{aligned}
$$

See for instance, Meyer EJC1997

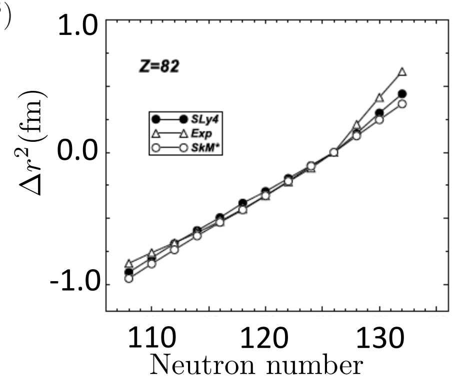
Infinite nuclear matter and Nuclear Masses



$$E_0,\ \left.\frac{\partial E}{\partial \rho}\right|_{\rho_0},\ \left.\frac{\partial^2 E}{\partial \rho^2}\right|_{\rho_0}$$

The ρ^α term
Is a trick to get
the curvature right!

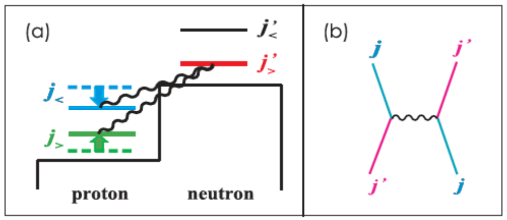
Surface effects

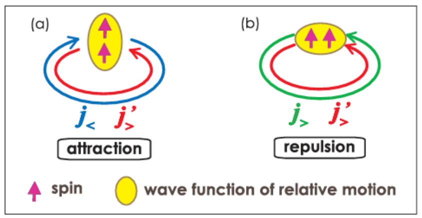


Tensor interaction

$$
\begin{aligned}
v^{\mathrm{t}}(\mathbf{r}) = \tfrac{1}{2}t_e\{&[3(\boldsymbol{\sigma}_1\cdot\mathbf{k}')(\boldsymbol{\sigma}_2\cdot\mathbf{k}')-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}'^2]\delta(\mathbf{r}) \\
&+\delta(\mathbf{r})[3(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}^2]\} \\
&+t_o[3(\boldsymbol{\sigma}_1\cdot\mathbf{k}')\delta(\mathbf{r})(\boldsymbol{\sigma}_2\cdot\mathbf{k})-(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\mathbf{k}'\cdot\delta(\mathbf{r})\mathbf{k}].
\end{aligned}
$$

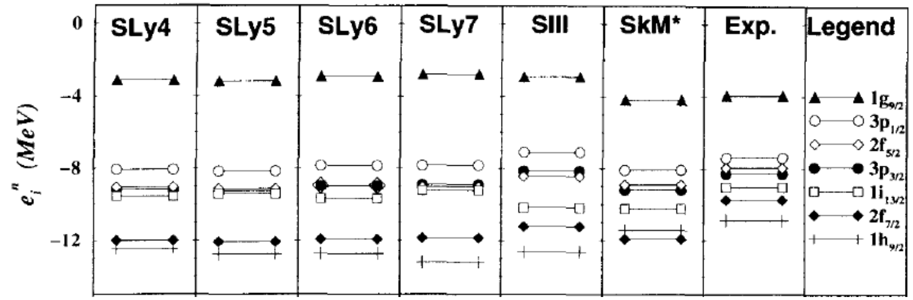
Lesinski et al, PRC76 (2007)



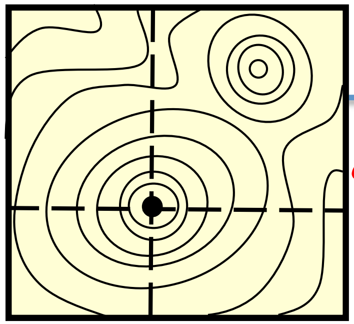


Otsuka et al, PRL95 (2005)

Shell effect



Chabanat et al, NPA (1998)



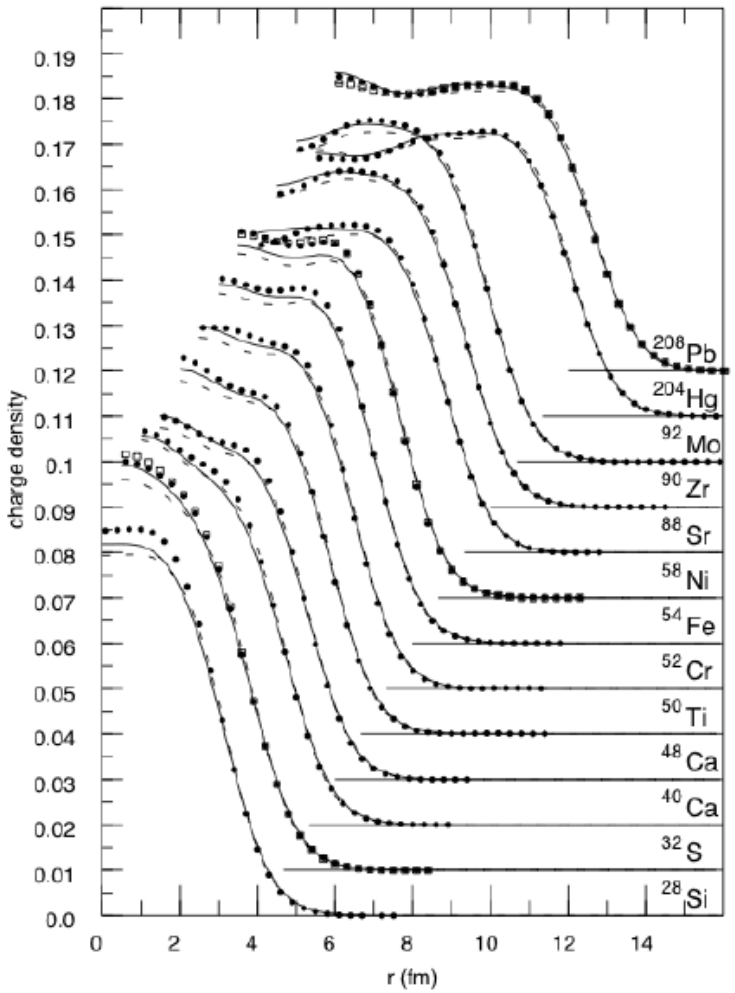
$\mathcal{E}[\rho]$

Energy Density Functional based on effective interaction

Success

Density Functional theory
Are firstly optimal for:
(i) Ground state energy
(ii) Local one-body density

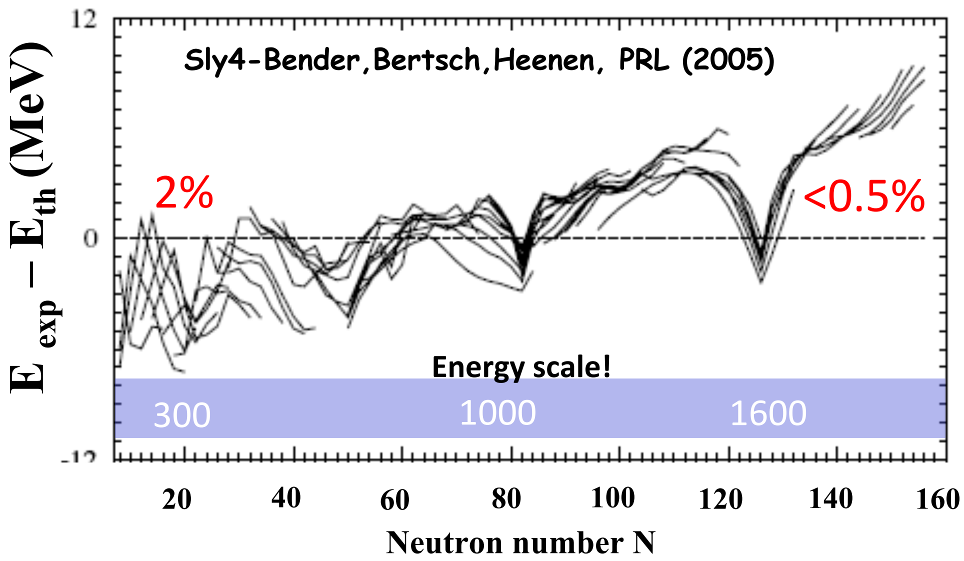
Charge density



(From A. Brown)

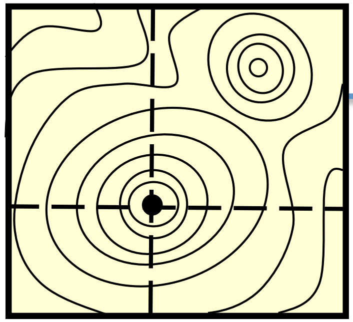
And it does it very well…

Ground state energy



Still some local fluctuations

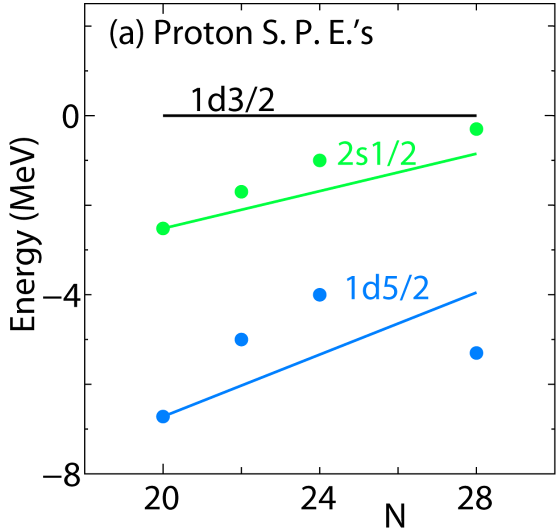
We ask EDF much more: shapes, spectroscopy, s.p. shell effects…



Fingerprints of spontaneous symmetry breaking

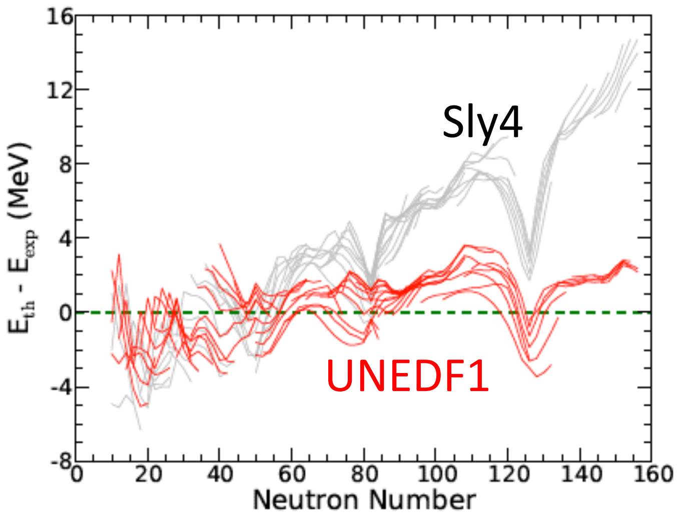
To improve the functional one can:

- Add new terms: better isospin dependence, tensor,...)



Otsuka et al., PRL 95 (2005)

- Make better fit on precise containing new information



N. Schunck et al, nuc-lth:1107.5005

- Use symmetry breaking in Functional theory

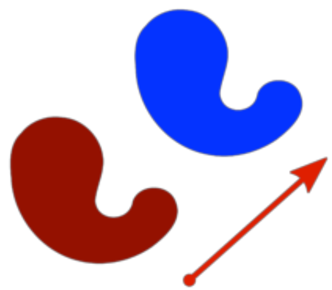
Let us restart from the nuclear Hamiltonian

$$H = \sum_{ij} \langle i|T|j\rangle a_i^+ a_j + \frac{1}{2} \sum_{ijkl} \langle ij|v_{12}|lk\rangle a_i^+ a_j^+ a_l a_k$$

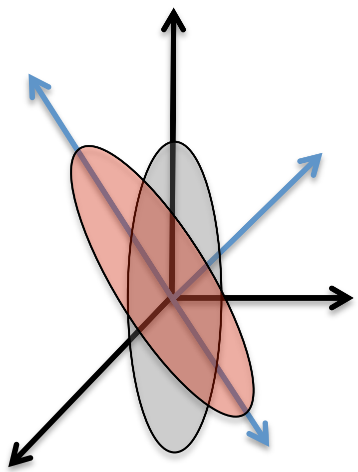
A symmetry is conserved if the Hamiltonian commute with the generator of the group of Transformation associated with this symmetry: $[H, G] = 0$

Some symmetries that are respected by the nuclear Hamiltonian:

- Translational invariance $[H, P_{\mathrm{c.m.}}] = 0$
- Rotational invariance $[H, J] = 0$
- Particle number $[H, N] = 0$
- Parity $[H, \pi] = 0$
- ...



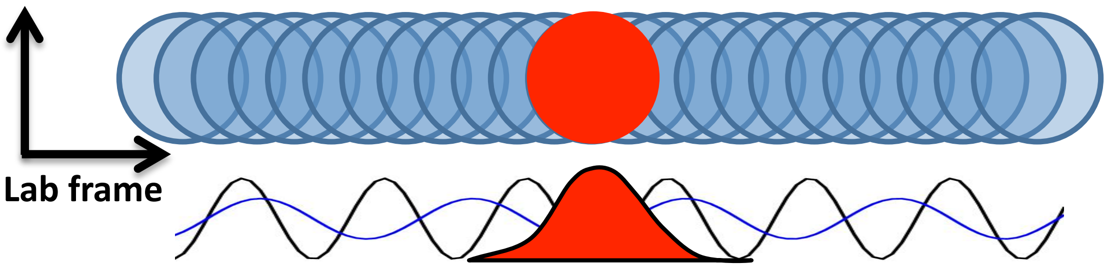
From wiki



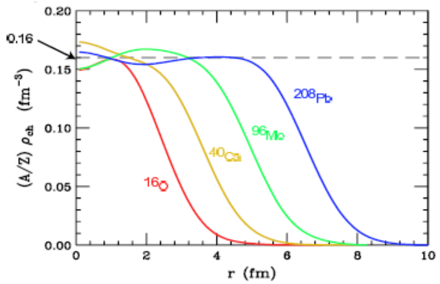
...

Fingerprints of spontaneous symmetry breaking and intrinsic frame

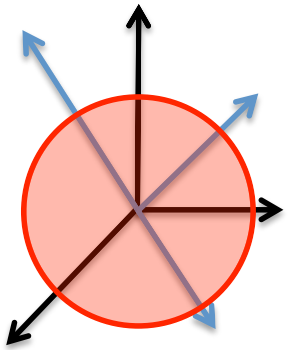
Translation:



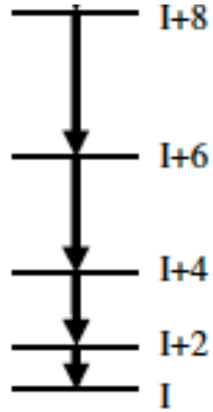
But nuclei have a finite size



Rotations:

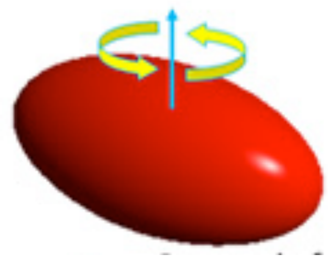


Nuclei do present rotational bands

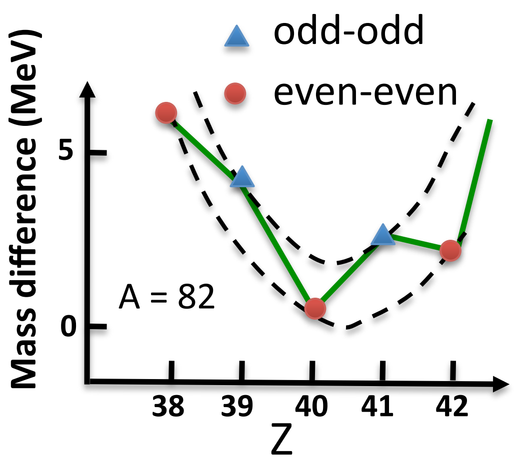


$$E_{\mathrm{rot}} = \frac{I(I+1)}{2J}\hbar^2$$

Nuclei might be deformed



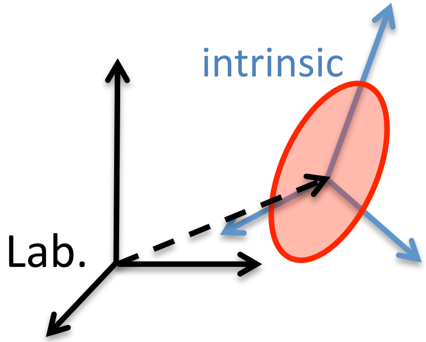
Particle number:



Onset of pairing

Breaking particle number symmetry

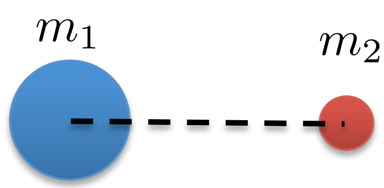
Theoretical approach to symmetry breaking

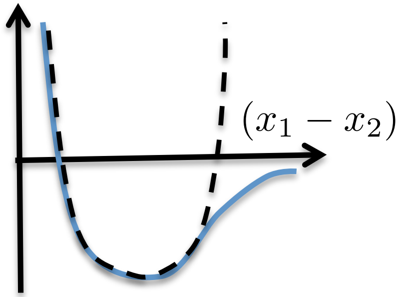


The many-body state respect all symmetries of H in the lab. frame

EDF is performed in the intrinsic frame and uses states that breaks symmetries

Simple example: two particles in interaction





Hamiltonian $\hat{H} = \frac{\hat{p}_1^2}{2m_1} + \frac{\hat{p}_2^2}{2m_2} + C(\hat{x}_1 - \hat{x}_2)^2$

Suppose $\Psi(x_1, x_2) = \varphi_1(x_1)\varphi_2(x_2)$

and minimize $\delta\Big(\langle\Psi|\hat{H}|\Psi\rangle - E\langle\Psi|\Psi\rangle\Big) = 0$

$\Rightarrow$ $\varphi_i(x_i) = \frac{1}{\sqrt{\pi b_i^2}} e^{-x_i^2/(2b_i^2)}$ $\qquad b_i^2 = \frac{\hbar}{\sqrt{2m_i C}}$

This is equivalent to solve

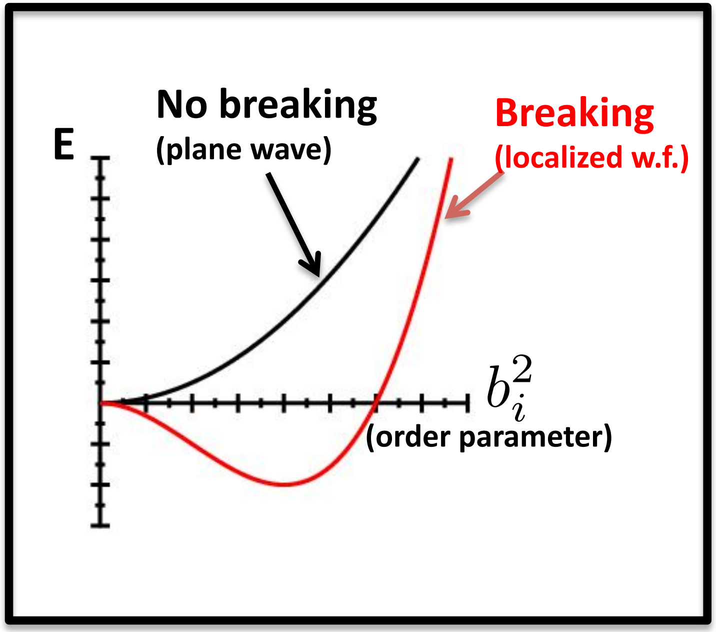
$$\hat{H}_{\mathrm{MF}} \simeq \hat{h}(\hat{x}_1) + \hat{h}(x_2)$$

with $\hat{h}(\hat{x}_i) = \frac{\hat{p}_i^2}{2m_i} + C\hat{x}_i^2$

Break translational Symmetry unless

$b_i \to 0$

$\Rightarrow$ $\varphi_i(x_i) \propto e^{\frac{i p_i x_i}{\hbar}}$



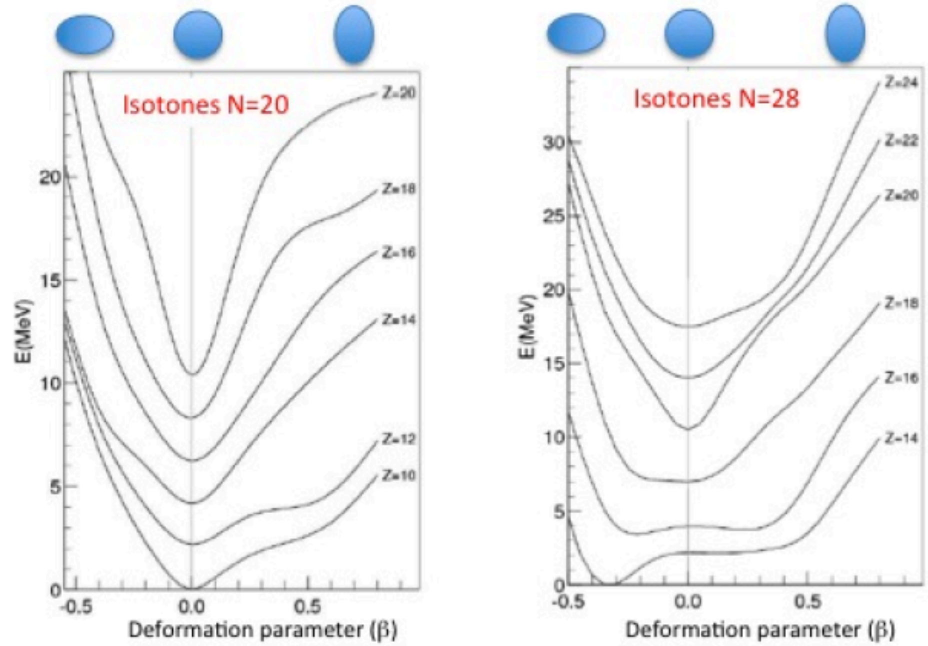
Success of EDF and broken symmetries

- Translational invariance is always broken

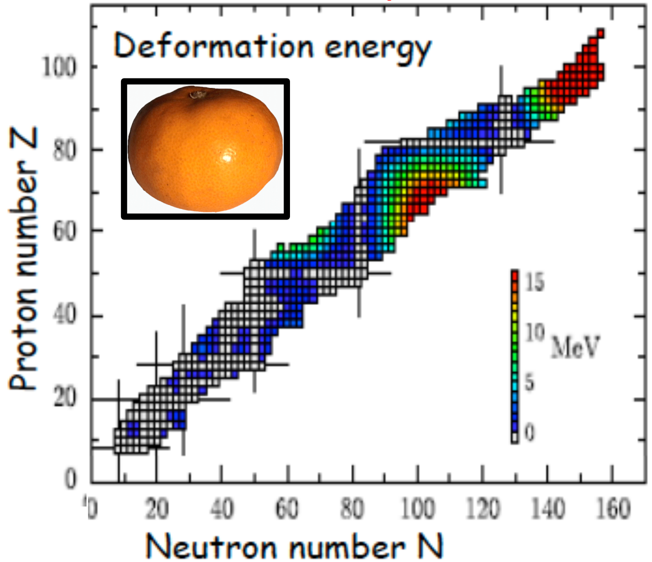
- To treat deformation spherical symmetry should be broken (3D codes)

Minimization under constraint

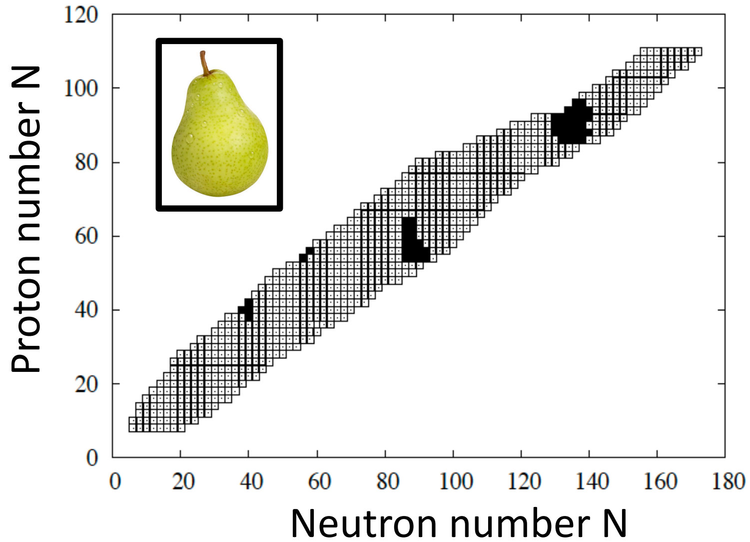
$$\delta\left(\mathcal{E}[\rho] - \lambda_2\langle Q_2\rangle + \cdots\right) = 0$$



Quadrupole



Octupole



Bender, Bertsch, Heenen, PRC73 (2006)

Robledo, Bertsch, arXiv:1107.3581

EDF: Pairing correlations in nuclei

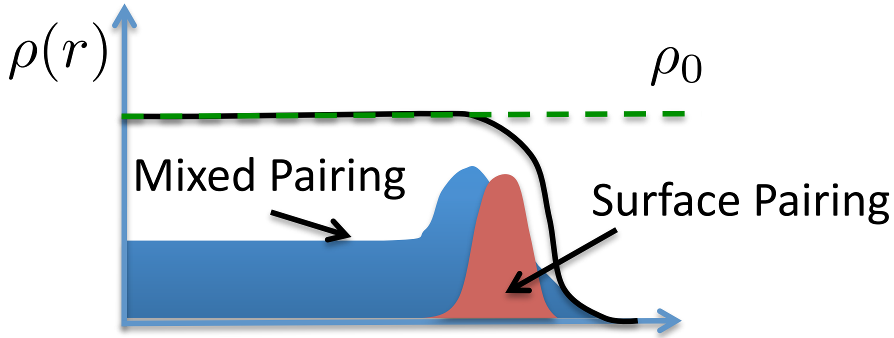
$$|\Phi_0\rangle = \Pi_i a_i^\dagger |-\rangle \Rightarrow |\Phi_0\rangle = \Pi \beta_\alpha^\dagger |-\rangle \quad \text{or} \quad |\Phi_0\rangle = \Pi_i (u_i + v_i a_i^\dagger a_{\bar{i}}^\dagger)|-\rangle$$

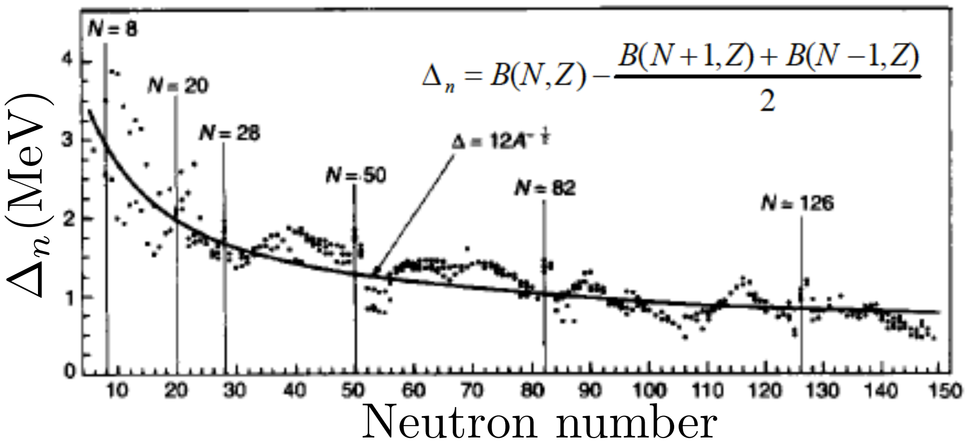
EDF: $$\mathcal{E}_{SR}[\rho, \kappa, \kappa^*] = \sum t_{ii}\rho_{ii} + \frac{1}{2}\sum \overline{v}_{ijij}^{\rho\rho}\rho_{ii}\rho_{jj} + \underbrace{\frac{1}{4}\sum \overline{v}_{i\bar{i}j\bar{j}}^{\kappa\kappa}\kappa_{i\bar{i}}^{*}\kappa_{j\bar{j}}}_{\text{Pairing channel}}$$

$$\Phi_0 \rightarrow \{\rho, \kappa\} \rightarrow \mathcal{E}_{SR}$$

Pairing interaction

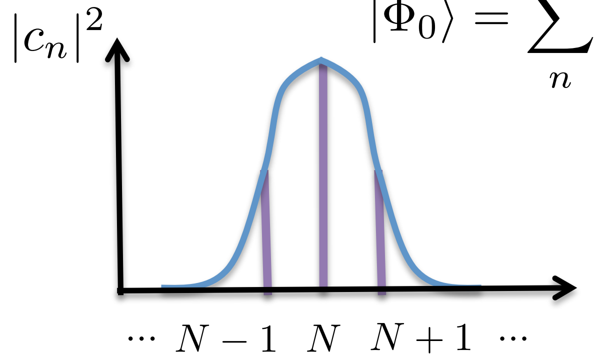
$$v^{\kappa\kappa} = v_0 \left(1 - \alpha \left[\frac{\rho}{\rho_0}\right]^\beta\right) \delta(r_1 - r_2)$$

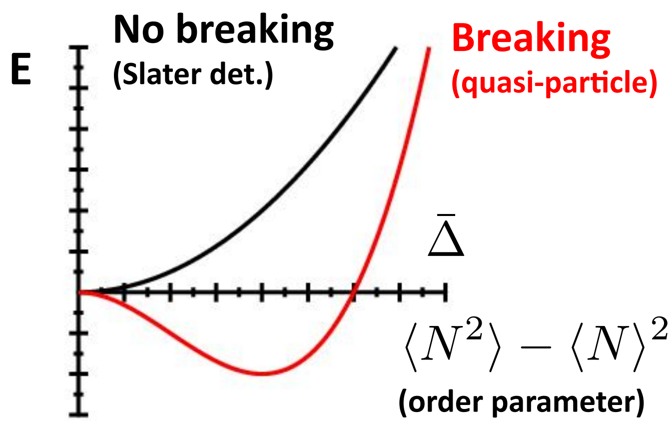




Particle number non-conservation

$$|\Phi_0\rangle = \sum_n c_n |\Psi_n\rangle$$





Third dilemma

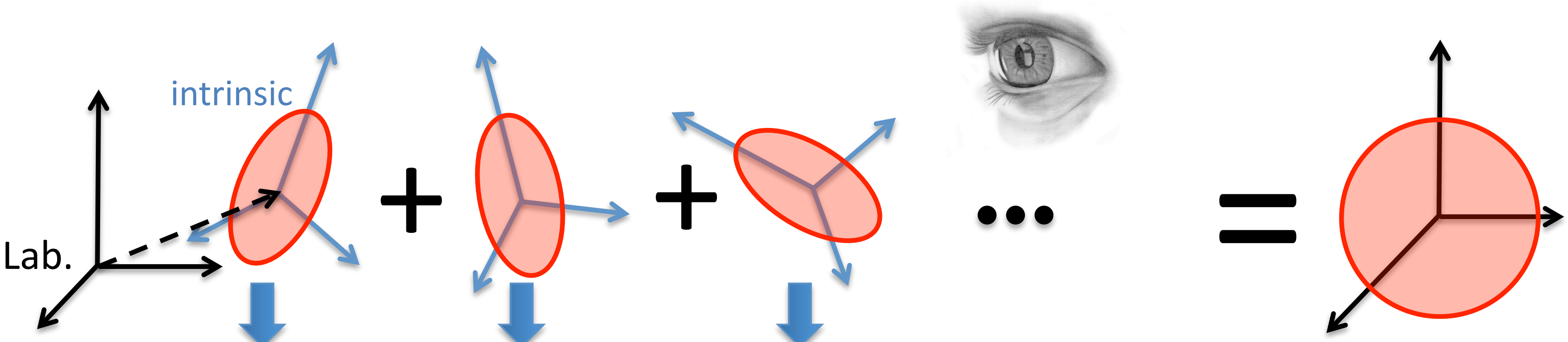
The many-body state respect all symmetries of H in the lab. frame

MB states should ultimately have good quantum numbers

EDF uses symmetry breaking to grasp correlation which are not easy to get without symmetry breaking

MB states do not have good quantum numbers

Symmetry Restoration : the rotation case



$|\Phi(\Omega_1)\rangle$ $\quad |\Phi(\Omega_2)\rangle$ $\quad |\Phi(\Omega_3)\rangle$ $\quad \dots$

All degenerated

Formulation with projectors

$$|\Psi^J\rangle = P_J|\Psi(0)\rangle \quad \text{with} \quad |\Psi^J\rangle = \int d\Omega f(\Omega)|\Phi(\Omega)\rangle$$

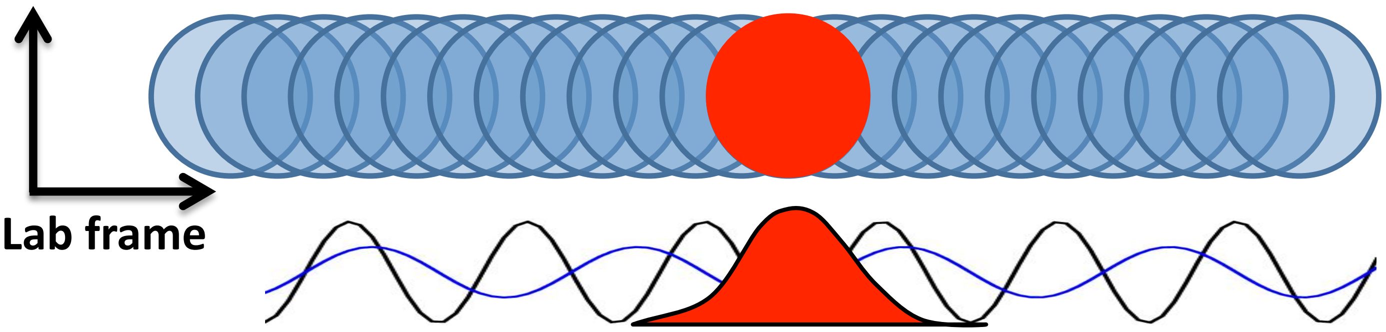
Terminology:
Generator Coordinate Method
Configuration mixing
Multi-Reference EDF

Some example of symmetry breaking restoration

Few examples

Physical effects

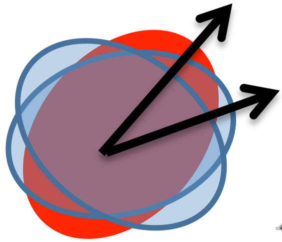
- translational invariance $|\Psi^{c.m.}\rangle = P_{c.m.}|\Psi(0)\rangle$

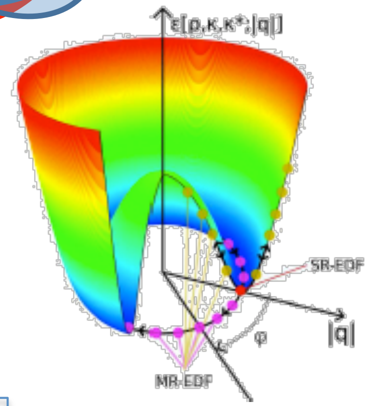


Surface Vibration

Quadrupole Correlation - Rotational Bands

- Rotational invariance $|\Psi^{J}\rangle = P_{J}|\Psi(0)\rangle$





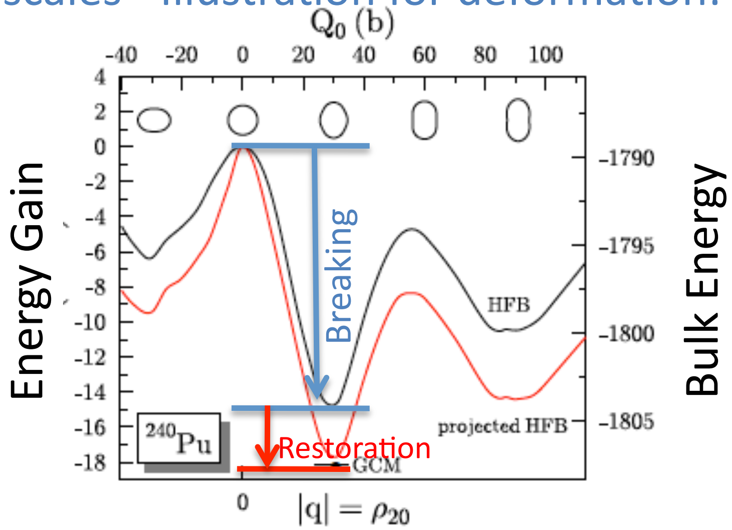
Pairing Correlations - Odd-even effects

- U(1) symmetry $|\Psi^{N}\rangle = P_{N}|\Psi(0)\rangle$

Ultimately

$$\boxed{|\Psi^{N,Z,J,\cdots}\rangle = P_N P_Z P_J \cdots |\Psi(0)\rangle}$$

Energy scales - Illustration for deformation:

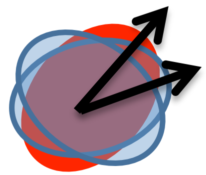


From T. Duguet, M. Bender (EJC 2009)

Bulk energy:	1800 MeV
Energy gain due to Symmetry breaking:	15 MeV
Energy gain due to Restoration:	Few MeVs

Dilemma: While data contains “fingerprints of symmetry breaking” the theory should be Symmetry conserving

$$|\Psi\rangle = \sum_{\mathrm{deg}} c_i |\Phi_i\rangle$$



The two steps nuclear Physics strategy

Step 1: Grasp the gross features of atomic nuclei

SR-EDF
(HF, mean-field…)

Mean-field

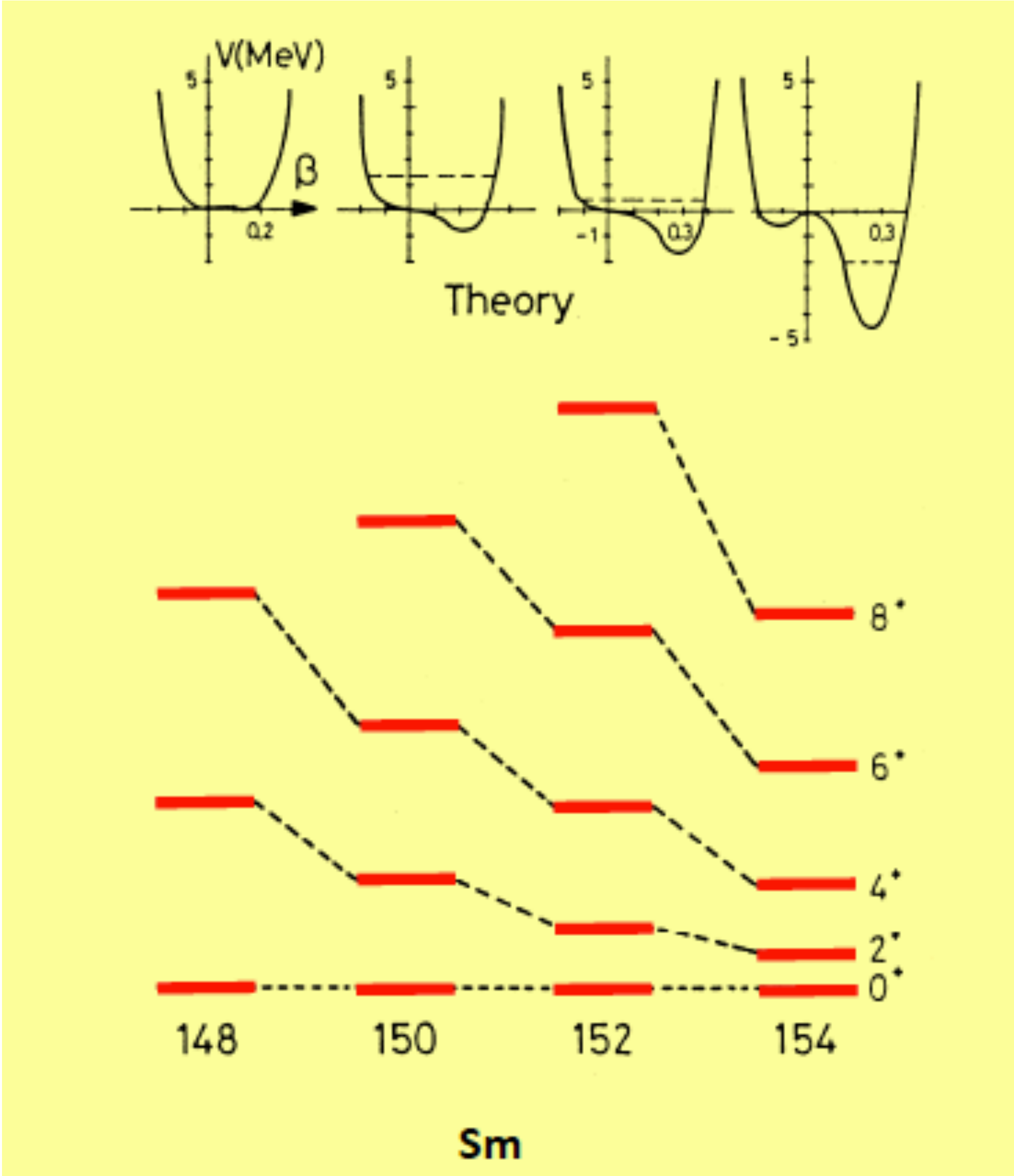
20-40 years…

Beyond Mean-field

Step 2: incorporate the local fine structure effects

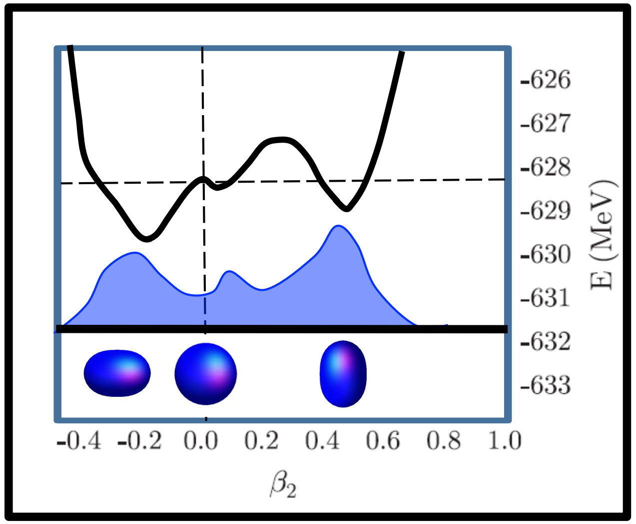
MR-EDF
(GCM, beyond mean-field…)

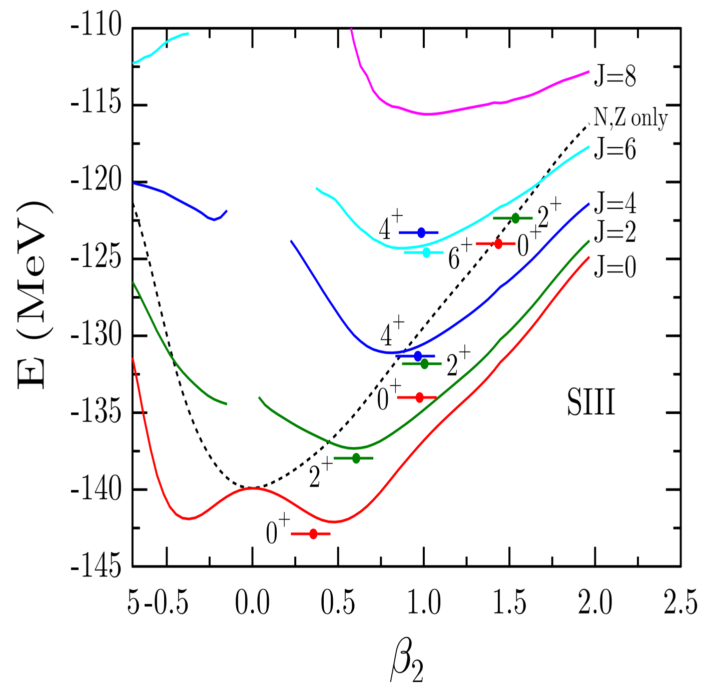
Interplay between single-particle and collective degrees of freedom



H. Goutte, EJC 2010

Configuration Mixing within EDF





Bender, Duguet, Heenen, Lacroix, arXiv:1011.4047

What we ask to a modern mean-field theory ?

- Ground state energy — MB
- Densities
- Deformation
- Shell evolution
- Spectroscopy — MB

Single Particle

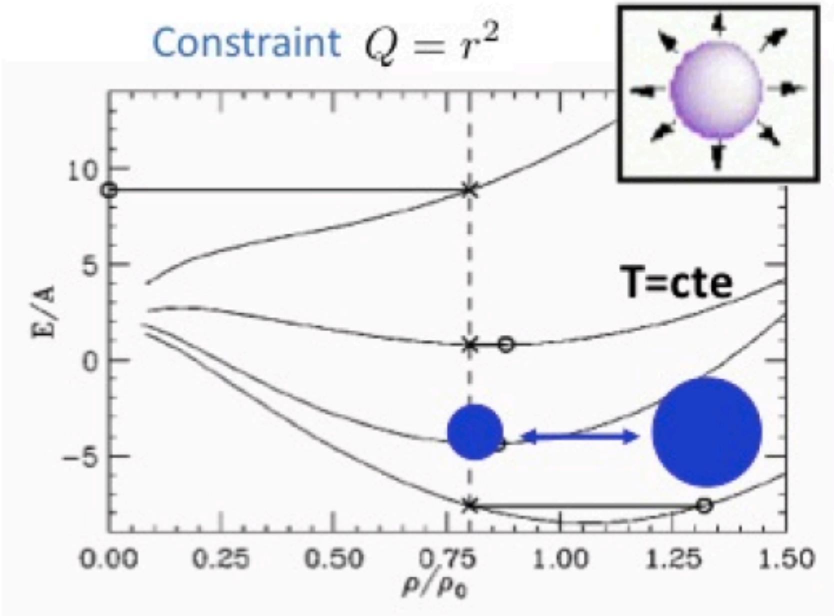
And much more…

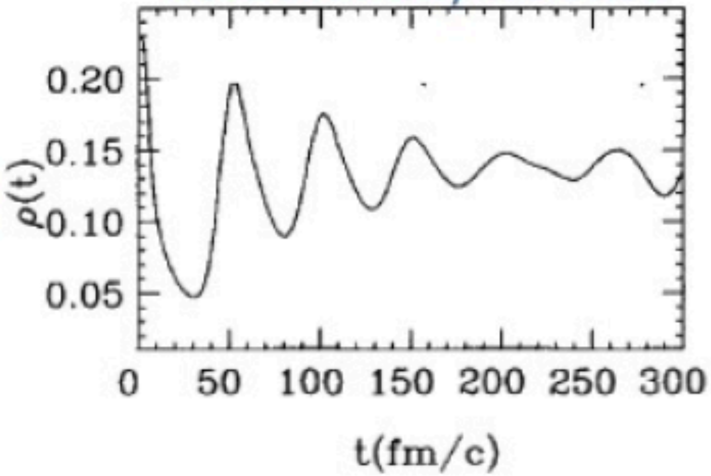
EDF, from ground state to thermodynamics, to dynamics

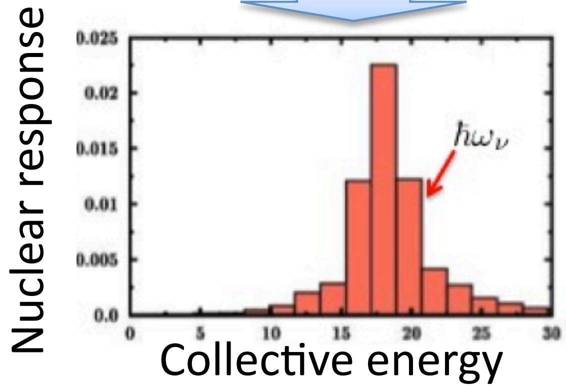
Constrained EDF at finite Temperature

$$\delta\{\mathcal{E}[\rho] - TS[\rho] - \mu \mathrm{Tr}(\hat{N}D)\} = 0$$

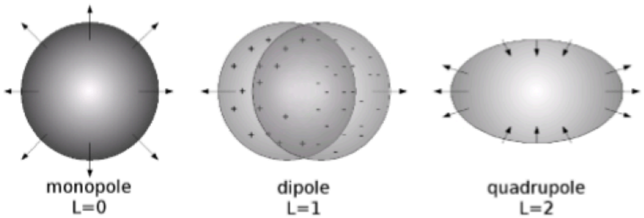
Constraint $Q = r^2$

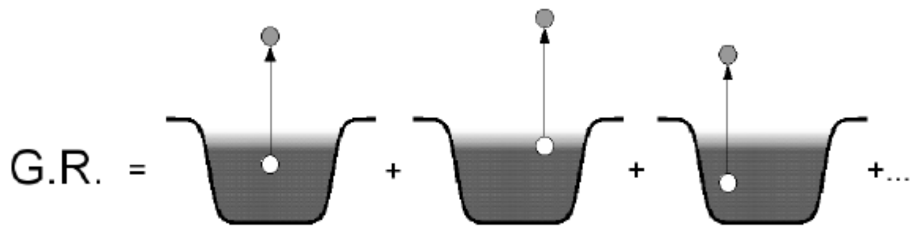






Giant Resonances

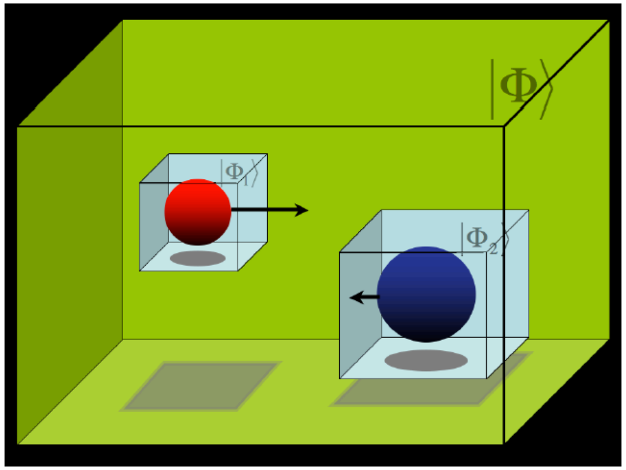


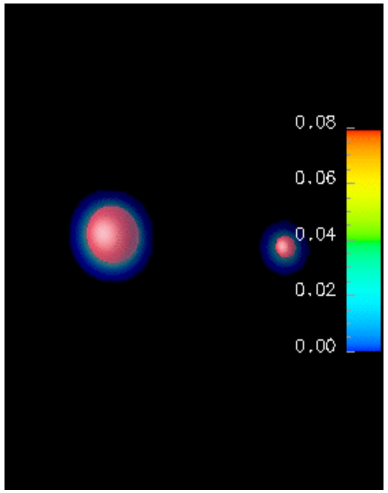


(see lecture M. Harakeh)

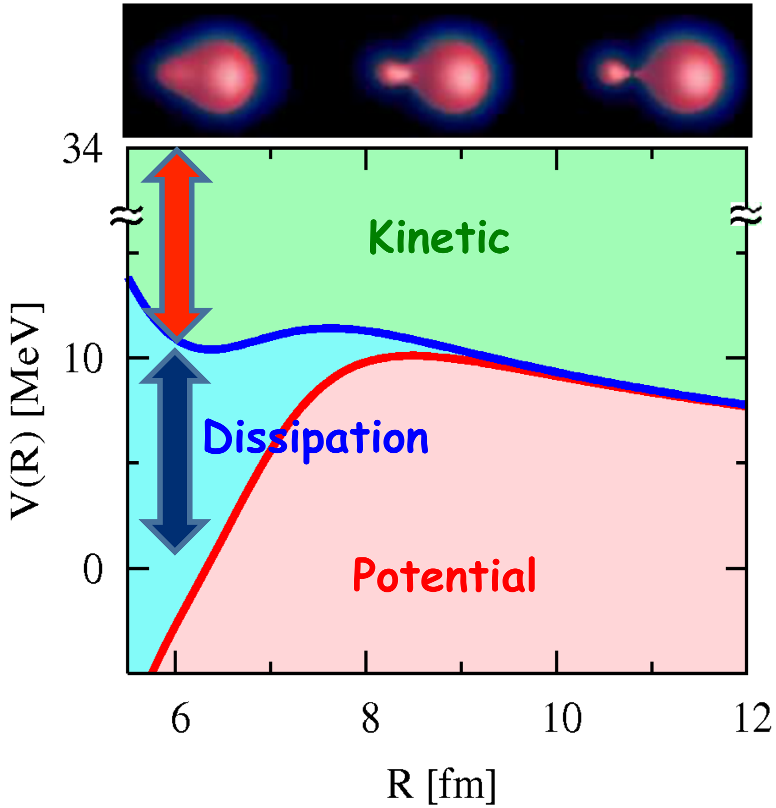
Nuclear Reactions

Interplay between nuclear reaction and nuclear structure





Example of study: the onset of dissipation



Simenel, Avez, Lacroix, arXiv:0806.2714

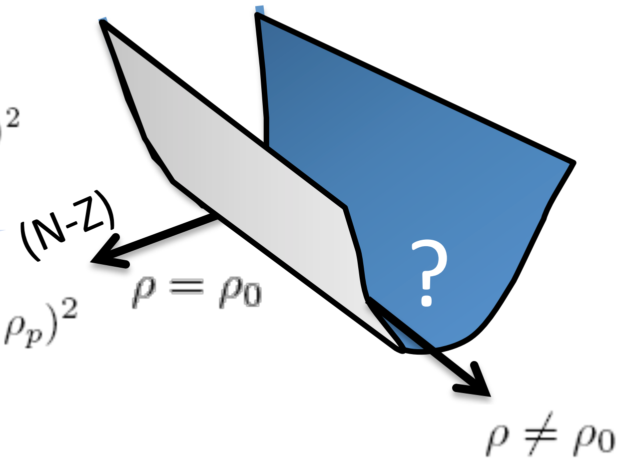
A unified theory for nuclear structure, reactions and stars

Strategy 1: exploring unknown region

Illustration with the Symmetry energy

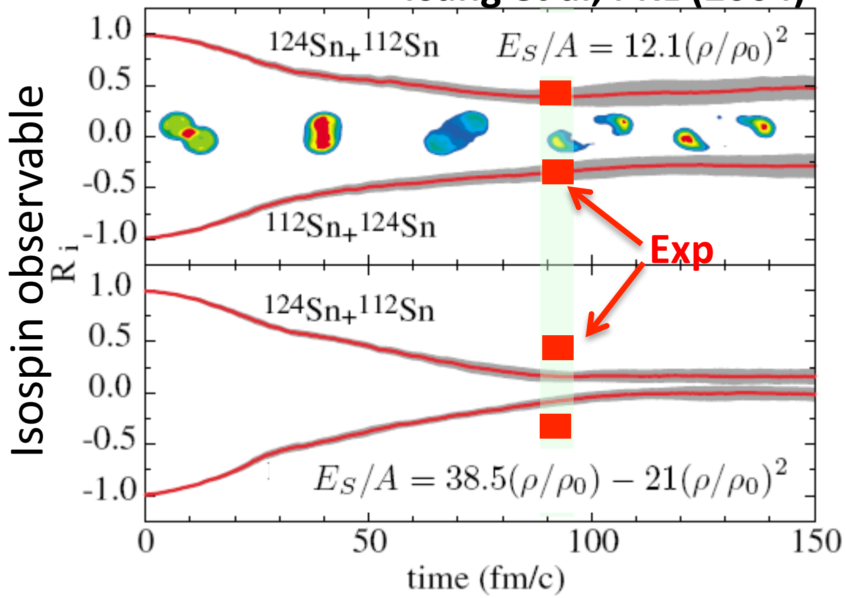
$$E_{LD} \propto a_S (N-Z)^2$$

$$E_{EDF} \propto a_S[\rho](\rho_n - \rho_p)^2$$



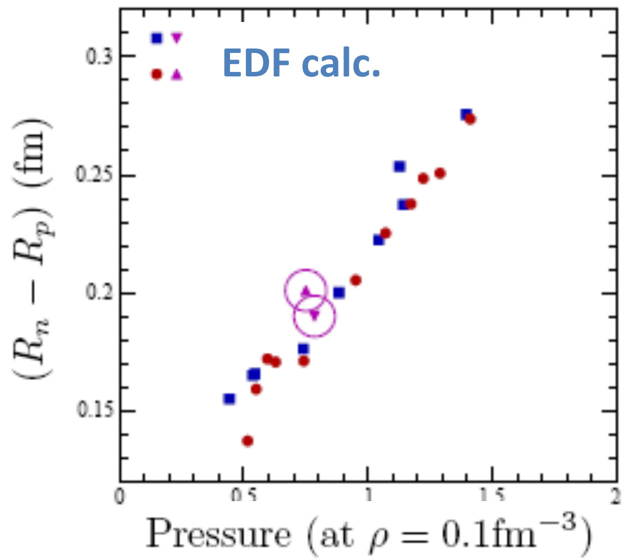
Constraint from Heavy-Ion reactions

Tsang et al, PRL (2004)



Constraint from Astrophysics

Steiner et al, Phys. Rep. (2005)



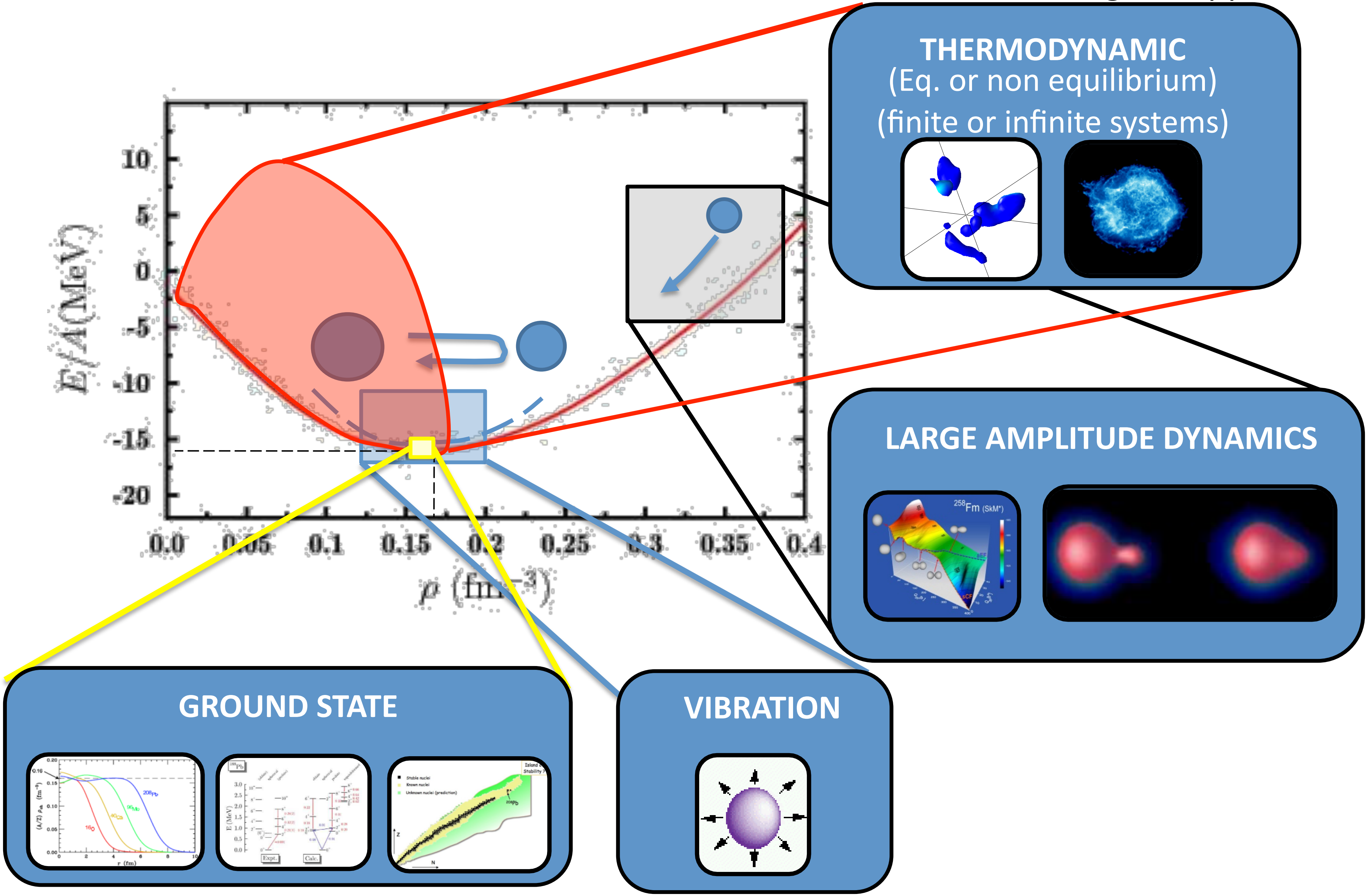
$$\text{Radius} \times \text{Pressure}^{-1/4} \simeq \text{cte}$$

Lattimer et al, Phys. Rep. (2000)

- **Measurement of neutron star radii constraint the asymmetry energy**
- **Cooling of proto-neutron star**
- **Exotic phases**

A unified theory for nuclear structure, reactions and stars

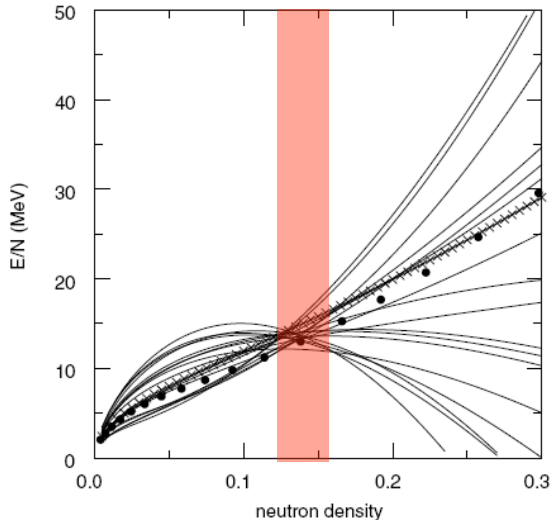
Range of application



A unified theory for nuclear structure, reactions and stars

Uncertainties and Predicting power in conventional EDF

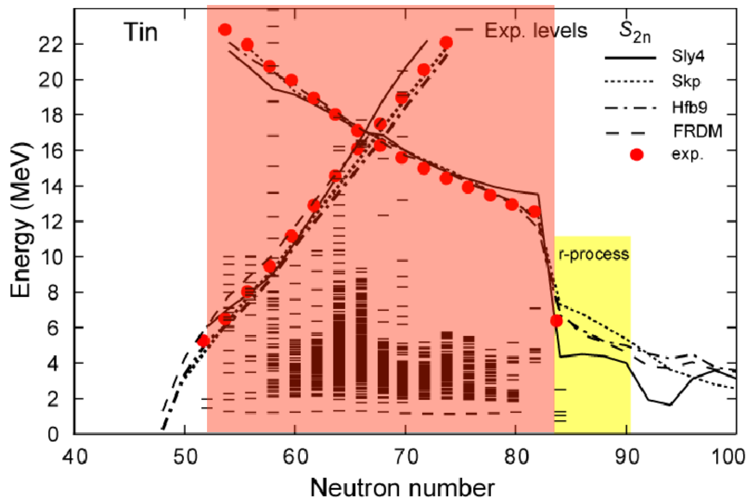
EOS of pure neutron matter



Brown, PRL85 (2000).

Conventional EDF are adjusted on a selected set of experimental data in finite and infinite systems

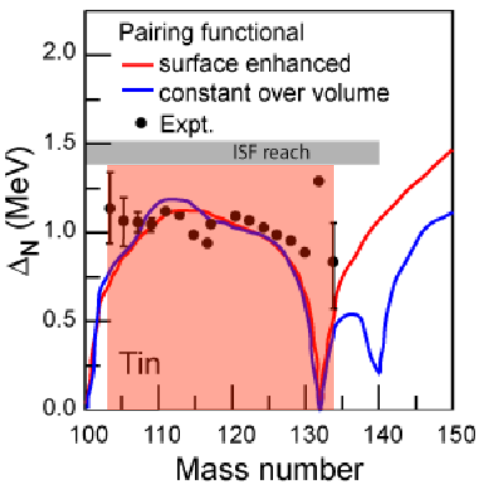
S_{2n} and S_{2p} in Tin isotopic chain



http://www.nscl.msu.edu/future/isf

Experimentally known

Pairing gap



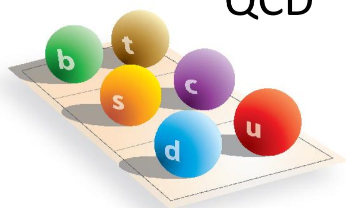
Large uncertainty in unknown region

Strategy to improve

Exp./Th. **Add more data (RIB, reactions, stars)**

Th./Exp. **Use the “bottom-up” approach**

QCD



Effective Field Theory
Chiral Perturbation theory

Epelbaum et al, Rev. Mod. Phys. 81 (2009)

Renormalization Group

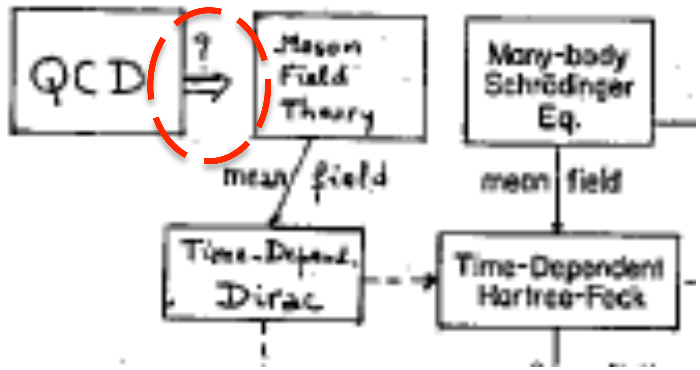
Bogner et al,
Prog. Part. Nucl. Phys. 65 (2010)

New soft interaction (2-3 body)
Many-Body Pert. Theory

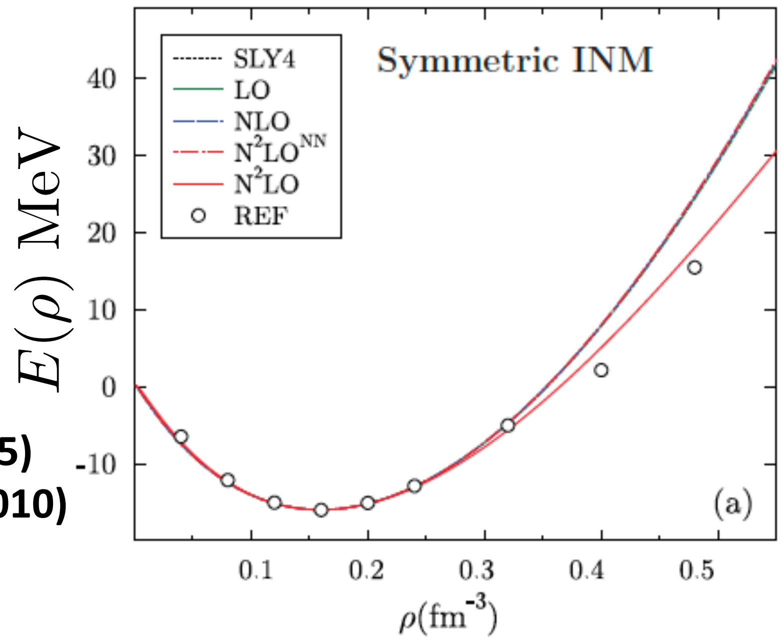
Density Matrix Expansion

Negele, Vautherin, Phys. Rev.C11 (1975)
Gebremariam et al, Phys. Rev. C 82 (2010)

Non-empirical functional



Cugnon, EJC 1985



Stoitsov et al, Phys. Rev. C82 (2010)

(that all folks…)